A Joint Optimal Algorithm Based on Relay Nodes Selection and Power Allocation

Zhao Shen[1,2,⋆], Daiyu Fu[2], and Yu Jin[2]

[1] Coll. Elect. Engn., Naval Univ. Engn.
Wuhan, China
[2] Department of Elect. & Informat. Engn., Huazhong Univ. of Sci. & Technol.,
Wuhan, China
{clingerlisa,fudaiyu}@gmail.com, shenzhao_0@163.com

Abstract. In this paper, we proposed a joint optimal algorithm based on relay nodes selection and power allocation, which utilizes the theory utility maximization modeling system by adopting convex optimization, and obtains the optimal power allocation between source node and relay nodes by applying the Karush-Kuhn-Tucker (KKT) condition. In addition, the relay nodes allocated with power unequal to zero are the selected relay nodes, therefore, the selection of relay nodes is realized. Moreover, the optimal number of relay nodes is discussed. Simulation results show that the proposed algorithm achieves a better performance than the traditional equal power allocation algorithm.

Keywords: Relay, convex optimization, nodes selection, power allocation.

1 Introduction

In the wireless communication network, forwarding by the assistance of relay can reduce the transmission path loss and improve overall system throughput, which makes the overall system performance significantly enhanced[1-2]. Therefore, the researches on relay-assisted transmission make a significant contribution to the study of the future wireless communication system.

At present, the research about relay communication is not mature. The correlative research focuses on the multi-user two-hop relay-assisted network with one source, one destination and signal or multiple relay. The two main problems that need to be solved are the selection of relay nodes and the power allocation[3-4]. So far, there are a variety of relay transmission strategies, such as Amplify-Forward (AF), Decode-Forward (DF), partial DF, etc[5-7]. In this paper, we utilize AF for the system modeling. While the system power is limited, the power allocation among the relay nodes transmitting simultaneity is a very important problem

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that affects the system performance greatly. Nowadays, the studies about the problem are mainly on the power allocation between relay node and source in signal relay network, and the power allocation among relay nodes in multiple relay networks. In the previous studies, while selecting signal relay node among multiple relay nodes, the power allocation between source and relay nodes are considered at the same time. However, while multiple relay nodes selected, the power allocation among relays nodes transmitting simultaneity is not considered. In order to reduce the complexity, the equal power allocation among relay nodes is taken into account in most of the existing researches.

By utilizing the Network Utility Maximization (NUM) idea and applying the convex optimization theory, a joint optimal algorithm based on relay nodes selection and power allocation is proposed in this paper. NUM is the method that utilizing the description of consumer income during accepting service in economics, defining the service satisfaction provided by the wireless communication network for the node (user) as network utility, making the utility maximization model for transmission and allocation problem in the wireless network, and applying the optimization tool for optimal system resource allocation. After applying NUM thinking to obtain the optimization modeling, we can utilize convex optimization to solve complicated optimization problem. Simulation results show that the proposed algorithm can acquire greater performance gain than the traditional equal power allocation algorithm.

The rest of this paper is organized as follows. Section 2 provides the system model. Section 3 presents the details of the Optimization algorithm based on relay nodes selection and power allocation. In section 4 the simulation results are provides. Finally, Section 5 concludes the paper.

