Distributed Fast Convergent Power Allocation Algorithm in Underlay Cognitive Radio Networks

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Abstract. In underlay cognitive radio networks, secondary users can share the same frequency band with primary users under the condition of meeting interference temperature constraint. In order to improve their spectrum efficiency, we consider a market competitive equilibrium (CE) model to formulate the multi-channel power allocation problem. In this paper, we prove the existence and uniqueness of CE. We simplify the CE to Nash equilibrium (NE) first, which exists and is unique under weak-interference conditions, for the fixed price; we then prove that the prices converge to the equilibrium price and present the sufficient condition of unique CE solution. Furthermore, we propose a distributed fast convergent power allocation algorithm (FCPAA) with round robin rules. The simulation results show that FCPAA can satisfy the interference temperature constraint perfectly and converge faster than the one in literature [9].

Keywords: Cognitive Radio, power allocation, competitive equilibrium, Nash equilibrium, distributed algorithm.

1 Introduction

With the development of wireless communication technology, the conflict is increasing between the fast growing demand for wireless spectrum resources and the scarcity of spectrum resources can be allocated. However, the actual measuring results show that a lot of the allocated spectrum resources are unutilized or underutilized [1]. Cognitive radio [2] which is considered as a promising technology to solve the current spectrum plight is being extensively studied. Typically, cognitive radio systems based on spectrum reuse approach is divided into three categories: underlay, overlay and interweave. In the underlay cognitive radio system [3], the primary users (authorized users) share the spectrum resources with the secondary users (non-authorized users), and the interference caused by secondary users must be lower than the interference temperature threshold, so secondary users' transmit powers need to be effectively controlled.

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In the underlay cognitive radio, power allocation is an important issue due to the interference temperature regulation of the primary system. This multichannel power allocation problem has been formulated as a non-cooperative game [4-6] and the Nash equilibrium is considered as the corresponding solution. However, the power allocation in a NE may be inefficient or not socially optimal [12]. Some literatures [7, 8] use Stackelberg game mechanism for power allocation. They maximize the primary user's benefits, but do not consider that the interference caused by secondary users to the primary users must be lower than interference threshold. Therefore we propose to use the competitive market model presented by literatures [9, 10] to solve this problem. In the competitive market model, a primary user sets a set of prices for channels to leverage the power allocation, which can satisfy its interference temperature constraint. The competitive equilibrium (CE) of this game is a set of prices and power allocations of all secondary users, which is a two-variable optimization problem and it is difficult to prove the existence and uniqueness of the solution. Moreover, wasting opportunity [15] may be caused when secondary user adjust their strategies, which is affecting the convergent speed.

So, this paper focuses on solving these problems. We first prove the existence and the uniqueness of CE, by predigesting the two-variable optimization problem to a simple one, when the price is fixed. Second, we derive the convergence of price and sufficient conditions of unique CE solution. Lastly, a distributed fast convergent power allocation algorithm (FCPAA) with round robin rules is proposed through which all the secondary users' power allocation can quickly converge to the unique CE while the interference temperature constraints of primary system are satisfied.

The rest of the paper is organized as follows. The competitive market model is formulated in Section 2. Section 3 investigates the existence and uniqueness of CE for this model. The distributed FCPAA with round robin rules is described in Section 4. Section 5 presents the simulation results to show that the proposed FCPAA meets the interference temperature constraint and converges fast.

2 System Model and Problem Formulation

2.1 System Model

We consider a cognitive radio system that consists of one primary user, a set of $\boldsymbol{N}=\{1,2,\ldots,n\}$ secondary users, and a set of $\boldsymbol{M}=\{1,2,\ldots, m\}$ channels, as shown in Fig. 1. All the users coexist in each channel and may access multiple channels at the same time. As in [11], the primary user sets a spectrum mask c_j for the j^{th} ($j\in \boldsymbol{M}$) channel, and requires the interferences caused by secondary users to be less than this level. Supposed that the power allocated by user i ($i\in \boldsymbol{N}$) to channel j is x_{ij}>0, the interference constraint implemented by primary user in the j^{th} channel is:

$$\textstyle\sum_{i=1}^{n} b_{ij} x_{ij} < c_j \quad . \tag{1}$$

where b_{ij} is the interference coefficients of the i^{th} secondary user on the j^{th} channel, that experienced by the primary user. To achieve an efficient allocation of spectrum, we associate a price p_j>0 with each channel j.

