Leasing and Pricing Strategies for Wireless Service Providers in Dynamic Spectrum Sharing

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Abstract. In dynamic spectrum sharing, Wireless Service Providers (WSPs) can dynamically acquire spectrum by leasing from spectrum broker and sell spectrum to users. In this paper, we model the interactions between secondary WSPs and users as a three-stage game with objective of maximizing WSPs' profits. The competitive WSPs make leasing strategies in stage I and pricing strategies in stage II. Users follow Wardrop's principle and choose WSP with respect to price and quality of service (QoS) in stage III. We analyze the static game by means of backward induction. Given the users' equilibrium, the pricing sub-game and leasing full game for competitive WSPs both have a unique Nash equilibrium. The situation without complete information is also studied by dynamic game. The short term pricing dynamic game converges to the Nash equilibrium of the pricing sub-game, while the long term leasing dynamic game converges to the Nash equilibrium of the full game.

Keywords: Dynamic spectrum leasing, Leasing and pricing, Nash equilibrium, Three-stage game.

1 Introduction

Spectrum is an indispensable resource in wireless communications. Spectrum has been statically allocated by regulatory agency (e.g., FCC in USA, Ofcom in UK) to prevent the signals from interfering with each other. As wireless services increase dramatically these years, spectrum seems to be scarce. However, recent researches [1], [2] show that most of spectrum bands are underutilized because demand changes with time and location. Dynamic spectrum access can improve the spectrum utilization by allowing primary Wireless Service Providers (WSPs)/users to share the spectrum with secondary WSPs/users. With cognitive radio technology, dynamic spectrum access comes true. Devices with cognitive capability can reconfigure themselves according to the circumstances they sensed. Based on the manners of spectrum acquisition, we divide dynamic spectrum access mechanisms into opportunistic spectrum access and hierarchical spectrum access [3]. In opportunistic spectrum access, secondary WSPs /users acquire spectrum by spectrum sensing. In hierarchical spectrum access, secondary WSPs/users acquire spectrum by leasing.

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Recently, increasing researches focus on dynamic spectrum leasing. Literatures [4]-[6] have studied the secondary WSPs' strategies of obtaining spectrum from spectrum owners. Concerning secondary WSPs' competitive service providing, most existent works (e.g., [7], [8]) focus on pricing interaction between competitive WSPs. However, secondary WSPs' leasing strategies are tightly related with service pricing strategies. Only a few works (e.g., [9], [10]) jointly considered WSPs' strategies for spectrum acquiring and service pricing. In [9], the demand function of users is derived directly from economics area, which can not reflect the users' quality of service (QoS) requirements in communication. A concrete wireless spectrum sharing model is used in [10], which causes that the result is not general.

The key difference here is that we give an analytical study of leasing and pricing game for non-cooperative secondary WSPs with objectives of maximizing their own profit. Profit is the difference between the revenue earned by selling services and cost of leasing spectrum. The secondary users follow the Wardrop's principle [11] and choose WSP based on price and QoS. We use a three-stage (leader-follower) game to model the interactions between the WSPs and users as well as the WSPs' leasing and pricing decisions. Nash equilibrium of the game is solution to our problem. In the first stage, the WSPs lease bandwidth from spectrum broker. In the second stage, the WSPs make decision on service pricing to attract users. In the third stage, users decide which WSP to access.

The structure of this paper is as follows. In section 2, system model is described. In section 3, we give the static game formulation, and solve the game by backward induction. In section 4, we model WSPs' interactions as dynamic game with incomplete information, and analyze the convergence of the dynamic game. In section 5, through simulation results, some insights of static and dynamic game of non-cooperative WSPs are discussed. In section 6, conclusion is given.