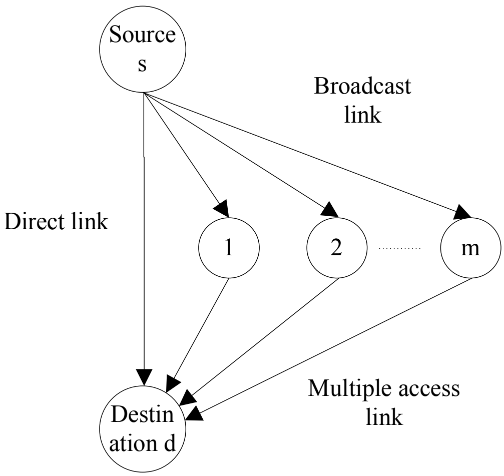

Fig. 1. System model

2 System Model

We consider a two-hop multi-relay assisted system as depicted in (Fig. 1), which consists of $m+2$ nodes: a source $(s), adestination$(d), and m relay $(1,2,\ldots,m)$. The source communicates with the destination by the assistance of relay. The transmission channel is orthogonal by time division. Therefore, the communication between the source and the destination consists of $m+1$ time slots. Let the time slot is equal to unit length. In addition, full channel state information is assumed. We suppose that all channels are Gaussian White noise the mean is zero, variance is $N0$. The power gain of the node i to node j is denoted by $|h_{i,j}|^2$. During the first time slot, the source transmits signal to destination and m relays. The signal at destination and m relays are given:

$$y_{s,d} = \sqrt{E_s} h_{s,d} + n_{s,d}$$

$$y_{s,i} = \sqrt{E_s} h_{s,i} + n_{s,i} \,, i = 1, 2, \ldots, m \tag{1}$$

Where E_s denotes transmitting power, $n_{s,d}$, $n_{s,i}$ are the noise.

3 Optimization Algorithm Based on Relay Nodes Selection and Power Allocation

For AF, m relay nodes receive the signal transmitted by source, normalize the receiving signal, and transmit a new signal $tildey_{s,i} = \gamma y_{s,i}$ to the destination, the normalization coefficient γ satisfies $\gamma = 1/\sqrt{E|y_{s,i}|^2}$.

After normalization, the receiving signal for m relay nodes turns into:

$$\begin{aligned} \tilde{y}_{s,i} &= \frac{\sqrt{E_s} h_{s,i}}{\sqrt{E|y_{s,i}|^2}} s + \frac{n_{s,i}}{\sqrt{E|y_{s,i}|^2}} \\ &= \frac{\sqrt{E_s} h_{s,i}}{\sqrt{E_s|h_{s,i}|^2 + N_0}} s + \frac{n_{s,i}}{\sqrt{E_s|h_{s,i}|^2 + N_0}} \end{aligned} \tag{2}$$

Where E_i denotes the transmitting power for ith relay node, and satisfies $E_s + \sum_{i=1}^{m} E_i \leq E$, E denotes the system total power. During the next m time slots, m relay nodes transmit the normalized signal to the destination with power E_i. in turn. Hence, the receiving signal at destination can be expressed as follows:

$$\begin{aligned} y_{i,d} &= \sqrt{E_i} h_{i,d} \tilde{y}_{s,i} + n_{i,d} \\ &= \sqrt{E_i} h_{i,d} \left(\frac{\sqrt{E_s} h_{s,i}}{\sqrt{E_s|h_{s,i}|^2 + N_0}} s + \frac{n_{s,i}}{\sqrt{E_s|h_{s,i}|^2 + N_0}} \right) + n_{i,d} \\ &= \sqrt{\frac{E_s E_i}{E_s|h_{s,i}|^2 + N_0}} h_{i,d} h_{s,i} s + \sqrt{\frac{E_s E_i}{E_s|h_{s,i}|^2 + N_0}} h_{i,d} n_{s,i} + n_{i,d} \end{aligned} \tag{3}$$

The signal that forwarded by m relay nodes at the destination can be normalized as follows:

$$\tilde{y}_{i,d} = \frac{y_{i,d}}{\sqrt{\frac{E_i|h_{i,d}|^2}{E_s|h_{s,i}|^2+N_0}+1}} \tag{4}$$

Adopting maximal ratio combining for the $m+1$ signals at the destination, the total SNR is given:

$$SNR = \frac{E_s|h_{s,d}|^2}{N_0} + \sum_{i=1}^{m} \frac{E_sE_i\frac{|h_{s,i}|^2}{N_0}\cdot\frac{|h_{i,d}|^2}{N_0}}{E_s\frac{|h_{s,i}|^2}{N_0}+E_i\frac{|h_{i,d}|^2}{N_0}+1} \tag{5}$$

