# Performance Analysis of Opportunistic Spectrum Sharing System 

Wanbin Tang, Yanfeng Han, Hua Jin, and ShaoQian Li<br>National Key Laboratory of Science and Technology on Communications, University of Electronic Science and Technology of China, Chengdu, Sichan, China<br>wbtang@uestc.edu.cn


#### Abstract

Frequency spectrum is a limited resource for wireless communications. Cognitive radio provides an approach to efficient utilization of spectrum. Spectrum handoff is an indispensable component in cognitive radio networks to provide resilient service for the secondary users. In this paper, we analyze the handoff performance of an opportunistic spectrum sharing systems over a coverage area. In the OSS systems, If an active secondary user detects the arrival of a primary user on a given channel, it releases the channel and switches to another idle channel. If no channel is available, the call waits in a buffer until either one channel becomes available or a maximum waiting time is reached. We examine the performances in terms of the link maintenance probability and mean handoff numbers of the secondary user's. In the last, we prove the simulation result in session V .


Keywords: Opportunistic spectrum sharing, Markov process, Handoff.

## 1 Introduction

Frequency spectrum is a limited resource for wireless communications. Cognitive radio provides an approach to efficient utilization of spectrum and become a important research issue [1].

According to Federal Communications Commission (FCC), temporal and geographical variations in the utilization of the assigned spectrum range from $15 \%$ to $85 \%$ [2]. Then, frequency agile radios (FARs) have attracted more interest in the research community. In a scenario of opportunistic spectrum sharing (OSS), the FARs is called secondary users (SUs) and the owners of the allocated spectrum are the primary users (PUs).

In the OSS system, SU opportunistically use channels that are not occupied by PU. In order not to cause harmful interference to the PU, if an active SU detects that a PU will accesses the channel, then it moves to another idle channel, or moves to a waiting buffer. In the latter case, the SU waits in a buffer until either a new channel becomes available or a timeout occurs after a predefined maximum waiting time. So spectrum handoff is a major difficulty and also an inherent capability to support reliable service. On detecting a PU appearance, the SU has to vacate the channel for the PU. After the channel release, the SU will re-construct the communications. During this

[^0]procedure, the SU may search the idle channel and transfer its communications to this channel. This procedure is referred as spectrum handoff.

In this paper, we model an opportunistic spectrum sharing system and evaluate its handoff performance. The rest of the paper is organized as follows. Section 2 describes the system model and assumptions. Section 3 develops a Markov model of the system, while Section 4 derives the performance metrics. In section 5, We present the numerical results to illustrate the performance of the OSS system. Finally, the paper is concluded in Section 6.

## 2 Model and Assumptions

In this paper, we shall assume an SU can perfectly detect the arrival of a new PU, and an arriving SU also can detect that the given channel is idle. We also assume that there are a total of N channels in the OSS systems. Each channel is assumed to be of equal bandwidth. We further assume that each user occupy one channel for simplicity. This is explained with the help of Fig. 1[7]. There are a total five channels of which two are occupied by PUs and one by an SU. When a new SU arrives as shown in Fig. 1a, it chooses a random free channel. A PU can choose any random channels and as shown in Fig. 1b, if it chooses a secondary occupied channel, the SU jumps to a different free channel. If there is no other channel available, the SU will be queued in the buffer in Fig. 1c.


Fig. 1. The model of channel occupy in OSS system ( $\mathrm{N}=5$ )

In Fig. 1d, the queued SUs are served in first-come first-served (FCFS) order. The head-of-line SU will reconnect to the system when a channel becomes idle before the maximum waiting time expires. We set the maximum waiting time of a SU equal to its residence time in the cell.