2 System Model

As shown in Fig.1, we consider a duopoly case in which two secondary WSPs deploy their infrastructures in the same geography area. Secondary WSPs and users are equipped with cognitive capabilities. Primary WSPs put their temporarily unused spectrum into spectrum broker, gain extra revenue from leasing the spectrum to secondary WSPs and draw back spectrum after a specific period of time. Secondary WSPs can lease spectrum from spectrum broker and configure their infrastructures. Secondary users no longer have long term contract with specific WSPs and have the ability to choose WSP according to QoS and price. The WSPs can dynamically adjust their bandwidth leasing and service pricing decisions to attract users for maximizing their individual profit. We would like to study the behaviors of competitive secondary WSPs in making decisions of leasing spectrum and pricing service with the consideration of users' behavior. If not special specified, we refer WSP as secondary WSP and users as secondary users in remnant paper. As in [9] and [12], we consider the case where users pay for the capacity (resources) that they use instead of the services they receive.

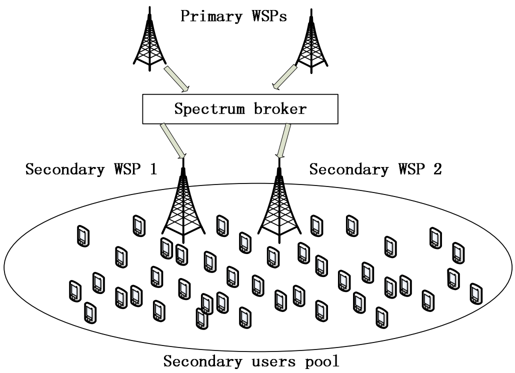

Fig. 1. System model

To acquire spectrum, the WSPs have to decide the amount of leasing bandwidth b_i ($i \in \{1,2\}$). The spectrum broker charges WSPs according to the unit price function $F(b_1+b_2)$ [13], in which $F(b)$ is strictly positive, non-decreasing and convex for $b>0$. The WSPs are charged at the same unit price. The price function describe the characteristic of primary WSPs' cost of spectrum providing, since the more bandwidth leased to secondary WSPs the more influence introduced into their own services. In our paper, we use the following format for $F(b)$,

$$F(b) = C(b_1+b_2) \tag{1}$$

in which C is a positive constant. We can see from (1) that spectrum cost for each WSP not only depends on its own leasing bandwidth but also depends on the other WSP's. To maximize its own profit, WSP i should consider its opponent's leasing strategy.

The system capacity s_i provided by WSP i is determined by the leasing spectrum,

$$s_i = k_i b_i \tag{2}$$

where k_i is WSP i's spectrum efficiency. Here, capacity is simply interpreted as the maximum amount of throughput a WSP can support. With adaptive modulation, the transmission rate can be dynamically adjusted based on the channel quality. Spectral efficiency can be obtained from [14],

$$k_i = \log_2(1+K_i\gamma_i), \tag{3}$$

where $K_i = 1.5/\left(\ln 0.2/BER_i^{tar}\right)$, γ_i denotes the signal to noise ratio, and denotes the target bit error rate BER_i^{tar}.

We model the interaction between WSPs and end users as a three-stage game ($i \in \{1,2\}$).

Stage I (Leader): WSP decides the leasing bandwidth (b_i).

Stage II (follower): WSP decides the service pricing (p_i).

Stage III (follower): Users make decision on which WSP to access, and give the demand distribution (d_i).

3 Backward Induction of the Three-Stage Game

When the complete information is available for WSPs, we analyze the static game by means of backward induction.

3.1 User Sub-game in Stage III

(1) Definition of Effective Price
We assume d_i is WSP i 's demand ($i \in \{1,2\}$). If $d_i \le s_i$, all packets are served. If $d_i > s_i$, demand exceeds WSP i 's system capacity, then the packets in excess are lost which are uniformly chosen among the sent ones. Hence, a packet is correctly sent with probability $q_i = \min\left(1, s_i/d_i\right)$. Here we use congestion pricing as in [15]. In congestion period, the users are charged higher to prevent the situation from deteriorating.

Definition 1. we define the effective price of WSP i as $\overline{p}_i = p_i/q_i = p_i \max\left(1, d_i/s_i\right)$.

