Fast Blind Spectrum Sensing Method Based on Multi-stage Wiener Filter

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Abstract. Spectrum sensing is the key problem for cognitive radio systems. A fast blind sensing method on Multi-Stage Wiener Filter (MSWF) of the received signals is proposed to sense the available spectrum for the cognitive users with the help of the multiple antennas at the receiver of the cognitive users. The greatest advantage of the new method is that it requires no information of the noise power and without any eign-decomposition (or SVD). Both the simulation and the analytical results demonstrate that the proposed method is effective, and almost the same performance compare with the eigen-value based methods.

Keywords: Spectrum sensing, subspace projection, cognitive radio, random matrix theory.

1 Introduction

Cognitive radio (CR) [1] has recently emerged as a promising technology to increase the spectrum utilization in wireless communications. In a CR network, secondary users (SU) continuously sense the spectral environment, reliably detect weak primary signals over a targeted wide frequency band, and adapt transmission parameters (such as the transmitting power, modulation and coding scheme, carrier frequency, etc.) to opportunistically use the available spectrum. The typical sensing methods include the energy detector, the matched filter, the cyclostationarity feature detection, and so on.

The typical sensing methods required the knowledge of noise power, License Users' (LU) waveform or known patterns and signal cyclostationary feature. All the above methods need a subjectively pre-defined threshold, which affects the robustness of the methods.

Recently, some blind sensing algorithms are derived from the eigen-values of the covariance matrix. Among them, the detectors based on the sample covariance matrix, including the MME detector [2], MET detector [3], the information theoretic detector [4], [5], and DMM detector [6], have been recently proposed. All of them work well in the case of noise uncertainty, and can even perform better than the ideal ED (with perfect noise power estimate) when the detected signals are highly correlated.

P. Ren et al. (Eds.): WICON 2011, LNICST 98, pp. 243–253, 2012.

However, these methods suffer from the heavily computational load of the eigen-decomposition, which may be unacceptable in real-time signal processing and large-dimension array system. To deal with this problem, a fast blind sensing method based on Multi-Stage Wiener Filter is proposed, which requires no information of the noise power and without any eigen-value decomposition (EVD). We also derive the threshold of our detector based on the random matrix theory.

2 Blind Spectrum Sensing Based on Multi-stage Wiener Filter

2.1 Array Model and Blind Sensing Algorithm Based on EVD

Multi-antenna is widely used in wireless communication due to its ability in improving the performance of the system. Here the multi-antenna is also served for sensing the LU signal.

Assume a uniform linear array is employed at the CR receiver side with M antennas. The array output data are

$$\boldsymbol{X}(k) = \boldsymbol{A}(\theta)\boldsymbol{S}(k) + \boldsymbol{N}(k) = \sum_{i=1}^{P} \boldsymbol{a}(\theta_i)\boldsymbol{s}_i(k) + \boldsymbol{N}(k) \tag{1}$$

where $\boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1)\ \boldsymbol{a}(\theta_2) \cdots \boldsymbol{a}(\theta_P)]$ is the steering vector of the array, $\boldsymbol{S}(k) = [\boldsymbol{s}_1(k)\ \boldsymbol{s}_2(k) \cdots \boldsymbol{s}_P(k)]^{\mathrm{H}}$ is the signal-vector, and $\boldsymbol{N}(k) = [\boldsymbol{n}_1(k)\ \boldsymbol{n}_2(K) \cdots \boldsymbol{n}_M(k)]$ is the noise-vector. The covariance Matrix of output data is

$$\boldsymbol{R}_{XX} = E[\boldsymbol{X}(k)\boldsymbol{X}^{\mathrm{H}}(k)] = \boldsymbol{A}(\theta)\boldsymbol{R}_{SS}\boldsymbol{A}(\theta)^{\mathrm{H}} + \boldsymbol{R}_{NN} \tag{2}$$

where $\boldsymbol{R}_{SS} = E[\boldsymbol{S}(k)\boldsymbol{S}^{\mathrm{H}}(k)]$ is the covariance of the signals, and H denotes the Hermitian Transpose. $\boldsymbol{R}_{NN} = E[\boldsymbol{N}(k)\boldsymbol{N}^{\mathrm{H}}(k)]$ is the noise covariance equal to $\sigma_{\mathrm{n}}^2\boldsymbol{I}$ in Gaussian white noise. Here, we only consider the Gaussian white noise situation.