## 3 Performance Analysis

In this section, we analyze the OSS system performance in a given service area. Arrivals of the PU and SU are assumed to form independent Poisson processes with
rates $\lambda_{1}$ and $\lambda_{2}$, respectively. The holding times of the PU and SU are assumed to be exponentially distributed with means $h_{1}^{-1}$ and $h_{2}^{-1}$, respectively. The residence times for the PU and SU in the service area are also assumed to be exponentially distributed with means $r_{1}^{-1}$ and $r_{2}^{-2}$, respectively. Hence, the channel holding times for the PU and SU are exponentially distributed with means $\mu_{1}^{-1}=\left(r_{1}+h_{1}\right)^{-1}$ and $\mu_{2}^{-1}=\left(r_{2}+h_{2}\right)^{-1}$, respectively.

Let $X_{1}(t)$ denote the number of PUs in the OSS system at time $t$. Similarly, let $X_{2}(t)$ be the number of SUs in the system at time $t$, including the SUs being served and those waiting in the buffer. The process $\left(X_{1}(t), X_{2}(t)\right)$ is a two-dimensional Markov process with state space $S=\left\{\left(n_{1}, n_{2}\right) \mid 0 \leq n_{1} \leq N, 0 \leq n_{2} \leq N\right\}$.


Fig. 2. State diagram of OSS system

We take $N=3$ for example to analyze the State diagram of OSS system, the state transition diagrams of the OSS system can be presented in Fig. 2 [8].

The transition rate from state $\left(n_{1}, n_{2}\right)$ to $\left(n_{1}^{*}, n_{2}^{*}\right)$ denote by $T_{n_{1}, n_{2}}^{n_{1}, n_{2}^{*}}$ is given by:

$$
\begin{gather*}
T_{n_{1}, n_{2}}^{n_{1}+1, n_{2}}=\lambda_{1} 1_{\left\{0 \leq n_{1}<N, 0 \leq n_{2} \leq N\right\}}  \tag{1}\\
T_{n_{1}, n_{2}}^{n_{1}-1, n_{2}}=n_{1} \mu_{1} 1_{\left(1 \leq n_{1} \leq N, 0 \leq n_{2} \leq N\right\}}  \tag{2}\\
T_{n_{1}, n_{2}}^{n_{1}, n_{2}+1}=\lambda_{2} 1_{\left\{0 \leq n_{1} \leq N-1,0 \leq n_{2}<N-n_{1}\right\}}  \tag{3}\\
T_{n_{1}, n_{2}}^{n_{1}, n_{2}-1}=\left[\left(n_{2}-N+n_{1}\right) r_{2}+\left(N-n_{1}\right) \mu_{2}\right] 1_{\left\{1 \leq n_{1} \leq N, N-n_{1}<n_{2} \leq N\right\}} \\
+n_{2} \mu_{2} 1_{\left\{0 \leq n_{1} \leq N-1,1 \leq n_{2} \leq N-n_{1}\right\}} \tag{4}
\end{gather*}
$$

where $1_{\{x\}}$ is an indicator function defined as 1 if $x$ is true and 0 otherwise.

Let $\pi\left(n_{1}, n_{2}\right)$ denote the steady-state probability. The steady-state system probability vector can be represented as $\pi=\left(\pi_{0}, \pi_{1}, \ldots, \pi_{N}\right)$, where

$$
\begin{equation*}
\pi_{n}=(\pi(n, 0), \pi(n, 1), \ldots \pi(n, N)), 0 \leq n \leq N . \tag{5}
\end{equation*}
$$

The vector $\pi$ is the solution of the following equations:

$$
\pi Q=0 \bigcup \pi e=1
$$

where $e$ and 0 are column vectors of all ones and zeros, respectively. The infinitesimal generator, $Q$, of the two dimensional Markov process is given by:

$$
Q=\left[\begin{array}{ccccccc}
E_{0} & B_{0} & 0 & \cdots & 0 & 0 & 0  \tag{6}\\
D_{1} & E_{1} & B_{1} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & D_{N-1} & E_{N-1} & B_{N-1} \\
0 & 0 & 0 & \cdots & 0 & D_{N} & E_{N}
\end{array}\right]
$$