Note that p_i is the price that WSP i decided for each packet sent in its network. As a consequence, the effective price $\overline{p}_i$ denotes average price to pay for successfully sending a packet. The effective price works just like the congestion price.
(2) Users' Behavior

We assumed that users are infinitesimal, and their behavior follows Wardrop's principle [12]: users always choose the WSP with lowest effective price. If $\overline{p}_1 > \overline{p}_2$, then $d_1 = 0$, and Vice versa. All users perceive the same effective price is $\overline{p} = \min\left(\overline{p}_1, \overline{p}_2\right)$.

We defined total demand as the total number of packets for which the willingness to pay is larger than or equal to the effective price $\overline{p}$. Hence, total demand can be represented by a function $D(\cdot)$ of the effective price, and it is assumed to be continuous and strictly decreasing,

$$D\left(\overline{p}\right) = \left[\alpha - \beta \overline{p}\right]^+ , \alpha > 0 , \beta > 0 , \tag{4}$$

which means that the users would rather not to transmit any data when the effective price is larger than α/β .

$$d_1 + d_2 = D\left(\overline{p}\right) \tag{5}$$

(3) Users Distribution Equilibrium

In Fig.2, we show the users equilibrium characterization for specific p_1 and p_2. Here we discuss the situation when $p_1 < p_2$. Meanwhile, we give some demand functions in 4 different cases. We can get the users' distribution according to the intersection of demand function and effective price curves.

If $D \le s_1$, the effective price that all users perceive is p_1, and all users choose WSP 1 ($d_1 = D$, $d_2 = 0$), and the case d represents this situation.

If $s_1 < D \le s_1 p_2 / p_1$, the effective price is $p_1 d_1 / s_1$. Although the needed capacity is larger than WSP 1's system capacity, all users still access to WSP 1 ($d_1 = D$, $d_2 = 0$), since $\overline{p} < p_2$. Case c corresponds to this situation.

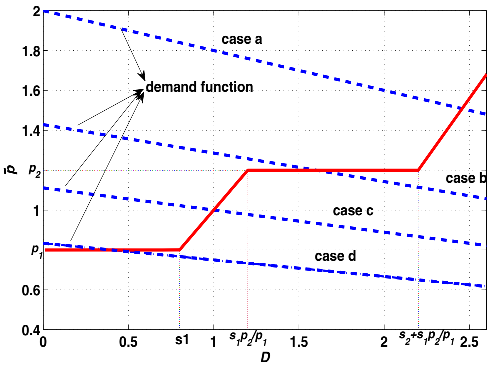

Fig. 2. Users' equilibrium demand with effective price

If $D > s_1 p_2 / p_1$, WSP 2 begins to get some demand. In case b, $s_1 p_2 / p_1 < D \le s_1 p_2 / p_1 + s_2$, the effective price all users perceive equals to p_2, and $d_1 = s_1 p_2 / p_1$, then $d_2 = D - s_1 p_2 / p_1$. In case a, $D > s_1 p_2 / p_1 + s_2$, all WSPs are saturated. The effective price the users perceived is

$$\overline{p} = p_1 d_1 / s_1 = p_2 d_2 / s_2 \quad . \tag{6}$$

The situation when $p_1 > p_2$ can be analyzed as the same as above. In special situation $p_1 = p_2$, we supposed that the demand distribution is proportioned to the system capacity, which means that

$$d_1 / s_1 = d_2 / s_2 \, . \tag{7}$$

3.2 WSP's Pricing Sub-game in Stage II

In this stage, the pricing strategies of non-cooperative WSPs are investigated. The WSPs have the information of the demand distribution as described in preceding section.

(1) Pricing Sub-game Model

The WSPs gain revenue by providing service to users, and the revenue is defined as

$$R_i\left(p_i, p_j\right) = p_i d_i \quad \text{for } i, j \in \{1,2\},\ i \neq j. \tag{8}$$

The pricing sub-game between WSPs is modeled as below.

Players: two WSPs.