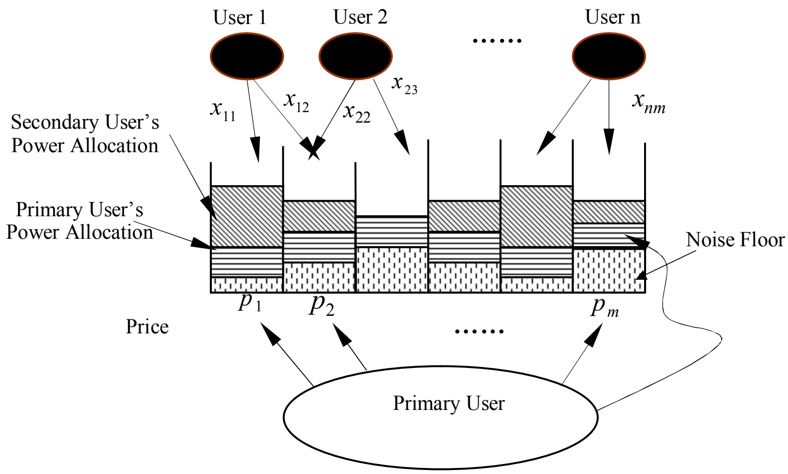

Fig. 1. Competitive spectrum market model

For a given vector of prices, $\boldsymbol{p}=[p_1, \ldots, p_m]^T$, each user i chooses the power allocation $\boldsymbol{x}_i=[x_{i1}, \ldots, x_{im}]^T$ that maximizes its utility function subject to its budget w_i. The i^{th} secondary user has its utility (i.e., sum data rate) as:

$$u_i(\boldsymbol{x}_i, \overline{\boldsymbol{x}}_i) = \sum_{j=1}^{m} \log(1 + \frac{x_{ij} h_{ii}^j}{\sigma_{ij} + \sum_{k \neq i} h_{ik}^j x_{kj}}) \quad . \tag{2}$$

In (2), $\overline{\boldsymbol{x}}_i = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_k \mid k \in \boldsymbol{N}, k \neq i\}$ are other n-1 users' power allocations. The interference experienced by the secondary users, are characterized by coefficients $h_{ik}^j > 0$, on channel j from user $k \neq i$ to user i. $\{h_{ii}^j\}_{j=1}^m$ is the channel gain of the direct link for each secondary pair i, here we define $\{h_{ii}^j\}_{j=1}^m = 1$. The noise level and interference caused by primary user to i^{th} secondary users in the j^{th} channel is σ_{ij}.

Meanwhile, the power allocation of each secondary user is limited by its own budget, that is, the total payment of "purchased" power spectral density does not exceed its endowed budget w_i. Here we describe as:

$$\sum_{j=1}^{m} p_j x_{ij} \leq w_i \quad . \tag{3}$$

2.2 Problem Formulation

As stated above, the optimization problem of power allocation is formulated as a competitive equilibrium problem: Each secondary user aims at maximizing its own utility $u_i(\boldsymbol{x}_i, \overline{\boldsymbol{x}}_i)$ under the local budget constraints in (3) and the interference constraints in (1).

So, this CE problem can be expressed as follows:

$$\begin{aligned}&\text{maximize } \ u_i(\boldsymbol{x}_i, \overline{\boldsymbol{x}}_i)\\&\text{subject to: } \ \sum_{j=1}^{m} p_j x_{ij} \le w_i,\, x_{ij} \ge 0 \ .\end{aligned} \tag{4}$$

Specifically, for each secondary user i, its strategy space can be expressed as:

$$\chi_i = \{\boldsymbol{x}_i \in \Omega_i, \sum_{i=1}^{n} b_{ij} x_{ij} < c_j \mid \Omega_i = (x_{i1}, x_{i2}, \cdots, x_{im})^T \in \Re^m,$$
$$\sum_{j=1}^{m} p_j x_{ij} \le w_i, x_{ij} \ge 0, \quad \forall i \in \boldsymbol{N}, \forall j \in \boldsymbol{M}\}$$

Consequently the entire strategy profile space becomes:

$$\chi = \{\boldsymbol{x}_i \in \chi_{i\in N} \Omega_i, \forall i \in \boldsymbol{N}\} \ . \tag{5}$$

The secondary users are ignorant of the spectrum mask of the primary user when making their power allocations. So to keep the interference level low, the primary user adjusts its price p_j until (1) holds.

The competitive equilibrium (CE) [12] of this model is the vector of prices $\boldsymbol{p}^*$ and the corresponding optimal power allocations of the secondary users $\boldsymbol{x}_i^*$. The optimal power allocation of spectrum is when the total interference level from secondary users reaches the spectrum mask, thus all spectrum opportunities have been utilized. Our goal is to find such a competitive equilibrium.