where each sub matrix is defined by:

$$
\begin{gather*}
B_{i}=\lambda_{1} I_{N+1}, 0 \leq i<N  \tag{7}\\
D_{i}=i \mu_{1} I_{N+1}, 1 \leq i \leq N  \tag{8}\\
E_{i}=A_{i}-\bar{\delta}(i) D_{i}-\bar{\delta}(N-i) B_{i}, 0 \leq i \leq N \tag{9}
\end{gather*}
$$

where $I_{n}$ denotes an $n$-by- $n$ identity matrix, the matrix $A_{i}$ has the same size as $E_{i}$.The $(j, k)$ element of the matrix $A_{i}$ is given by:

$$
A(j, k)=\left\{\begin{array}{cc}
\left(1-p_{f}-\bar{\delta}(i) p_{m}\right) \lambda_{2} & 0 \leq i \leq N-1,0 \leq j<N-i, k=j+1,  \tag{10}\\
\lambda_{2} & 0 \leq i \leq N, N-i \leq j<N, k=j+1, \\
j \mu_{2} & 0 \leq i \leq N-1,1 \leq j \leq N-i, k=j-1, \\
(N-i) \mu_{2}+(j-N+i) r_{2}, & 1 \leq i \leq N, N-i<j \leq N, k=j-1, \\
-A(j, j-1)+A(j, j+1)] & 0 \leq i \leq N, 0 \leq j \leq N, k=j, \\
0 & \text { othemise },
\end{array}\right.
$$

## 4 Performance Metrics

Next, we obtain various performance measures of interest.

## A. Blocking Probabilities

According to preference [4], we can obtain the blocking probability of PUs and SUs.

$$
\begin{gather*}
P_{1}=\sum_{n_{2}=0}^{N} \pi\left(N, n_{2}\right)=\pi_{0} \prod_{i=1}^{N}\left[B_{i-1}\left(-C_{i}\right)^{-1}\right] e  \tag{11}\\
P_{2}=\sum_{n_{1}=0}^{N} \sum_{n_{2}=N-n_{1}}^{N} \pi\left(n_{1}, N\right) \tag{12}
\end{gather*}
$$

## B. Mean Reconnection Probability

According to preference [4], we can obtain the mean reconnection probability.

$$
\begin{equation*}
\gamma=\frac{\sum_{n_{1}=1}^{N} \sum_{j=0}^{n_{1}-1} \pi\left(n_{1}, N-n_{1}+j+1\right) \beta(j)}{\sum_{n_{1}=1}^{N} \sum_{j=0}^{n_{1}-1} \pi\left(n_{1}, N-n_{1}+j+1\right)} \tag{13}
\end{equation*}
$$

and $\beta(j)$ is defined as following:

$$
\begin{equation*}
\beta(j)=\frac{n_{1} \mu_{1}+\left(N-n_{1}\right) \mu_{2}}{n_{1} \mu_{1}+\left(N-n_{1}\right) \mu_{2}+(j+1) r_{2}} \tag{14}
\end{equation*}
$$

## C. The Number of Spectrum Handoff

When a PU appears there are four consequences on SU's behaviour: (a) the SU need not to release its channel; (b) the SU releases the channel and comes into another idle channel; (c) the SU releases the channel and comes into the buffer, and in the last, returns a channel within the maximum waiting time; and (d) the SU in the buffer leaves the system. Because of behaviour (b) and behaviour (c), there are two patterns of spectrum handoff, Let $P_{V}$ [6]denote the probability that an SU leaves its channel when a PU appears. This probability is equal to the probability that a particular channel is reclaimed by a PU . When the number of PU is i , the probability that a particular channel is reclaimed by the PU is given by $1 /(N-i)$. Then, we have