The EVD based methods are based on eigen-decomposition of the covariance matrix R, and the signal and noise subspace are obtained from the eigenvectors of eigen-values, $\boldsymbol{U}_S = [\boldsymbol{u}_1\ \boldsymbol{u}_2\ \cdots \boldsymbol{u}_P\ ]$ and $\boldsymbol{U}_N = [\boldsymbol{u}_{P+1}\ \boldsymbol{u}_{P+2}\ \cdots \boldsymbol{u}_M\ ]$, where $\boldsymbol{u}_i\ (i = 1, \cdots, M)$ is the eigenvectors of $\boldsymbol{R}_{XX}$. So $\boldsymbol{R}_{XX}$ can be expressed as

$$\boldsymbol{R}_{XX} = \boldsymbol{U}_S\boldsymbol{\Sigma}_S\boldsymbol{U}_S^{\mathrm{H}} + \boldsymbol{U}_N\Sigma_N\boldsymbol{U}_N^{\mathrm{H}} = \sum_{i=1}^{M} \lambda_i\boldsymbol{u}_i\boldsymbol{u}_i^{\mathrm{H}} \tag{3}$$

where $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_p > \lambda_{P+1} = \lambda_{P+2} = \cdots = \lambda_M = \sigma_n^2$,if there are P LUs' signal impinging on the uniform linear array.

Once the eigen-values are obtained, some blind sensing algorithms can be derived[2], [3], [10].

A) MME detector: the ratio of largest eigen-value compared with the smallest eigen-value.

$$\mathrm{T}_{\mathrm{MME}} = \frac{\lambda_1}{\lambda_M} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \gamma_{MME} \tag{4}$$

B) MET detector: the ratio of largest eigen-value compared with the mean of all eigen-values.

$$\mathrm{T}_{\mathrm{MME}} = \frac{M\lambda_1}{Trace(\boldsymbol{R}_{XX})} \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \gamma_{MET} \tag{5}$$

C) DMM detector: the difference of largest eigen-value with the smallest eigen-value.

$$\mathrm{T}_{\mathrm{DMM}} = \lambda_1 - \lambda_M \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}} \gamma_{DMM} \tag{6}$$

In noise only case, the covariance matrix R_{xx} is a Wishart random matrix. We can use the random matrix theory (RMT) to approximate the true CDF of detector based on largest and smallest eigen-values.

2.2 Fast Blind Sensing Algorithm Based on MSWF

The eigen-value based sensing methods for spectrum sensing has outstanding performance compared with other algorithms. However, these methods suffer from the heavily computational load of the eigen-decomposition. To deal with this problem, we use the Multi-Stage Wiener Filter technology to develop the fast and blind sensing algorithm. The multi-stage wiener filter (MSWF) technique is based on orthogonal projections, which was successfully used in adaptive beam-forming, adaptive reduced-rank interference suppression and space-time adaptive processing (STAP) . In our early work, it was adopted to develop the low complexity bearing estimation algorithms [11] successfully.

The MSWF was proposed by Goldstein et al [7] to find an approximate solution to the Wiener-Hopf equation which does not need the inverse of the covariance matrix. The MSWF algorithm is given by the following set of recursions:

Step1. Initialization: $\boldsymbol{d}_0(k)$ and X0(k) = X(k)

Step2. Forward recursion: For $i = 1, 2, \cdots, M$

$$\boldsymbol{h}_i = E[\boldsymbol{d}_{i-1}^*(k)\boldsymbol{X}_{i-1}(k)] \Big/ \sqrt{\left|E\left[\boldsymbol{d}_{i-1}^*(k)\boldsymbol{X}_{i-1}(k)\right]\right|^2} \tag{7}$$

$$\boldsymbol{d}_i(k) = \boldsymbol{h}_i^{\mathrm{H}} \boldsymbol{X}_{i-1}(k) \tag{8}$$

$$\boldsymbol{B} = null\{\boldsymbol{h}_i\} = \boldsymbol{I}_M - \boldsymbol{h}_i \boldsymbol{h}_i^H \tag{9}$$

$$\boldsymbol{X}_i(k) = \boldsymbol{B}_i^{\mathrm{H}} \boldsymbol{X}_{i-1}(k) \tag{10}$$