3 Existence and Uniqueness of Equilibrium Point

In this section we prove the existence and the uniqueness of CE for the proposed competitive market model. Competitive equilibrium (CE) is a set of prices and corresponding power allocations of secondary users. When the price $\boldsymbol{p}$ is fixed, CE can be simplified to NE which is a optimization problem easily to solve. So the existence and uniqueness of NE is proved when the price is fixed. The existence of NE follows Lemma 1.

Lemma 1. There exists an equilibrium point over the strategy profile space χ for the competitive market mode if the price is fixed and the weak-interference condition ($\boldsymbol{h}_j^i < 1$) is satisfied.

Proof: We use Theorem 4.4 in [13] to prove the existence of the equilibrium point. First, the strategy profile space χ in (5) is a closed and bounded set. Meanwhile it is convex which is derived as follows. Now we consider the complete necessary and sufficient conditions that characterize the optimal problem (4), they can be summarized as:

$$\begin{aligned} & w_i \cdot \nabla_{\boldsymbol{x}_i} u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i) \le (\nabla_{\boldsymbol{x}_i} u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i)^T \boldsymbol{x}_i) \cdot \boldsymbol{p}, \quad \forall i \\ & \sum_{j=1}^{m} p_j x_{ij} = w_i \quad, \forall i \\ & \sum_{i=1}^{n} b_{ij} x_{ij} = c_j, \quad \forall j \\ & x_{ij} \ge 0 \quad, \quad \forall i, j \quad . \end{aligned} \tag{6}$$

where $\nabla_{\boldsymbol{x}_i} u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i) \in \Re^n$ denotes any sub-gradient vector of $u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i)$ with respect to $\boldsymbol{x}_i$. The inequalities and equalities in (6) are all linear, except the first:

$$w_i \cdot \nabla_{\boldsymbol{x}_i} u(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i) \le (\nabla_{\boldsymbol{x}_i} u(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i)^T \boldsymbol{x}_i) \cdot \boldsymbol{p} \quad . \tag{7}$$

Now we use $\sum_i \boldsymbol{x}_i = \boldsymbol{x}_{tot}$ ($\boldsymbol{x}_{tot}$ is total power provided by primary user) to prove that (7) is actually a convex inequality. Let $\boldsymbol{h}_j^i = \boldsymbol{h}_i^{j^T} = \{h_{i1}^j, h_{i2}^j, \cdots, h_{ik}^j \mid k \in \boldsymbol{N}, k \ne i\}^{\boldsymbol{T}}$, the partial derivative of $u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i)$ to x_{ij} is:

$$(\nabla_{\boldsymbol{x}_i} u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i))_j = \frac{1}{\sigma_{ij} + \boldsymbol{h}_j^i (\sum_{k \ne i} x_{kj}) + x_{ij}}, \forall j \, . \tag{8}$$

So that: $\nabla_{\boldsymbol{x}_i} u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i)^T \boldsymbol{x}_i = \sum_{j=1}^{m} \frac{x_{ij}}{\sigma_{ij} + \boldsymbol{h}_j^i (\sum_{k \ne i} x_{kj}) + x_{ij}}$ and $\sum_{k \ne i} x_{kj} = (\boldsymbol{x}_{tot} - \boldsymbol{x}_i)_j$.

Thus:

$$(\nabla_{\boldsymbol{x}_i} u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i))_j = \frac{1}{\sigma_{ij} + \boldsymbol{h}_j^i \boldsymbol{x}_{tot} + (1 - \boldsymbol{h}_j^i) \boldsymbol{x}_i} \quad , \forall j$$

$$\nabla_{\boldsymbol{x}_i} u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i)^T \boldsymbol{x}_i = \sum_{j=1}^{m} \frac{x_{ij}}{\sigma_{ij} + \boldsymbol{h}_j^i \boldsymbol{x}_{tot} + (1 - \boldsymbol{h}_j^i) \boldsymbol{x}_i} \, .$$

Then, using the logarithmic transformation, we can rewrite the nonlinear inequality (7) as:

$$\begin{aligned} & \log(\sigma_{ij} + \boldsymbol{h}_j^i \boldsymbol{x}_{tot} + (1 - \boldsymbol{h}_j^i) \boldsymbol{x}_i) + \log(p_j) + \log(\sum_{j=1}^{m} \frac{x_{ij}}{\sigma_{ij} + \boldsymbol{h}_j^i \boldsymbol{x}_{tot} + (1 - \boldsymbol{h}_j^i) \boldsymbol{x}_i}) \, . \\ & \ge \log(w_i), \quad \forall ij \end{aligned} \tag{9}$$

We know the price $\boldsymbol{p}$ is fixed, so (9) is actually a convex inequality for $1 - \boldsymbol{h}_j^i \ge 0$ constantly. Therefore, under the weak-interference and fixed price condition, the strategy profile space χ is convex.