$$
\begin{equation*}
P_{V}=\frac{\sum_{i=0}^{N-1} \sum_{j=0}^{N} \frac{1}{N-i} P_{r}(i, j)}{1-P_{1}} \tag{15}
\end{equation*}
$$

here, the item $\left(1-P_{1}\right)$ shows the probability that the PU can insert the channel. Let $P_{N V}$ denote the probability that an SU need not vacate its channel. So we can conclude:

$$
\begin{equation*}
P_{N V}=\frac{\sum_{i=0}^{N-1} \sum_{j=0}^{N}\left(1-\frac{1}{N-i}\right) P_{r}(i, j)}{1-P_{1}} \tag{16}
\end{equation*}
$$

Because there are two patterns of spectrum handoff, we let $q_{s 1}$ denote the link maintenance probability of spectrum handoff which is caused by behaviour (b). So the probability $q_{f 1}$ that the SU fails to return the channel is 0 . Let $q_{s 2}$ denote the link maintenance probability of spectrum handoff which is caused by behaviour (c), and let $q_{f 2}$ denote the probability of the spectrum handoff is failed. Link maintenance probability refers to the probability that link is successfully maintained when the SU vacates the channel. So we can conclude:

$$
\begin{gather*}
q_{s 1}=P_{V}\left[P_{r}(i, j) \delta(i+j<N)\right]  \tag{17}\\
q_{f 1}=0  \tag{18}\\
q_{s 2}=P_{V}\left[P_{r}(i, j) \delta(i+j \geq N)\right] \gamma  \tag{19}\\
q_{f 2}=P_{V}\left[P_{r}(i, j) \delta(i+j \geq N)\right](1-\gamma) \tag{20}
\end{gather*}
$$

Let $H$ denote the number of spectrum handoff for an SU from its beginning of service to the end of the service. In this section, we will develop the probability mass function of the discrete random variable $H$. Let $t_{c s}$ denote the SU call holding time with the average $1 / \mu_{2}$, pdf $f_{t_{c s}}(t), \mathrm{CDF} F_{t_{c s}}(t)$, and Laplace Transform of pdf is $f_{t_{c s}}^{*}(s)$. Let $\overline{F_{t_{c s}}}(t)$ denote the complementary cumulative distribution function (CCDF), $\overline{F_{t_{c s}}}(t)=1-F_{t_{c s}}(t)$. Let $t_{p u, j}$ denote the PU inter-arrival time between $(j-1) t h$ and $j t h \mathrm{PU}$ with the generic form $t_{p u}$. Here, the first PU refers to the immediate next PU after SU admission in the system. Denote $\tau_{k}=\sum_{j=1}^{k} t_{p u, j}$. For poisson process, $\tau_{k}$ follows Erlang distribution with pdf

$$
\begin{equation*}
f_{\tau_{k}}(t)=\frac{\lambda_{1}\left(\lambda_{1} t\right)^{k-1}}{(k-1)!} e^{-\lambda_{1} t} \tag{21}
\end{equation*}
$$

## 1) Zero Spectrum Handoff

For an accepted SU, there are two situations leading to zero spectrum handoff. If the SU call holding time is smaller than the PU inter-arrival time, the SU can complete its service before a PU appears. On the other hand, there are several PU arrivals when the

SU using the channel, but these PUs use different channels from the one used by the SU. In this case, the SU need not to vacate the channel to perform spectrum handoff. Considering these two conditions leading to zero spectrum handoff, we have:

$$
\begin{equation*}
P(H=0)=P\left(t_{c s}<t_{p u}\right)+\sum_{j=1}^{\infty} P\left(\tau_{j}<t_{c s}<\tau_{j+1}\right) P_{N V}^{j} \tag{22}
\end{equation*}
$$

We first compute the first item in the right-side of (22).

$$
\begin{align*}
& P\left(t_{c s}<t_{p u}\right) \\
& =\int_{0}^{\infty} f_{t_{c s}}(x) \int_{x}^{\infty} f_{t_{p u}}(y) d y d x \\
& =f_{t_{c s}}^{*}\left(\lambda_{1}\right) \tag{23}
\end{align*}
$$