Let $a_0 = \frac{|h_{s,d}|^2}{N_0}, a_i = \frac{|h_{s,i}|^2}{N_0}, b_i = \frac{|h_{i,d}|^2}{N_0}$, we get:

$$SNR = E_sa_0 + \sum_{i=1}^{m} \frac{E_sE_ia_ib_i}{E_sa_i+E_ib_i+1} \tag{6}$$

For the system as above, in order to minimize the outage probability, the simplest solution is maximizing the total SNR. Here we consider the total SNR as the system utility. Utilizing NUM to model the system, the problem becomes how to allocate power between the source and relay nodes for maximizing the total SNR when the total system power satisfied $E_s + \sum_{i=1}^{m} E_i \leq E$.Mathematically, this constrained optimization problem can be formulated as:

$$\begin{aligned} \text{maximize} \quad & E_sa_0 + \sum_{i=1}^{m} \frac{E_sE_ia_ib_i}{E_sa_i+E_ib_i+1} \\ \text{subject to} \quad & E_s + \sum_{i=1}^{m} E_i \leq E \\ & 0 \leq E_s \leq E_{s\,max} \\ & 0 \leq E_i \leq E_{i\,max} \end{aligned} \tag{7}$$

The above optimization problem can be distributed into two parts: the first part is object function, namely the total SNR, the object is to maximize the total SNR; the second part is constrained conditions including three parts: the first condition is the total system power constrain, the second condition is the power constrain at source, the third condition is the power constrain at each relay node. The variables of the above optimization problem (7) are E_s,$E_i$where $i = 1, \ldots, m$. From (7), we can know that there are coupling relationship among the variables of object function Es and Ei ,for $i = 1, \ldots, m$. Therefore, the solution process of the problem will be very complex. Then we will consider the problem from another point of view, because:

$$SNR = E_sa_0 + \sum_{i=1}^{m} \frac{E_sE_ia_ib_i}{E_sa_i+E_ib_i+1} = E_s\sum_{i=0}^{m} a_i - \sum_{i=1}^{m} \frac{E_s^2a_i^2+E_sa_i}{E_sa_i+E_ib_i+1}$$

Therefore, the optimization problem (7) can be expressed as:

$$\begin{aligned} \text{maximize} \quad & E_s \sum_{i=0}^{m} a_i - \sum_{i=1}^{m} \frac{E_s^2 a_i^2 + E_s a_i}{E_s a_i + E_i b_i + 1} \\ \text{subject to} \quad & E_s + \sum_{i=1}^{m} E_i \leq E \\ & 0 \leq E_s \leq E_{s\,max} \\ & 0 \leq E_i \leq E_{i\,max} \end{aligned}$$

If E_s is fixed, the first part of the object function will transmit into a constant. Hence, the above problem is equivalent to:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{m} \frac{E_s^2 a_i^2 + E_s a_i}{E_s a_i + E_i b_i + 1} \\ \text{subject to} \quad & \sum_{i=1}^{m} E_i \leq E - E_s \\ & 0 \leq E_i \leq E_{i\,max} \end{aligned} \tag{8}$$

The transformed problem (8) is equivalent to transmit signal with a fixed power at source, and allocate power among relay nodes. The optimal value must be satisfied $\sum_{i=1}^{m} E_i = E - E_s$, therefore, the variable of the optimization problem are E_i, for $i = 1, \ldots, m$. The optimal value must be satisfied $\sum_{i=1}^{m} E_i = E - E_s$, therefore, the variable of the optimization problem are Ei, for $i = 1, \ldots, m$. By utilizing convex theory, it is easy to verify that the object function is satisfied to the second order condition of convex function. In addition, the equality and inequality constrain conditions are affine. Therefore, the problem is a convex optimization problem. Moreover, the constrain conditions are linear, the problem can be transformed into a convex optimization problem in standard form as follows:

$$\begin{aligned} \text{minimize} \quad & f_0(x) = \sum_{i=1}^{m} \frac{E_s^2 a_i^2 + E_s a_i}{E_s a_i + E_i b_i + 1} \\ \text{subject to} \quad & g(x) = 1^T x - (E - E_s) = 0 \\ & h_i(x) = -x_i \leq 0 \\ & p_i(x) = x_i - E_{i\,max} \leq 0 \text{for} \quad i = 1, \ldots, m \end{aligned} \tag{9}$$