Second, the utility function u_i is continuous on the strategy profile space χ. Meanwhile, given a fixed $\bar{\boldsymbol{x}}_i$, u_i is strictly concave in $\boldsymbol{x}_i$, because $\nabla^2_{x_i x_i} u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i)$ is negative definite:

$$\nabla^2_{x_i x_i} u_i(\boldsymbol{x}_i, \bar{\boldsymbol{x}}_i) = diag\{-\frac{1}{(\sigma_{ij} + \boldsymbol{h}^i_j(\sum_{k\neq i} x_{kj}) + x_{ij})^2} \forall j\}.$$

Therefore, according to Theorem 4.4 in [13] there exists a equilibrium point NE. ■ To investigate the uniqueness of NE, we use the similar method in [14] and the uniqueness follows Lemma 2.

Lemma 2. The equilibrium point is unique over the strategy profile space χ for the competitive market mode if the price is fixed and the weak-interference condition is satisfied.

Proof: we know if $x_{ij}>0$, for all i and j:

$$(\nabla_{x_i^*} u(\boldsymbol{x}_i^*, \bar{\boldsymbol{x}}_i^*)^T \boldsymbol{x}_i^*) \cdot \boldsymbol{p} - w_i \cdot \nabla_{x_i^*} u(\boldsymbol{x}_i^*, \bar{\boldsymbol{x}}_i^*) = 0 \quad . \tag{10}$$

(i.e., every user only purchases most valuable power). Now we use (9) and (10) to derive:

$$\log(\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} + (1-\boldsymbol{h}^i_j)\boldsymbol{x}_i^*) + \log(p_j) + \log(\sum_{j=1}^{m} \frac{x_{ij}^*}{\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} + (1-\boldsymbol{h}^i_j)\boldsymbol{x}_i^*}) = \log(w_i), \forall ij, x_{ij}^* > 0$$

Assume there are two different NEs denoted by points $\boldsymbol{A}=\{\boldsymbol{a}_i\}_{i=1}^n$ and $\boldsymbol{B}=\{\boldsymbol{b}_i\}_{i=1}^n$, so that:

$$\Gamma(\boldsymbol{A}) = \log(\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} + (1-\boldsymbol{h}^i_j)\boldsymbol{a}_i) + \log(p_j) + \log(\sum_{j=1}^{m} \frac{a_{ij}}{\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} + (1-\boldsymbol{h}^i_j)\boldsymbol{a}_i}) - \log(w_i), \forall ij$$

$$\Gamma(\boldsymbol{B}) = \log(\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} + (1-\boldsymbol{h}^i_j)\boldsymbol{b}_i) + \log(p_j) + \log(\sum_{j=1}^{m} \frac{b_{ij}}{\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} + (1-\boldsymbol{h}^i_j)\boldsymbol{b}_i}) - \log(w_i), \forall ij \cdot$$

We have both $\Gamma(\boldsymbol{A})=0$ and $\Gamma(\boldsymbol{B})=0$, it follows that $\Gamma(\boldsymbol{A})-\Gamma(\boldsymbol{B})=0$. So we define $\Gamma(\boldsymbol{A})-\Gamma(\boldsymbol{B})$ as formula (11).

$$\begin{aligned}\Gamma(\boldsymbol{A})-\Gamma(\boldsymbol{B}) = {} & \log(\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} + (1-\boldsymbol{h}^i_j)\boldsymbol{a}_i) + \log(\sum_{j=1}^{m} \frac{a_{ij}}{\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} + (1-\boldsymbol{h}^i_j)\boldsymbol{a}_i}) \\ & - \log(\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} - (1-\boldsymbol{h}^i_j)\boldsymbol{b}_i) - \log(\sum_{j=1}^{m} \frac{b_{ij}}{\sigma_{ij} + \boldsymbol{h}^i_j \boldsymbol{x}_{tot} + (1-\boldsymbol{h}^i_j)\boldsymbol{b}_i}), \forall i, j\end{aligned} \; . \tag{11}$$

When $\boldsymbol{h}_j^i < 1$, (7) is a convex inequality, so the left of (9) is a monotonous function, that is: for distinct equilibrium points $\boldsymbol{A}$ and $\boldsymbol{B}$, we have $\Gamma(\boldsymbol{A})-\Gamma(\boldsymbol{B})\neq 0$, so (11) is not equal to 0. Therefore, the previous assumption is invalid.