Strategy space: WSP i can choose price p_i from the feasible set $\mathrm{P}_i = [0, +\infty)$, $i \in \{1,2\}$.

Payoff function: $R_i\left(p_i, p_j\right) = p_i d_i$, for $i, j \in \{1,2\}$, $i \neq j$.

For pricing game, Nash equilibrium is a point of price strategies $\left(p_1^*, p_2^*\right)$, each WSP maximizes its revenue assuming that the other WSP chooses the equilibrium price, and no WSP can increase its revenue by unilaterally changing its price [16], $\forall i, j \in \{1,2\}, i \neq j$,

$$R_i\left(p_i^*, p_j^*\right) \geq R_i\left(p_i, p_j^*\right). \tag{9}$$

According to the analysis about demand distribution, we find that the payoff functions in (8) are not derivable. To determine the existence and uniqueness of Nash equilibrium is difficult for this model, therefore we analyze it numerically as [15]. The parameters are $s_1 = 1$, $s_2 = 2$, and $\alpha = 10$, $\beta = 3$.

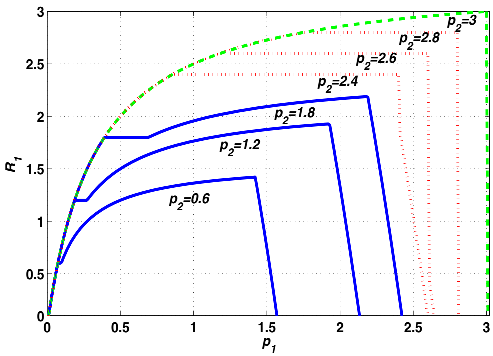

Fig. 3. Revenue of WSP 1 as a function of p_1

For given parameters as above and fixed values of p_2, we show the revenue of WSP 1 with changing p_1 in Fig.3. All the curves are the same when $p_1 > p_2 s_1 / D(p_2)$ is low enough, which means that all users choose WSP 1 as case d in Fig. 2. When $p_1 > p_2 s_1 / D(p_2)$, the curves have different characteristics.

For $p_2 \le (\alpha - s_2 - s_1)/\beta$, both WSPs are saturated at the user equilibrium as case a in Fig. 2. The revenue of WSP 1 would reach the maximum when $p_1 = (10 - s_1)/(s_2/p_2 + 3)$ as the blue curves in Fig.3.

For $(\alpha - s_2 - s_1)/\beta < p_2 \le (\alpha - s_1)/\beta$, only WSP 1 is saturated at the user equilibrium as case b in Fig. 2. The WSP 1's best price strategy p_1 is not unique, but an interval in $\left[s_1 p_2/D(p_2), s_1 p_2/(D(p_2) - s_2)\right]$ as the red curves in Fig.3.

For $(\alpha - s_1)/\beta < p_2 \le \alpha/\beta$, all users choose WSP 1 and $d_2 = 0$ at the user equilibrium as case c and d in Fig.2. The revenue of WSP 1 reaches the maximum when $p_1 = (10 - s_1)/\beta$ as the green curve in Fig. 3.

(2) Nash Equilibrium of Pricing Sub-game

The best reply function of WSP i's revenue is

$$BR_i(p_j) = \arg\max_{p_i \ge 0} R_i(p_i, p_j), \text{ for } i, j \in \{1,2\}, i \ne j \tag{10}$$

The best reply is the price value which maximizes the revenue of the WSP i while the other WSP's price is fixed. Nash equilibrium can be represented by the set of points (p_1^*, p_2^*), in which $p_1^* \in BR_1(p_2^*)$ and $p_2^* \in BR_2(p_1^*)$.

Fig.4 gives an example of best replies for non-cooperative WSPs, the model parameters we used are the same as in Fig.3.When $(\alpha - s_2 - s_1)/\beta \le p_2 \le (\alpha - s_1)/\beta$, best replies are not unique which is easy to understand from the analysis in Fig. 3. The intersection zones of best replies are $(0,0)$ and the range $p_1 = p_2 \in \left[(\alpha - s_1 - s_2)/\beta, \min\left((\alpha - s_2)/\beta, (\alpha - s_1)/\beta\right)\right]$.