Hence, the solution process for the problem (9), is just the process of allocation power among the relay nodes. In addition, since $x_i = 0$ is satisfied to the constrain condition. For any relay node, if the transmitting power is 0, the relay node will not forward the signal. We can select the relay nodes that will forward signal when allocation the power. The convex optimization problem can be solved by utilizing Lagrange duality. Then we will introduce the solution process. The Lagrange function corresponding to problem (9) is:

$$L(x,\lambda,\mu,\nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i h_i(x) + \sum_{i=1}^{m} \mu_i p_i(x) + \nu g(x)$$

Because the object function and constrain condition of problem (9) are differentiable, and the constrain condition is satisfied to Slater condition, therefore, we can utilize the KKT condition to obtain the optimal value. The KKT condition for the problem is as follows.

$$h_i(x^*) = -x_i^* \le 0$$
$$\lambda_i^* \ge 0$$
$$p_i(x^*) = x_i^* - E_{i\,max} \le 0$$
$$\mu_i^* \ge 0$$
$$g(x^*) = 1^T x^* - (E - E_s) = 0 \tag{a}$$
$$\lambda_i^* h_i(x^*) = 0 \tag{b}$$
$$\mu_i^* p_i(x^*) = 0 \tag{c}$$
$$\triangledown f_0(x^*) + \sum_{i=1}^{m} \lambda_i^* \triangledown h_i(x^*) + \sum_{i=1}^{m} \mu_i^* \triangledown p_i(x^*) + \nu^* \triangledown g(x^*) = 0$$
$$\text{That is} \quad -\frac{(E_s^2 a_i^2 + E_s a_i) b_i}{(E_s a_i + x_i^* b_i + 1)^2} - \lambda_i^* + \mu_i^* + \nu^* = 0 \tag{d}$$

From (d), we can get $\mu_i^* = \frac{(E_s^2 a_i^2 + E_s a_i) b_i}{(E_s a_i + x_i^* b_i + 1)^2} + \lambda_i^* - \nu^*$.Take it into (c), we can obtain:

$$\left[\frac{(E_s^2 a_i^2 + E_s a_i) b_i}{(E_s a_i + x_i^* b_i + 1)^2} + \lambda_i^* - \nu^*\right] \cdot (x_i^* - E_{i\,max}) = 0 \tag{10}$$

In order to simplify the problem, from $\lambda_i^* x_i^* = 0$, we can tighten the constrain condition $x_i^* \ge 0$ to $x_i^* \ge 0$, then λ_i^* for $i = 1, \ldots, m$, and (10) can be simplified as:

$$\left[\frac{(E_s^2 a_i^2 + E_s a_i) b_i}{(E_s a_i + x_i^* b_i + 1)^2} - \nu^*\right] \cdot (x_i^* - E_{i\,max}) = 0 \tag{11}$$

$$\nu^* \le \frac{(E_s^2 a_i^2 + E_s a_i) b_i}{(E_s a_i + x_i^* b_i + 1)^2} \tag{12}$$

Formula (11) can be discussed in three parts as following:

$$x_i^* = \begin{cases} 0 &, \quad \nu^* \ge \frac{E_s a_i b_i}{E_s a_i + 1} \\ \sqrt{\frac{E_s^2 a_i^2 + E_s a_i}{\nu^* b_i}} - \frac{E_s a_i + 1}{b_i} &, \quad \frac{(E_s^2 a_i^2 + E_s a_i) b_i}{(E_s a_i + E_{i\,max} b_i + 1)^2} \le \nu^* \le \frac{E_s a_i b_i}{E_s a_i + 1} \\ E_{i\,max} &, \quad \nu^* \le \frac{(E_s^2 a_i^2 + E_s a_i) b_i}{(E_s a_i + E_{i\,max} b_i + 1)^2} \end{cases}$$

x_i^* can be written as:$x_i^* = min\{\max\{0, \sqrt{\frac{E_s^2 a_i^2 + E_s a_i}{\nu^* b_i}} - \frac{E_s a_i + 1}{b_i}\}, E_{i\,max}\}$,from $\sum_{i=1}^{m} E_i = E - E_s$, that is $\sum_{i=1}^{m} x_i = E - E_s$the optimal solution must satisfy:

$$\sum_{i=1}^{m} min\left\{max\left\{0, \sqrt{\frac{E_s^2 a_i^2 + E_s a_i}{\nu^* b_i}} - \frac{E_s a_i + 1}{b_i}\right\}, E_{i\,max}\right\} = E - E_s \qquad (13)$$