Therefore, under the weak-interference and the fixed price condition, there exists a unique equilibrium point over the strategy profile space χ. ■

Lemma 3. Under the weak-interference condition, the price p can converge to the optimal p^*, so there exists a unique equilibrium point CE over the strategy profile space χ for the competitive market mode.

Proof: Now we prove the only optimal $\boldsymbol{p}^*$ which can satisfy (4) and (1). To meet the formula (1), we have the complementary condition:

$$0 \le \{\boldsymbol{p}^* \perp (c_j - \sum_{i=1}^{n} b_{ij}x_{ij})_{j=1}^{m}\} \ge 0 .$$

where the compact notation $0 \le a \perp b \ge 0$ meas ab=0, $a \ge 0$, and $b \ge 0$. We have:

$$\Phi(\boldsymbol{p}^*) = \sum_{j=1}^{m} p_j^*(c_j - \sum_{i=1}^{n} b_{ij}x_{ij}) \quad . \tag{12}$$

The strategy profile of $\boldsymbol{p}$ can be seen as a closed convex set and Φ is continuous, so price equilibrium $\boldsymbol{p}^*$ exists.

Supposed that $\boldsymbol{p_a}^*$ and $\boldsymbol{p_b}^*$ are two distinct equilibrium points, we can get $\Phi(\boldsymbol{p_a}^*) = \Phi(\boldsymbol{p_b}^*) = 0$. Let:

$$\begin{aligned}\Phi(\boldsymbol{p_a}^*)-\Phi(\boldsymbol{p_b}^*) &= \sum_{j=1}^{m} p_{aj}^*(c_j - \sum_{i=1}^{n} b_{ij}x_{ij}(p_{aj}^*)) - \sum_{j=1}^{m} p_{bj}^*(c_j - \sum_{i=1}^{n} b_{ij}x_{ij}(p_{bj}^*)) \\ &= \sum_{j=1}^{m}(p_{aj}^* - p_{bj}^*)c_j - \sum_{j=1}^{m}\sum_{i=1}^{n} b_{ij}[p_{aj}^*x_{ij}(p_{aj}^*) - p_{bj}^*x_{ij}(p_{bj}^*)] \\ &= \sum_{j=1}^{m}(p_{aj}^* - p_{bj}^*)c_j - \sum_{j=1}^{m}\sum_{i=1}^{n} b_{ij}[w_i - w_i] = \sum_{j=1}^{m}(p_{aj}^* - p_{bj}^*)c_j \quad .\end{aligned} \tag{13}$$

We know formula (13) is not equal to 0 constantly for different $\boldsymbol{p_a}^*$ and $\boldsymbol{p_b}^*$. It is contradicts with $\Phi(\boldsymbol{p_a}^*) = \Phi(\boldsymbol{p_b}^*)$. Therefore our hypothesis does not hold. There exists the only price equilibrium $\boldsymbol{p}^*$. ■

In the condition of $\boldsymbol{h}_j^i \le 1$, the price $\boldsymbol{p}$ can converge to the optimal $\boldsymbol{p}^*$, so there exists a unique equilibrium point CE over the strategy profile space χ for the competitive market mode.

4 Distributed Power Allocation Algorithm

In this section we design a distributed algorithm (FCPAA) with round robin rules through which all the secondary users' power allocation can converge to the unique CE proved in the previous section, while the interference temperature constraints of the primary system are still satisfied. At the same time, round robin rules are used to prevent secondary users from wasting opportunity, so FCPAA has a higher convergent speed.

From the previous section, it is obtained that when the price $\boldsymbol{p}$ is fixed, a CE problem can be predigested to a NE problem which can be found by water filling. Specifically, $\overline{\boldsymbol{x}}_i$ are the power allocations of other users, we derive water filling solution for the power allocation problem (4) according to literature [6] as follows:

$$x_{ij}^{*} = WF_{ij}(\overline{\boldsymbol{x}}_i; p_j) = (\frac{v_i}{p_j} - \sigma_{ij} - \sum_{k \neq i} h_{ik}^{j} x_{kj})^{+} \quad . \tag{14}$$

That is, x_{ij}^{*} is the NE solution.

Secondly, we adjust the price $\boldsymbol{p}$ and have the optimal $\boldsymbol{p}^{*}$ to make the process converges to a CE. Here we use *Tâtonnement* process [12], which is described as: if the total demand $\sum_i x_{ij}$ exceeds the supply c_j on channel j, then increase the price p_j, if the demand falls short of supply, then decrease it.