END FOR

Step3.Backward recursion: $\boldsymbol{e}_M(k)=\boldsymbol{d}_M(k)$

For $i=M,M-1,\cdots,1$

$$w_i=E[\boldsymbol{d}_{i-1}^*(k)\boldsymbol{e}_i(k)]\big/E[|\boldsymbol{e}_i(k)|^2] \tag{11}$$

$$\boldsymbol{e}_{i-1}(k)=\boldsymbol{d}_{i-1}(k)-w_i^*\boldsymbol{e}_i(k) \tag{12}$$

END FOR

In this paper, we consider using the output data of an arbitrary array element as the reference signal which are easily obtained.

$$\boldsymbol{d}_0(k)=\boldsymbol{x}_i(k)=\boldsymbol{e}_i^T\boldsymbol{A}(\theta)\boldsymbol{S}(k)+\boldsymbol{n}_i(k) \tag{13}$$

where $\boldsymbol{e}_i=[\underbrace{0\,0\cdots1}_{i}\,0\cdots0]^T$.

The covariance matrix can be tri-diagonalized by match filters. That is

$$\boldsymbol{R}_d=\boldsymbol{H}^H\boldsymbol{R}_{X_0}\boldsymbol{H}=\begin{bmatrix}\sigma_{d_1}^2 & \delta_2^* & 0 & \cdots & 0\\ \delta_2 & \sigma_{d_2}^2 & \delta_3^* & \ddots & \vdots\\ 0 & \delta_3 & \sigma_{d_3}^2 & \ddots & 0\\ \vdots & \ddots & \ddots & \ddots & \delta_M^*\\ 0 & \cdots & 0 & \delta_M & \sigma_{d_M}^2\end{bmatrix} \tag{14}$$

where $\boldsymbol{H}=[\boldsymbol{h}_1\ \boldsymbol{h}_2\ \cdots\boldsymbol{h}_M]$ is the matrix of match filters, $\sigma_{d_i}^2=E[\boldsymbol{d}_i(k)\boldsymbol{d}_i^*(k)]=\boldsymbol{h}_i^H\boldsymbol{R}_{X_i}\boldsymbol{h}_i$ represents the covariance of reference signal and $\delta_{i+1}=\sqrt{|E[\boldsymbol{X}_i(k)\boldsymbol{d}_i^*(k)]|^2}=\|\boldsymbol{r}_{X_id_i}\|_2$ is the module of ith cross-correlation vector. $\sigma_{d_i}^2$ is similar with the eigen-value λ_i. According to the $\sigma_{d_i}^2$, we can derived our blind sensing algorithm.

$$\mathrm{T}_1=\frac{\max\{\sigma_{d_i}^2\mid i=1,2,\cdots M\}}{\frac{1}{\mathrm{M}-1}\left(\mathrm{trace}(\boldsymbol{R}_d)-\max\{\sigma_{d_i}^2\mid i=1,2,\cdots M\}\right)}\ \underset{\mathrm{H}_0}{\overset{\mathrm{H}_1}{\gtrless}}\ \gamma \tag{15}$$

But in low SNR, the difference between $\sigma_{d_i}^2$ and the eigen-values λ_i is too large to be used for the detector. And also the detection threshold cannot be derived from the conclusion of random matrix theory. So get the estimation the largest eigen-value from tri-diagonal matrix $\boldsymbol{R}_d$ is the key problem. Using the conclusion in [8], we can find the bound of largest eigen-value of tri-diagonal matrix $\boldsymbol{R}_d$, which has the same eigen-values of Rxx .

Theorem 1. Given a tri-diagonal matrix $A_{n\times n}$, the non-zero elements of A are all great than zero.

$$A=\begin{bmatrix} a_{11} & b_1 & 0 & \ddots & 0 \\ c_2 & a_{22} & b_2 & \ddots & \vdots \\ 0 & c_3 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & b_{n-1} \\ 0 & \cdots & 0 & c_n & a_{nn} \end{bmatrix} \tag{16}$$

Let $c_1 = b_n = 0$ and

(1) $a_i^{(m)} = c_i^{(m)} + a_{ii}^{(m)} + b_i^{(m)}, i = 1,2,\cdots n$

(2) $r^{(m)} = \min_i a_i^{(m)}, R^{(m)} = \max_i a_i^{(m)}, m = 1,2,\cdots$

(3) $D_m = diag\left(a_1^{(m)}, a_2^{(m)}, \cdots, a_n^{(m)}\right), m = 1,2,\cdots$

(4) $A_{m+1} = D_m^{-1} A_m D_m, m = 1,2,\cdots$ is the matrices sequence, which have the same eigen-values.