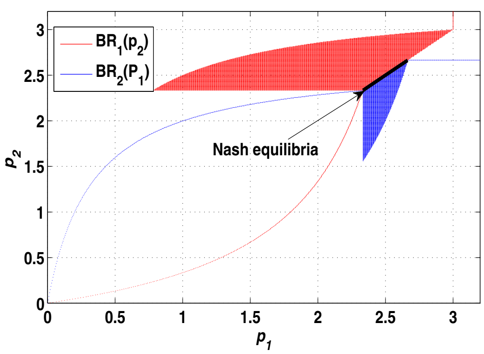

Fig. 4. Best reply curves of both WSPs

(3) Nash Equilibrium Discussion

$(0,0)$ is a trivial Nash equilibrium, because no WSP would set its price to be 0.

There exist infinite pricing best reply intersection points in the range $p_1 = p_2 \in \left[(\alpha - s_1 - s_2)/\beta, \min\left((\alpha - s_2)/\beta, (\alpha - s_1)/\beta\right)\right]$. In Fig.4, the solid black line represents this set of infinitely pricing intersection points. However, the intersection points $p_1 = p_2 \in ((\alpha - s_1 - s_2)/\beta, \min\left((\alpha - s_2)/\beta, (\alpha - s_1)/\beta\right)]$ correspond to the case that only one WSP is saturated while the other is not. The unsaturated WSP i ($i \in \{1,2\}$) would like to decrease its price p_i unilaterally by a small amount to attract more users and gain more revenue. Obviously, there is no Nash equilibrium in that zone. Both WSPs are exactly saturated at the point $p_1 = p_2 = \left((\alpha - s_1 - s_2)/\beta, (\alpha - s_1 - s_2)/\beta\right)$ and neither WSP would like to change its price unilaterally. As a result, $\left(p_1^*, p_2^*\right) = \left((\alpha - s_1 - s_2)/\beta, (\alpha - s_1 - s_2)/\beta\right)$ is the unique Nash equilibrium of pricing sub-game that we are looking for.

Since both WSPs are saturated, then according to (4) (5) (6) (8), we get

$$R_i = \frac{\alpha p_i p_j s_i}{s_i p_j + p_i s_j + \beta p_i p_j}. \tag{11}$$

We can get that $\partial R_i / \partial p_i > 0$, R_i increases monotonously when both WSPs are in saturated range. As the analysis of the blue curves in Fig.3 and $BR_i\left(p_j\right) = \arg\max_{p_i}\left(R_i\right)$, the best response function can be described as following. For $i, j = \{1,2\}, i \neq j$,

$$p_i^B\left(p_j\right) = \frac{\alpha - s_i}{s_j / p_j + \beta}. \tag{12}$$

Take the Nash equilibrium $\left(p_1^*, p_2^*\right)$ into (11), the demand distribution can be presented as

$$d_i^* = \frac{\alpha p_j^* s_i}{\beta p_i^* p_j^* + p_i^* s_j + p_j^* s_i}. \tag{13}$$

From above, we can get $d_i^* = s_i$. If WSPs decide the prices according to the Nash equilibrium $\left(p_1^*, p_2^*\right)$, leasing bandwidth can exactly satisfy the demand which means that the dynamic spectrum market reaches the market equilibrium [14].

3.3 WSP's Leasing Full Game in Stage I

(1) Leasing Full Game Model

In the first stage (full game leader), WSPs have to decide the leasing amount of the spectrum. Taking the demand distribution $\left(d_1^*, d_2^*\right)$ in stage III and pricing strategy $\left(p_1^*, p_2^*\right)$ in stage II into consideration, we give the leasing bandwidth decision $\left(b_1, b_2\right)$ to maximize the WSPs' profits.