Finally, we use the round robin rules proposed in [15] for a distributed network to accelerate the convergence speed. The round robin rules are as follow: we assume the number of secondary users involved in the game is n, at the beginning each user in the game adjusts its strategy with probability of $1/n$; After a stage game if the utility of i^{th} user is $u_i^t = u_i^{t-1}$ at the stage t, then the user i exits the game, and update the number n=n-1.Therefore the rules will let the user who can not increase its utility exit the game in time, which avoid wasting opportunity and increase the convergent speed.

Here is the FCPAA to all secondary users for getting the unique CE, which is as follow:

Step 1: Set $\boldsymbol{p}^{(0)} \geq 0$, initialize iterations: t_2=0.

Step 2: Set $\boldsymbol{x}^{0}(\boldsymbol{p}^{(t_2)}) \geq 0$, initialize iterations: t_1=0.

Step 3: If $\boldsymbol{x}_i^{(t_1)}(\boldsymbol{p}^{(t_2)})$ meets the stop condition, stop, and the output is: $\boldsymbol{x}_i^{*}(\boldsymbol{p}^{(t_2)}) = \boldsymbol{x}_i^{(t_1)}(\boldsymbol{p}^{(t_2)})$.

Step 4: According to (13), all users (i=1,..,n) update in phase $\boldsymbol{x}_i^{(t_1+1)}(\boldsymbol{p}^{(t_2)})$ as follows:

$$\boldsymbol{x}_i^{(t_1+1)}(\boldsymbol{p}^{(t_2)}) = WF_i(\overline{\boldsymbol{x}}_i^{(t_1+1)}; \boldsymbol{p}^{(t_2)}) \quad \forall i = 1, \ldots, n$$

Step 5: Determine if utility $u_i^{t_1+1}$ of the user i is equal to $u_i^{t_1}$, if it is, update n=n-1; otherwise, return to Step 4.

Step 6: Increase t_1 to t_1+1, return step 3.

Step7: If $\boldsymbol{p}^{(t_2)}$ meet the stop condition, stop, then output: $\boldsymbol{p}^* = \boldsymbol{p}^{(t_2)}$, $\boldsymbol{x}^* = \boldsymbol{x}^*(\boldsymbol{p}^{(t_2)})$.

Step 8: Primary user gets power allocation of all secondary users, and updates the price:

$$\boldsymbol{p}_j^{t_2+1} = \boldsymbol{p}_j^{t_2} + f_j(y_j(\boldsymbol{p}^{t_2})) \quad . \tag{15}$$

Where $f_j(y_j(\boldsymbol{p}^{t_2})$ is the marginal pricing function: $f_j(y_j(\boldsymbol{p}^t) = \dfrac{\partial p_j(t)}{\partial t}$.

Step 9: Increase t_2 to t_2+1, return step 3.

5 Simulation Results

5.1 Parameter Setting

The simulation results are presented to validate our proposed algorithm (FCPAA). Let us consider a cognitive radio environment with one primary and six secondary users coexist in ten channels (i.e. n=6 and m=10). In order to meet weak interference condition, we let the interference coefficients h_{ik}^j are randomly generated from $[1, 1/(n-1)]$. Furthermore, the noise levels are symmetric, σ_j in each channel is independent and identically distributed with uniform distribution [0, 1]. For all secondary users, their budgets are $\boldsymbol{w}=[w_1,\ldots,w_n]=1$, the spectrum mask is $\boldsymbol{c}=[c_1,\ldots,c_m]=1$. The Nash equilibrium is solved by iterative water filling with $\boldsymbol{p}$=1. The number of iteration is 100.

5.2 Convergence of FCPAA

Figure 2 shows the convergence of FCPAA with 6 users' utilities. We can see that their utility functions approach the steady state after 40 iterations. Figure 3 demonstrates the price variations of 10 channels are all less than 10^{-2} after 40 iterations. That is said that they converge to the optimal solution when reach to 40 iterations.