The left part of (13) is the piecewise monotone function of ν^*,which has two inflection points $\nu^* = \frac{(E_s^2 a_i^2 + E_s a_i) b_i}{(E_s a_i + E_{i\,max} b_i + 1)^2}$ and $\nu^* = \frac{E_s a_i b_i}{E_s a_i + 1}$, so we can get the only solution of the formula (13) easily using water filling.

4 Simulation Results

In the section, we compare the proposed algorithm with traditional equal power allocation algorithm, and the modulation is 16QAM. Figure 2 shows the bit error rate (BER) when $m = 10$ and $Es = 0.75E$. The result shows the proposed algorithm outperforms the equal power algorithm with $2\,dB$ gain.

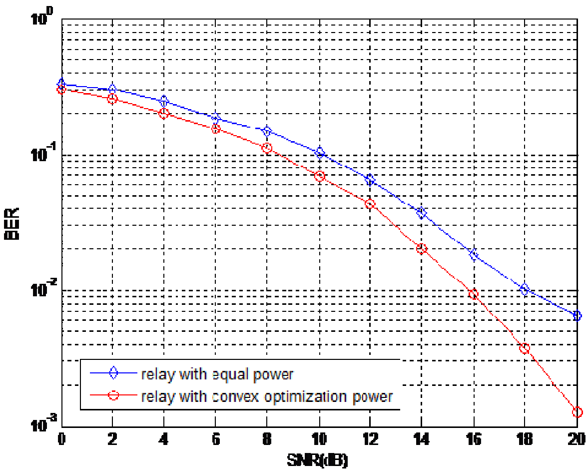

Fig. 2. Comparison of the proposed and equal power algorithm

The proposed algorithm fixes E_s, actually E_s/E with different values will affect the system performance.Fig.3 and Fig.4 show the BER when E_s/E has different values and $SN = 15dB$.

Figure 3 shows that when Es/E has small values, the performance will decrease sharply. When $SNR = 15dB$, $Es/E = 0.7$ is optimal. From figure 3 to figure 7, we can see the number of relay node should be 3.