The WSP i 's profit is defined as: $\forall i, j \in \{1,2\}, i \neq j$,

$$\pi_i(b_i, b_j) = R_i(k_i b_i, k_j b_j) - b_i C(b_i + b_j). \tag{14}$$

The leasing game can be modeled as follows.

Players: two WSPs.

Strategy space: WSP i can choose bandwidth amount b_i from the feasible set $b_i = [0, +\infty)$, $i \in \{1,2\}$. We assume that the bandwidth provided by spectrum broker is enough.

Payoff function: $\pi_i(p_i, p_j)$, for $i, j \in \{1,2\}$, $i \neq j$.

(2) Nash equilibrium of leasing full game

As the payoff function is derivable, Nash equilibrium of the leasing game exists and can be acquired through solving the function $\partial\pi_i / \partial b_i = 0$, for $i \in \{1,2\}$, $i \neq j$,

$$\pi_i = R_i(k_1 b_1, k_2 b_2) - b_i C(b_1 + b_2),$$

$$\frac{\partial \pi_i}{\partial b_i} = \left(\alpha k_i - 2k_i^2 b_i - k_i k_j b_j\right) / \beta - 2Cb_i - Cb_j = 0.$$

The best response function can be written as

$$b_i^B(b_j) = \frac{\alpha k_i - k_i k_j b_j - \beta C b_j}{2k_i^2 + 2\beta C} \tag{15}$$

Set $m = k_1 k_2 + \beta C$, $n = 2k_1^2 + 2\beta C$, $l = 2k_2^2 + 2\beta C$, we can get the unique Nash equilibrium as (b_1^*, b_2^*).

$$b_1^* = \frac{\alpha l k_1 - \alpha m k_2}{nl - m^2} \tag{16}$$

$$b_2^* = \frac{\alpha n k_2 - \alpha m k_1}{nl - m^2} \tag{17}$$

4 Dynamic Game of WSPs

In section 3, WSPs are assumed to have the complete information about the strategies and payoff functions for both WSPs. Hence WSPs can make simultaneous decisions, and the three-stage game is static. However, the situation is different in reality. The strategies and payoff functions of the other WSP are not fully available. WSP can only observe each other's history of strategies, and we present dynamic game model for the competitive WSPs. We investigate how WSPs interact in such a dynamic game, and give an iterative algorithm to achieve its dynamic equilibrium. As in [9], we assume that the pricing sub-game at the users' side is a short term dynamic game, while leasing game at the spectrum broker's side is a long term dynamic game. This is because that these two competitions are done separately. We assume the following discussions are in the condition where users' market is stable.

4.1 Short Term Pricing Dynamic Game

With incomplete information, current decision of the opponent's pricing strategy is not available. We use the history record of the WSP j to decide the price of WSP i based on the best response function in (12). We give the pricing dynamic game as

$$p_i(t) = p_i^B\left(p_j(t-1)\right) \text{ for } i, j \in \{1,2\} \ , \ i \neq j . \tag{18}$$

We describe the iterative algorithm of the pricing dynamic game as below.

Step 1. Initially $t = 0$, WSPs set prices as $p_i(0)$, $p_j(0)$.

Step 2. In each iteration $t > 0$, WSPs update their prices according to (18).

Step 3. Stop until the criteria is met. The criteria can be the maximum number of iteration or the difference between the WSP i's prices of two consecutive iterations is less than a predefined threshold.

Theorem 1: Given leasing bandwidth, pricing dynamic game converges to the Nash equilibrium point and it is stable.

Proof: It is obvious that the best response function in (12) is monotonous and bounded, so the pricing dynamic game is convergent. For the stable point, it satisfies $p_i = p_i^B(p_j)$, which is just the Nash equilibrium of pricing sub-game.

4.2 Long Term Leasing Dynamic Game

Leasing dynamic game is relative long term with respect to pricing dynamic game. We assume that leasing strategy only updates after pricing dynamic game finished.