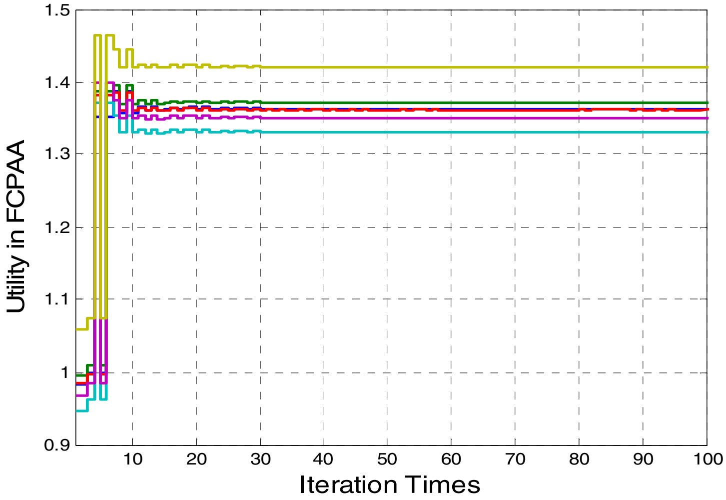

Fig. 2. Utility VS Iterations

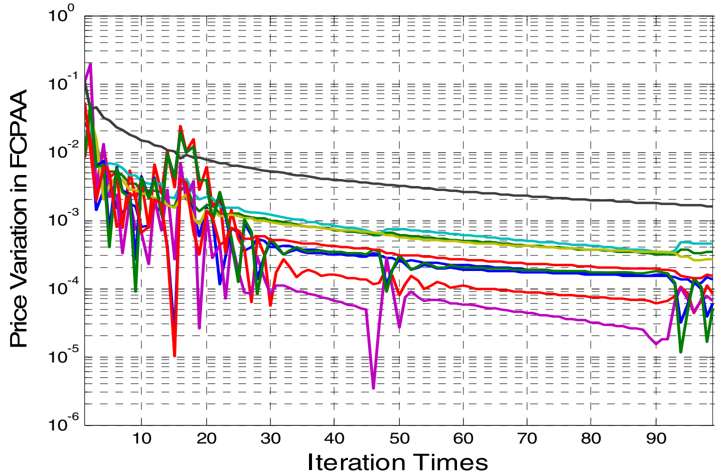

Fig. 3. Price VS Iterations

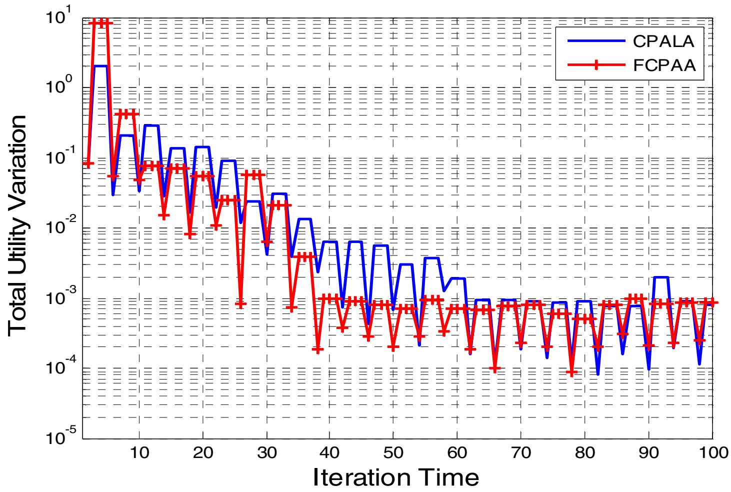

Fig. 4. Comparison of total utility of FCPAA and CPALA

Figure 4 compares the convergent speed of the proposed FCPAA and CPALA (competitive power allocation algorithm as LCP, proposed in literature [9]), we can see that FCPAA has reached convergence when the iteration time is 40. However, the total utility of CPALA converges after 60 iterations. That is said FCPAA converge faster than CPALA.

Figure 5 shows the relationship between noise levels with equilibrium prices. From the *Tâtonnement* processes, we know that it is necessary to reduce prices when noise level of each channel is higher than the spectrum mask. This simulation shows roughly in each channel (channel number is from 1 to 10) when the channel noise level is lower, the channel equilibrium price is higher as we expected.