$$A_m=\begin{bmatrix} a_{11}^{(m)} & b_1^{(m)} & 0 & \ddots & 0 \\ c_2^{(m)} & a_{22}^{(m)} & b_2^{(m)} & \ddots & \vdots \\ 0 & c_3^{(m)} & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & b_{n-1}^{(m)} \\ 0 & \cdots & 0 & c_n^{(m)} & a_{nn}^{(m)} \end{bmatrix} \tag{17}$$

If we set the stop condictions

$$\left|R^{(m)} - r^{(m)}\right| < \varepsilon \tag{18}$$

So the largest eigen-value of A can be estimated iteratively [8].

$$\lambda_{\max}(A) = R^{(m)} \tag{19}$$

The procedure of blind sensing based on MSWF is as followed:

Step1: Use the MSWF forward decomposition to compute the match filters $\boldsymbol{H} = [\boldsymbol{h}_1\ \boldsymbol{h}_2 \cdots \boldsymbol{h}_M]$

Step2: Use equation (20) to get the tri-diagonal matrix $\boldsymbol{R}_d$.

$$\begin{cases} \sigma_{d_i}^2 = E\left[\boldsymbol{d}_i(k)\boldsymbol{d}_i^*(k)\right], \quad i = 1,2,\cdots M \\ \delta_{i+1} = \sqrt{\left|E\left[\boldsymbol{X}_i(k)\boldsymbol{d}_i^*(k)\right]\right|^2} = \left\|\boldsymbol{r}_{X_i d_i}\right\|_2 \end{cases} \tag{20}$$

Step3: According to Theorem 1, get the estimation of the largest eigen-value of tri-diagonal matrix $\boldsymbol{R}_d$.
Step4: Compute the detection statistics and compare with the threshold from equation (21).

$$T_2 = \frac{R^{(m)}}{\frac{1}{M-1}\left(\text{trace}(\boldsymbol{R}_{XX}) - R^{(m)}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{21}$$

Remark 1. To estimate the largest eigen-value, our method merely requires $O(M^2N)$ complex products operations. However the EVD-based algorithms require $O(M^2N)$ complex products operations of estimating the covariance matrix and $O(M^3)$ decomposition operations. Thus the computation complexity of our method is significantly reduced.

2.3 Theoretic Analysis and the Threshold Determination

Practically, the statistical correlation matrix $\boldsymbol{R}_{XX}$ is estimated through a sample covariance matrix. Introduce N as the number of samples collected by each receiver during the sensing period. The $M \times M$ sample covariance matrix $\boldsymbol{R}_{XX}(\boldsymbol{N})$ is then defined as

$$\boldsymbol{R}_{XX}(N) = \frac{1}{N}\sum_{k=1}^{N} \boldsymbol{X}(k)\boldsymbol{X}^{\text{H}}(k) \tag{22}$$

Although $\boldsymbol{R}_{XX}(\boldsymbol{N})$ converges to $\boldsymbol{R}_{XX}$ as N tends to infinity, for finite N, its properties depart from those of the statistical covariance matrix, then the eigen-values of $\boldsymbol{R}_{XX}$ have the property that $\lambda_1 > \lambda_2 > \cdots > \lambda_p > \lambda_{P+1} > \lambda_{P+2} > \cdots > \lambda_M$. At low SNR, the performance of a sensing algorithm is very sensitive to the threshold. Since we have no information of the signal (actually we even do not know if there is signal or not) and noise, it is difficult to set the threshold based on the P_d. Hence, usually we choose the threshold based on the P_{fa}. We need to determine the behavior of eigen-values under null hypothesis, i.e., H_0.
In noise only case, the distribution of the largest eigen-value [9], [10] is described in the following Lemmas.

Lemma 1. Assume that the noise is complex.

Let $A(N) = \frac{N}{\sigma_n^2} R_{XX}(N)$, $\mu = \left(\sqrt{N} + \sqrt{M}\right)^2$ and $\nu = \left(\sqrt{N} + \sqrt{M}\right)\left(\frac{1}{\sqrt{M}} + \frac{1}{\sqrt{N}}\right)^{1/3}$.

Assume that $\lim_{N\to\infty} \frac{M}{N} = \rho (0 < \rho < 1)$. Then $\frac{\lambda_{\max}(A(N)) - \mu}{\nu}$ converges (with probability one) to the Tracy-Widom distribution of order 2 (TW2) [9]. For the analytical formula of TW_2 refer to [9], and for the tables of its CDF refer to [10].