Before computing the second item, we develop an identity which will be used frequently in the following.

$$
\begin{align*}
& P\left(\tau_{j}<t_{c s}<\tau_{j+1}\right) \\
& =\int_{0}^{\infty} f_{t_{t s}}(t) P\left(\tau_{j}<t<\tau_{j+1}\right) d t \\
& =\int_{0}^{\infty} f_{t_{\text {cs }}}(t) \int_{0}^{t} f_{\tau_{j}}(x) \int_{t-x}^{\infty} f_{t_{p u}}(y) d y d x d t \\
& =\int_{0}^{\infty} f_{t_{c s}}(t) \frac{\lambda_{1}^{j} t^{j} e^{-\lambda t}}{j!} d t \\
& =\frac{\left(-\lambda_{1}\right)^{j}}{j!} f_{t c s}^{*(j)}\left(\lambda_{1}\right) \tag{24}
\end{align*}
$$

where $f_{t_{c s}}^{*(j)}(s)$ denotes the derivative of $j t h$ order. We continue the second item in the right-side of (22).

$$
\begin{align*}
& \sum_{j=1}^{\infty} P\left(\tau_{j}<t_{c s}<\tau_{j+1}\right) P_{N V}^{j} \\
& =\sum_{j=1}^{\infty} \int_{0}^{\infty} f_{t_{c s}}(t) \frac{\left(\lambda_{1} t\right)^{j} e^{-\lambda_{1} t}}{j!} d t P_{N V}^{j} \\
& =\int_{0}^{\infty} f_{t_{s s}}(t) e^{-\lambda_{1} t}\left[\sum_{j=1}^{\infty} \frac{\left(\lambda_{1} t P_{N V}\right)^{j}}{j!}\right] d t \\
& =\int_{0}^{\infty} f_{t_{t s}}(t) e^{-\lambda_{1} t}\left(e^{\lambda_{1} P_{N V}}-1\right) d t \\
& =f_{t_{c s}}^{*}\left(\lambda_{1}\left(1-P_{N V}\right)\right)-f_{t_{c s}}^{*}\left(\lambda_{1}\right) \tag{25}
\end{align*}
$$

Substituting (23) (25) into (22), we obtain:

$$
\begin{equation*}
P(H=0)=f_{t_{c s}}^{*}\left(\lambda_{1}\left(1-P_{N V}\right)\right) \tag{26}
\end{equation*}
$$

## 2) $\mathbf{k}(\mathbf{k} \geqslant 1)$ Spectrum Handoff

During the SU call holding time $t_{c s}$, there are $k$ spectrum handoffs. And these $k$ spectrum handoff are caused by SU behavior (b) and (c). In this section, we consider two conditions, after $k$ spectrum handoff, the SU successfully completed connection or fail to complete connection. Let $\operatorname{succ}_{k}$ and fail denote the successful and failful events, respectively. Then, the probability for $k$ spectrum handoff is expressed by:

$$
\begin{equation*}
P(H=k)=P\left(\operatorname{succ}_{k}\right)+P\left(\text { fail }_{k}\right) \tag{27}
\end{equation*}
$$

The successful events include the following possibilities. During the SU service, there are $k+j \mathrm{PU}$ arrivals. Among these PU arrivals, $k \mathrm{PU}$ requests the same channel used by the SU and in this $k$ spectrum handoff, there are two patterns of spectrum handoff, and the other $j \mathrm{PU}$ arrivals requests different channels. Hence, the SU has to perform $k$ number of spectrum handoff and all these spectrum handoff are successful. Consider all possibilities on the variable $j$, we obtain the probability for the event $\operatorname{Succ}_{k}$.