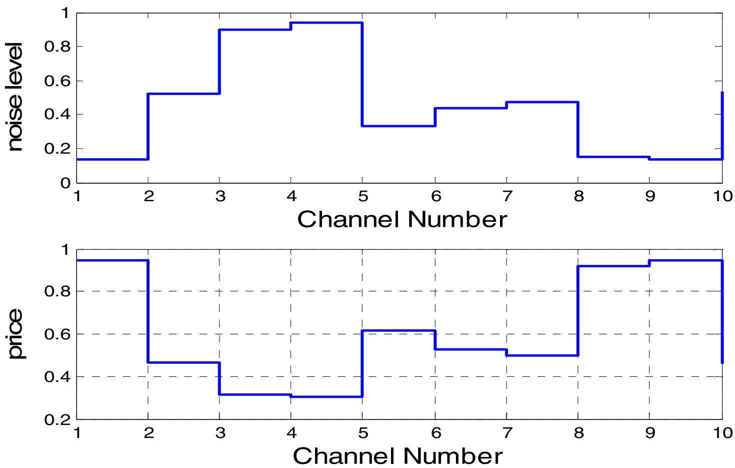

Fig. 5. Channel noise level VS price

5.3 Effectiveness of FCPAA

Figure 6 shows the total power allocation for each channel (or the channel load) in the proposed FCPAA and NE respectively. Here we know channel load of each channel (channel number is from 1 to 10) is lower than the spectrum mask in FCPAA. However, channel load of some channel in NE is higher than the spectrum mask, which affects the performance of the primary user.

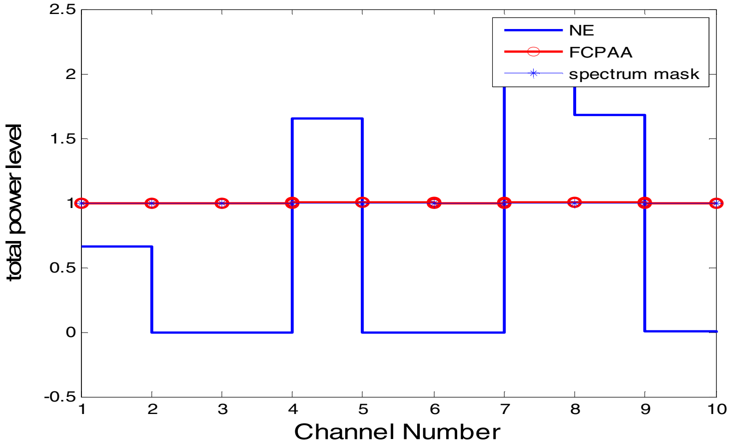

Fig. 6. Spectrum mask VS FCPAA VS NE

Figure 7 shows the convergence process of the system utility based on the proposed algorithm under different interfere threshold. From this figure we can see when interference threshold ***c***=1, the system utility converges after 40 iterations; When ***c***=3

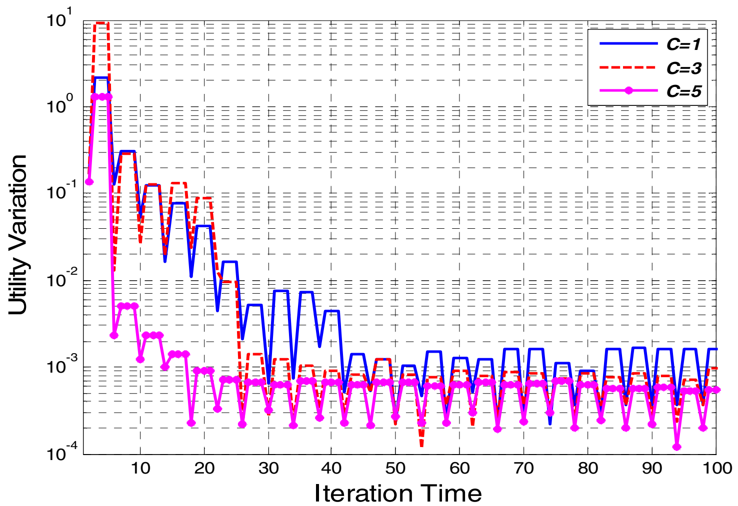

Fig. 7. System utility under different interfere thresholds VS Iterations

and ***c***=5, it converges after 25 and 15 iterations, respectively. That is said that the convergent speed of the proposed algorithm will be accelerated when the interference threshold increases. As shown in Table 1, when the interference threshold increases to a certain level, such as ***c***=5, here comes ***p***=0 due to the interference temperature constraint redundancy, then the algorithm is close to the performance of the algorithm based on the global optimal solution which don't consider the interference constraint.

Table 1. Comparison of system utility under different interfere thresholds and the global optimal solution

	Interfere thresholds			Global optimal solution
	c=1	***c***=3	***c***=5	
Total utility	8.181727	14.620557	19.030864	19.255959

6 Conclusion

In this paper, the multichannel power allocation problem in underlay cognitive radio networks is investigated. As in the literatures [9] and [10], we formulate this problem as a competitive market model and design a distributed fast convergent power allocation algorithm (FCPAA) with round robin rules. We prove the existence and the uniqueness of CE for the proposed FCPAA. Simulation results demonstrate that FCPAA is close to the global optimal solutions when the interference threshold increases and can converge to the CE solution more quickly than the algorithm in [9].

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