According to (18), we give the leasing dynamic game as

$$b_i(T) = \frac{\alpha k_i - \left(k_i k_j + \beta C\right) b_j(T-1)}{2k_i^2 + 2\beta C} . \tag{19}$$

Equation (15) has the same characteristic as equation (12), so the long term leasing dynamic game also converges to the Nash equilibrium which has the same reason as short term pricing dynamic game. We can describe the leasing dynamic game's iterative algorithm as below.

Step 1. Initially $T = 0$, WSPs set leasing bandwidth $b_i(0), i \in \{1,2\}$. Adjust the prices according to the short term pricing dynamic game and acquire the prices $p_i(0)$ ($i \in \{1,2\}$) which meet the criteria.

Step 2. In each iteration $T > 0$, WSPs update their leasing bandwidth according to (19). $p_i(T)$ ($i \in \{1,2\}$) can be obtained by short term pricing dynamic game.

Step 3. Stop until the criteria are met. The definition of stopping criteria is similar to previous part.

5 Simulation Results

(1) Static Game of Non-cooperative WSPs

The model parameters are $C=1$, $\alpha=10$, $\beta=3$. Nash equilibrium of pricing sub-game is only influenced by the total leasing bandwidth given the particular spectrum efficiency of the WSPs. In Fig.5, it shows that the pricing sub-game's Nash equilibrium decreases with the total leasing bandwidth in linear relation. Fig.6 shows that spectrum efficiency influences the WSP's leasing strategy. We can see that WSP with bigger spectrum efficiency gets more bandwidth. Fig.7 shows that for given k_1, WSPs' leasing bandwidth and profits change as k_2 increases. The WSP 2's leasing bandwidth firstly increases as k_2 grows because the WSP with bigger spectrum efficiency gets more bandwidth, and then it decreases because as spectrum efficiency increase the WSP needs less bandwidth to fulfill the demand. The profit of WSP 2 increases as k_2 grows, while the profit of WSP 1 decreases. Hence with the same objective to maximize the profit, every WSP will have the incentive to promote its own spectrum efficiency.

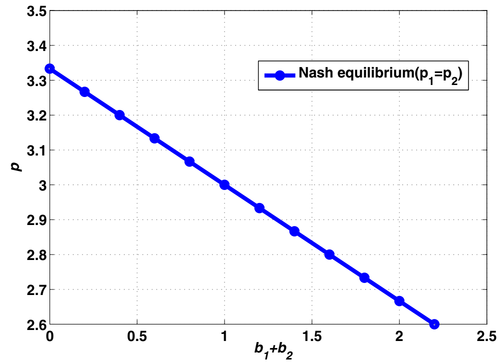

Fig. 5. Nash equilibrium of pricing with different leasing bandwidth

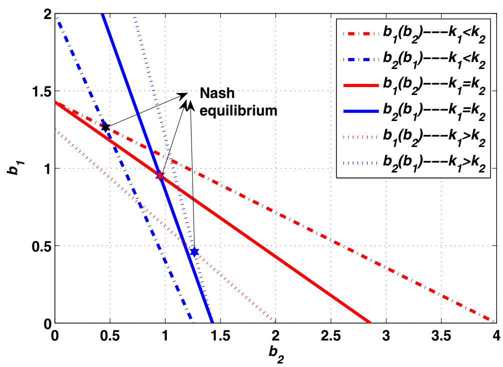

Fig. 6. Nash equilibrium of leasing with different spectrum efficiency

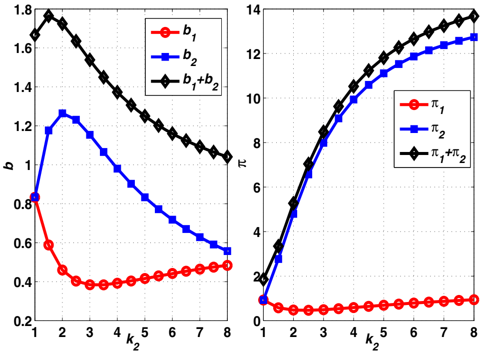

Fig. 7. With given $k_1 = 1$, the leasing bandwidth and profit varies with k_2

(2) Dynamic Game of Non-cooperative WSPs

We present some numerical results for the performance in dynamic spectrum market, and study the characteristic of the dynamic game. We set $s_1 = 1$, $s_2 = 2$, $\alpha = 10$, $\beta = 3$ in Fig.8 and $C = 1$, $k_1 = 1$, $k_2 = 2$, $\alpha = 10$, $\beta = 3$ in Fig.9.