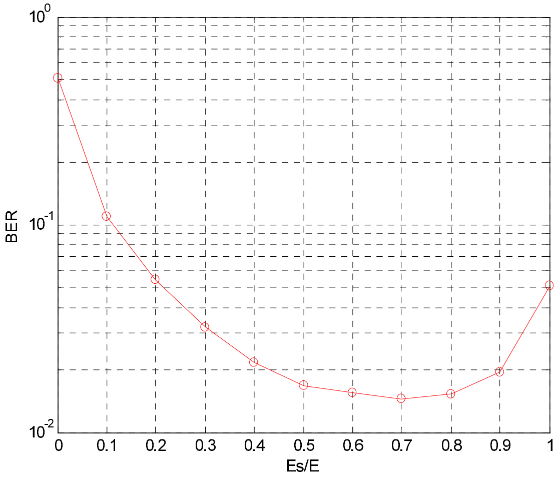

Fig. 3. Comparison of the proposed and equal power algorithm

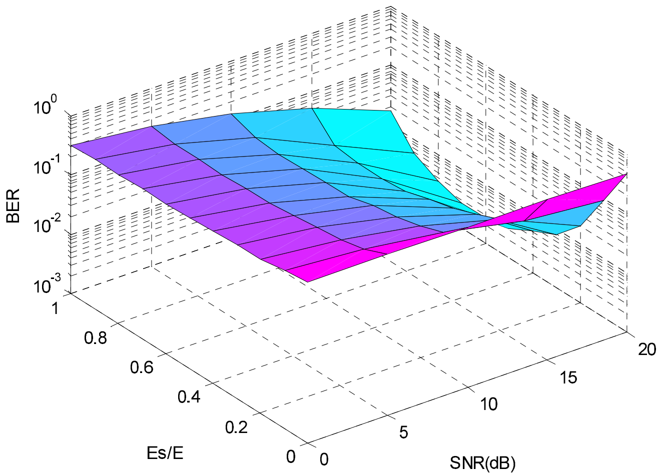

Fig. 4. The BER of the system with different E_s/E

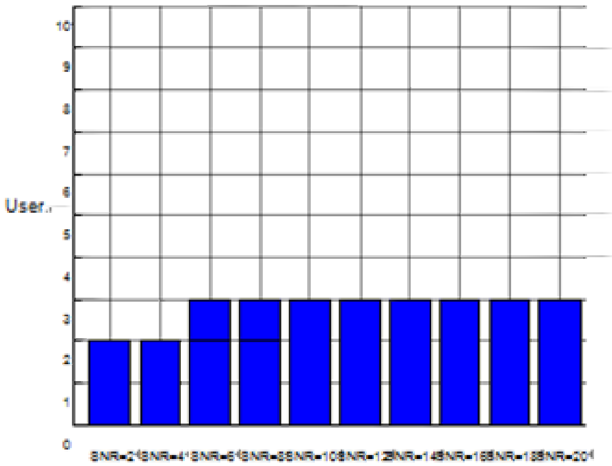

Fig. 5. The number of relay nodes when $E_s/E = 0.5$

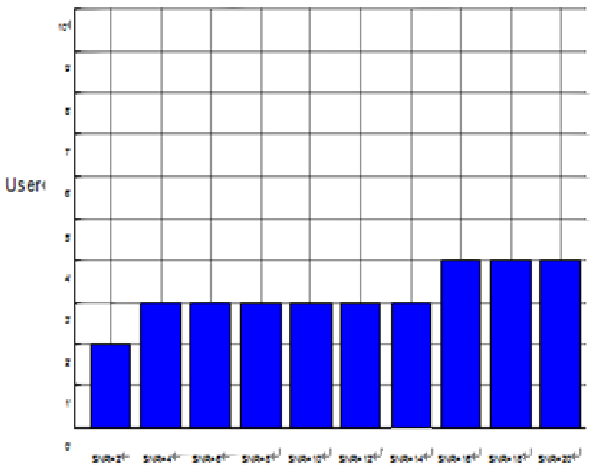

Fig. 6. The number of relay nodes when $E_s/E = 0.3$

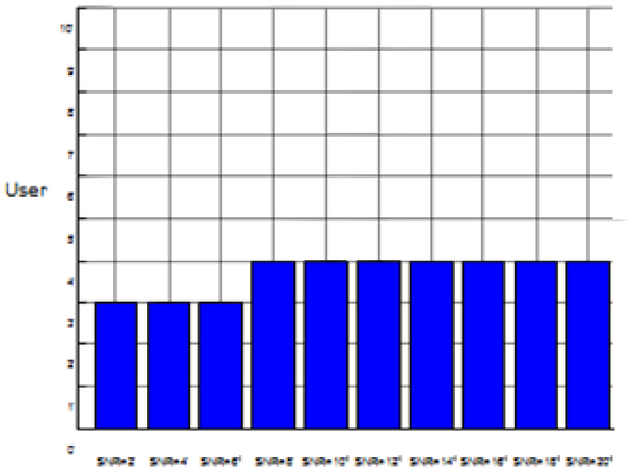

Fig. 7. The number of relay nodes when $E_s/E = 0.1$

5 Conclusion

In this paper, we propose a joint optimal algorithm based on relay nodes selection and power allocation, which utilizes the theory utility maximization modeling system by adopting convex optimization, and obtains the optimal power allocation between source node and relay nodes by applying KKT condition. Moreover, the relay nodes allocated with power unequal to zero are the selected relay nodes, therefore, the selection of relay nodes is realized. Simulation results show the proposed algorithm outperforms the traditional equal power allocation algorithm, and the number of the relay nodes should be 3, which is the optimal.

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