$$
\begin{align*}
& P\left(s u c c_{k}\right) \\
& =\sum_{j=0}^{\infty} P\left(\tau_{k+j}<t_{c s}<\tau_{k+j+1}\right)\binom{k+j}{j} P_{N V}^{j}\left(\sum_{i=0}^{k}\binom{k}{i} q_{s 1}^{i} q_{s 2}^{k-i}\right) \\
& =\sum_{j=0}^{\infty}\left[\int_{0}^{\infty} f_{t c s}(t) \frac{\left(\lambda_{1} t\right)^{k+j} e^{-\lambda_{1} t}}{(k+j)!} d t\right] \frac{(k+j)!}{j!k!} P_{N V}^{j}\left(q_{s 1}+q_{s 2}\right)^{k} \\
& \left.=\int_{0}^{\infty} f_{t_{c s}}(t) \frac{\left(\lambda_{1} t\right)^{k} e^{-\lambda_{1} t}}{k!}\right]\left[\sum_{j=0}^{\infty} \frac{\left(\lambda_{1} t\right)^{j}}{j!} P_{N V}^{j}\right] d t\left(q_{s 1}+q_{s 2}\right)^{k} \\
& =\int_{0}^{\infty} t^{k} f_{t_{c s}}(t) e^{-\lambda_{1}\left(1-P_{N V}\right) t} d t \frac{\lambda_{1}^{k}}{k!}\left(q_{s 1}+q_{s 2}\right)^{k} \\
& =\frac{\left(-\lambda_{1}\left(q_{s 1}+q_{s 2}\right)\right)^{k}}{k!} f_{t_{c s}}^{*(k)}\left(\lambda P_{V}\right) \tag{28}
\end{align*}
$$

The failure events include the following possibilities. During the SU service, there are $k+j(k \geq 1, j \geq 0) \mathrm{PU}$ arrivals. Compare with the successful events, the $k t h$ spectrum handoff is failed. Considering all possibilities on the variable $j$, we obtain the probability.

$$
\begin{align*}
& P\left(f a i l_{k}\right) \\
& =\sum_{j=0}^{\infty} P_{r}\left(\tau_{k+j}<t_{c s}\right)\binom{k+j-1}{j} P_{N V}^{j}\left[\sum_{i=0}^{k-1}\binom{k-1}{i} q_{s 1}^{i} q_{s 2}^{k-i-1}\right] q_{f 2} \\
& =\sum_{j=0}^{\infty} \int_{0}^{\infty} f_{\tau_{k+j}}(t)\left(1-F_{t_{s s}}(t) d t\binom{k+j-1}{j} P_{N V}^{j}\left(q_{s 1}+q_{s 2}\right)^{k-1} q_{f 2}\right. \\
& =\sum_{j=0}^{\infty} \int_{0}^{\infty} \frac{\lambda_{1}\left(\lambda_{1} t\right)^{k+j-1} e^{-\lambda_{1} t}}{(k+j-1)!} \overline{F_{t s s}}(t) d t \frac{(k+j-1)!}{j!(k-1)!} P_{N V}^{j}\left(q_{s 1}+q_{s 2}\right)^{k-1} q_{f 2} \\
& =\frac{\lambda_{1}^{k}}{(k-1)!} \int_{0}^{\infty} t^{k-1} \overline{F_{t_{c s}}}(t) e^{-\lambda \lambda_{1} P_{V} t} d t\left(q_{s 1}+q_{s 2}\right)^{k-1} q_{f 2} \\
& =\frac{\left[-\lambda_{1}\left(q_{s 1}+q_{s 2}\right)\right]^{k-1} \lambda_{1} q_{f 2}}{(k-1)!} \bar{F}_{t_{c s}(k-1)}^{*}\left(\lambda_{1} P_{V}\right) \tag{29}
\end{align*}
$$