For short term price adjustment in Fig.8, given the initial small value of prices, we can see that the iterative algorithm converges to the pricing Nash equilibrium very quickly. We also simulate the long term bandwidth adjustment in Fig.9, the convergence of leasing dynamic game is rather quick and it converges to the full game Nash equilibrium. Fig.9 also shows the corresponding prices at each stable point. The simulations show that the dynamic games converge to Nash equilibria.

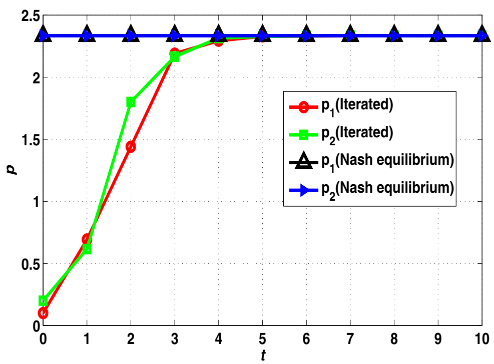

Fig. 8. Short term pricing dynamic game

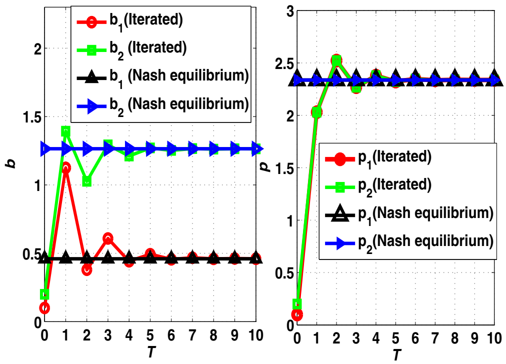

Fig. 9. Long term leasing dynamic game

6 Conclusion

In this paper, we use a three-stage game to model the interactions between competitive WSPs and users in dynamic spectrum leasing. The users choose WSP by taking price and QoS into considerations. The pricing sub-game has a unique Nash equilibrium. The pricing Nash equilibrium of WSPs is the same even when they lease different amount of bandwidth and have different spectrum efficiencies. Interestingly, with the pricing Nash equilibrium, the demand equals to the leasing bandwidth which means that the game reaches the market stable equilibrium. The pricing Nash equilibrium is only dependent on the total leasing bandwidth of WSPs. Leasing Nash equilibrium is unique and dependent on the spectrum efficiency. When information is incomplete, WSPs interact with each other in a dynamic game. We use the iterative algorithm based on the best response function as a solution to In this paper, we use a three-stage game to model the interactions between competitive WSPs and users in dynamic spectrum leasing. The users choose WSP by taking price and QoS into considerations. The pricing sub-game has a unique Nash equilibrium. The pricing Nash equilibrium of WSPs is the same even when they lease different amount of bandwidth and have different spectrum efficiencies. Interestingly, with the pricing Nash equilibrium, the demand equals to the leasing bandwidth which means that the game reaches the market stable equilibrium. The pricing Nash equilibrium is only dependent on the total leasing bandwidth of WSPs. Leasing Nash equilibrium is unique and dependent on the spectrum efficiency. When information is incomplete, WSPs interact with each other in a dynamic game. We use the iterative algorithm based on the best response function as a solution to the dynamic game. The short term pricing dynamic game converges to the Nash equilibrium of the pricing sub-game, while the long term leasing dynamic game converges to Nash equilibrium of the static full game.

There are many aspects to extend our results in this paper. For example, we can introduce the heterogeneous into WSPs for providing different services or into users with different QoS requirements in future works.

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