Substituting the two expressions into (26), we can obtain:

$$
\begin{align*}
P(H=k) & =\frac{\left[-\lambda_{1}\left(q_{s 1}+q_{s 2}\right)\right]^{k}}{k!} f_{t_{c s}}^{*(k)}\left(\lambda P_{V}\right) \\
& +\frac{\left[-\lambda_{1}\left(q_{s 1}+q_{s 2}\right)\right]^{k-1} \lambda_{1} q_{f 2}}{(k-1)!} \bar{F}_{t_{c s}}^{*(k-1)}\left(\lambda_{1} P_{V}\right) \tag{30}
\end{align*}
$$

## 3) Average Spectrum Handoff

In this section, we may conclude the expectation of the number of spectrum handoff $H$. Following the definition of expectation, we have:

$$
\begin{align*}
& E(H) \\
& =\sum_{k=0}^{\infty} k P(H=k) \\
& =\sum_{k=0}^{\infty} k P\left(\text { succ }_{k}\right)+\sum_{k=0}^{\infty} k P\left(\text { term }_{k}\right) \tag{31}
\end{align*}
$$

where $E(*)$ represents the expectation of a non-negative random variable.

$$
\begin{align*}
& \sum_{k=0}^{\infty} k P\left(s u c c_{k}\right) \\
& =\int_{0}^{\infty} f_{t_{c s}}(t) e^{-\lambda_{1} P_{v} t} \sum_{k=1}^{\infty} \frac{\left[\lambda_{1}\left(q_{s 1}+q_{s 2}\right) t\right]^{k}}{(k-1)!} d t \\
& =\lambda_{1}\left(q_{s 1}+q_{s 2}\right) \int_{0}^{\infty} t f_{t_{c s}}(t) e^{-\lambda_{1} q_{f 2} t} d t \\
& =-\lambda_{1}\left(q_{s 1}+q_{s 2}\right) f_{t_{c s}}^{*(1)}\left(\lambda_{1} q_{f 2}\right) \tag{32}
\end{align*}
$$

$$
\begin{align*}
& \sum_{k=1}^{\infty} k P\left(\text { term }_{k}\right) \\
& =\lambda_{1} \int_{0}^{\infty} \bar{F}_{t_{t s}}(t) e^{-\lambda_{1} P_{v} t}\left[\sum_{k=1}^{\infty} \frac{k\left(\lambda_{1} q_{s} t\right)^{k-1}}{(k-1)!}\right] d t q_{f} \\
& =\lambda_{1} \int_{0}^{\infty} \bar{F}_{t_{c s}}(t) e^{-\lambda_{1} P_{v} t}\left[\left(1+\lambda_{1} q_{s} t\right) e^{\lambda_{1} q_{s} t}\right] d t q_{f} \\
& =\lambda_{1} q_{f}\left[\bar{F}_{t_{c s s}}^{*}\left(\lambda_{1} q_{f}\right)-\lambda_{1} q_{s} \bar{F}_{t_{t s s}^{*(1)}}^{*}\left(\lambda_{1} q_{f}\right)\right] \\
& =\frac{\left(q_{s}+q_{f}\right)\left(1-f_{t_{c s}}^{*}\left(\lambda_{1} q_{f}\right)\right)}{q_{f}}+\lambda_{1} q_{s} f_{t_{c s}}^{*(1)}\left(\lambda_{1} q_{f}\right) \tag{33}
\end{align*}
$$

Substituting the two expressions into (30), we can obtain the theorem for the average number of spectrum handoff.

$$
\begin{equation*}
E(H)=\frac{\left(q_{s 1}+q_{s 2}+q_{f 2}\right)\left(1-f_{t_{s}}^{*}\left(\lambda_{1} q_{f 2}\right)\right)}{q_{f 2}} \tag{34}
\end{equation*}
$$

## 5 Numerical Results

In this section, we present numerical results for the OSS system model under the following parameter settings: $N=16, \mu_{1}=10, \mu_{2}=10, r_{2}=5$.

Fig. 3 shows the PU blocking probability $P_{1}$. We observe that $P_{1}$ increases with the PU intensity $\rho_{1}$, but don't depend on the SU intensity $\rho_{2}$. Fig. 4 shows the ST call blocking probability $P_{2}$. We observe that $P_{2}$ increases with $\rho_{1}$ or $\rho_{2}$.


Fig. 3. PU blocking probability


Fig. 4. SU blocking probability


Fig. 5. Mean reconnection probability of the queued SUs

Fig. 5 shows the mean reconnection probability. We observe that $\gamma$ decreases as $\rho_{1}$ increases and increases as the mean value $E[\tau]$ increases. The reason is as follows: the increase of $\rho_{1}$ results in the lower probability of the reconnection to system. While a longer maximum queueing time leads to a higher chance of reconnection.

Fig. 6 shows the function for the number of spectrum handoff in terms of $\lambda_{1} / \mu_{1}$. We observe that $H$ increases as $\rho_{1}$ increases, compare with literature [6], the probability and the mean number of spectrum handoff are higher than literature [6], the reason is as follows: after vacate the channel, the SU can come into the buffer, if a channel idle, the SU will reconnect the channel, this procedure is also spectrum handoff.


Fig. 6. The function for the number of spectrum handoff


Fig. 7. Mean number of spectrum handoff

## 6 Conclusion

We analyze the spectrum handoff performance of OSS system. In the OSS system, spectrum handoff is an inherent operation to support resilient and continuous communications. In this paper, the spectrum handoff procedure is characterized, and because of the buffer, the number of spectrum handoff is higher, but the blocking probability is lower, so these is a trade-off between blocking probability and handoff number.

Acknowledgments. This work is supported in part by High-Tech Research and Development Program (863 Program) of China under Grant No. 2009AA011801 and 2009AA012002, National Fundamental Research Program of China under Grant A1420080150, and National Basic Research Program (973 Program) of China under Grant No. 2009CB320405, Nation Grand Special Science and Technology Project of China under Grant No. 2008ZX03005-001, 2009ZX03007-004, 2009ZX03005-002, 2009ZX03005-004, 2010ZX03006-002, 2009ZX03004-001, 2010ZX03002-008-03.

## References

1. Akyildiz, I.F., Lee, W.Y., Vuran, M.C., et al.: NeXt generation/dynamic spectrum access/cognitive radio wireless networks: a survey. Computer Networks (50), 2127-2159 (September 2006)
2. McHenry, M.: Frequency agile spectrum access technologies. In: Proc. FCC Workshop (May 2003)
3. Tang, S., Mark, B.L.: Performance analysis of a wireless network with opportunistic spectrum sharing. In: Proc. IEEE Globecom 2007, Washington, D.C., USA (November 2007)
4. Tang, S., Mark, B.L.: Modeling and analysis of opportunistic spectrum sharing with unreliable spectrum sensing. IEEE Trans. Wireless Commun. (2009) (to appear)
5. Tang, S., Mark, B.L.: Analysis of opportunistic spectrum sharing with Markovian arrivals and phase-type service, Dept. of Electrical and Computer Eng., George Mason University, Tech. Rep.GMU-NAPL-Y08-N2 (December 2008)
6. Zhang, Y.: Spectrum Handoff in Cognitive Radio Networks: Opportunistic and Negotiated Situations. In: IEEE ICC 2009 (2009)
7. Kondareddy, Y.R., Andrews, N., Agrawal, P.: On the Capacity of Secondary Users in a Cognitive Radio Network. IEEE Xplore (December 9, 2009)
8. Capar, F., et al.: Comparison of bandwidth utilization for controlled and uncontrolled channel assignment in a spectrum pooling system. In: Proc. VTC Spring 2002 (2002)

[^0]:    P. Ren et al. (Eds.): WICON 2011, LNICST 98, pp. 411-423, 2012.
    © Institute for Computer Sciences, Social Informatics and Telecommunications Engineering 2012