According to the lemmas, we can derive the threshold of our algorithm based on false alarm probability.

$$
\begin{aligned}
\mathrm{P}_{\mathrm{f}\alpha} &= \Pr\left(\frac{\lambda_{\max}\left(\boldsymbol{R}_{XX}\right)}{\frac{1}{\mathrm{M}-1}\left(\operatorname{trace}\left(\boldsymbol{R}_{XX}\right)-\lambda_{\max}\left(\boldsymbol{R}_{XX}\right)\right)}>\gamma\right) \\
&= \Pr\left(\lambda_{\max}>\frac{\gamma \operatorname{trace}\left(\boldsymbol{R}_{XX}\right)}{\mathrm{M}-1+\gamma}\right) \\
&= \Pr\left(\frac{\sigma_{\mathrm{n}}^2}{\mathrm{N}}\lambda_{\max}\left(\mathrm{A(N)}\right)>\frac{\gamma \operatorname{trace}\left(\boldsymbol{R}_{XX}\right)}{\mathrm{M}-1+\gamma}\right) \\
&= \Pr\left(\frac{\lambda_{\max}\left(\mathrm{A(N)}\right)-\mu}{\nu}>\frac{\gamma \mathrm{NM}/\left(\mathrm{M}-1+\gamma\right)-\mu}{\nu}\right) \\
&= 1-\mathrm{F}_2\left(\frac{\gamma \mathrm{NM}/\left(\mathrm{M}-1+\gamma\right)-\mu}{\nu}\right)
\end{aligned}
\tag{23}
$$

where $\operatorname{trace}\left(\boldsymbol{R}_{XX}\right)=M\sigma_n^2$ is used.

So the threshold is

$$
\gamma=\frac{\left(\mathrm{M}-1\right)\left(\nu \mathrm{F}_2^{-1}\left(1-\mathrm{P}_{\mathrm{f}\alpha}\right)+\mu\right)}{\mathrm{MN}-\nu \mathrm{F}_2^{-1}\left(1-\mathrm{P}_{\mathrm{f}\alpha}\right)-\mu}
\tag{24}
$$

It can be seen that the threshold has nothing to do with the knowledge of noise power σ_{n}^2 and signal information, therefore our method is belong to blind sensing algorithm.

3 Simulation Results

To demonstrate the performance of the proposed methods, simulations are provided. A Uniform Linear Array (ULA) is used here with M = 8 sensors and half wavelength inter-element spacing. The QPSK signals are used in the simulations. The number of snapshots is 2048. In the following, all the results are averaged over 100000 Monte Carlo realizations.

Fig.1 shows our method, the T1 and T2 detectors' pdf of the received signal under H_1(SNR=-14dB) and H_0. From Fig.1 we can see that the pdf of T2 based on $R^{(m)}$ is more dispersed than $\sigma_{d_i}^2$ based T1 method, which demonstrates that our algorithm T2 works well at low SNR.

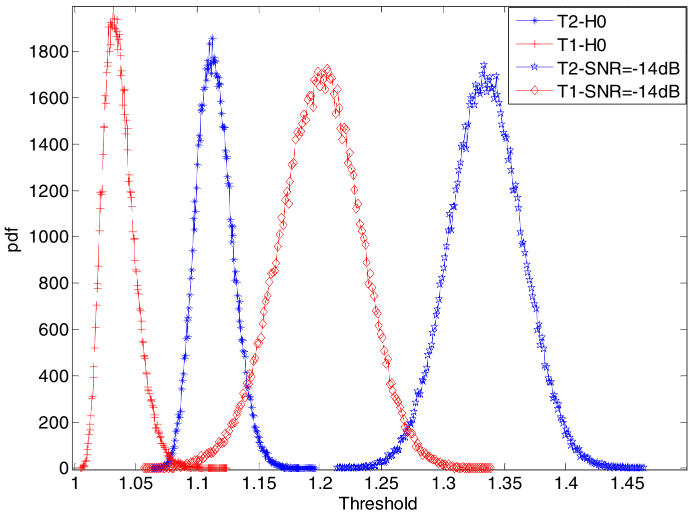


Fig. 1. Pdf of our method (T1 and T2 detectors)under H_0 and H_1

In fig.2, the ROC curves of of our method, MME [2] and DMM [6] method. are described at SNR=-16dB. It can be seen that our method (T2 detector) has outstanding performance compared with the eigen-value based algorithms.

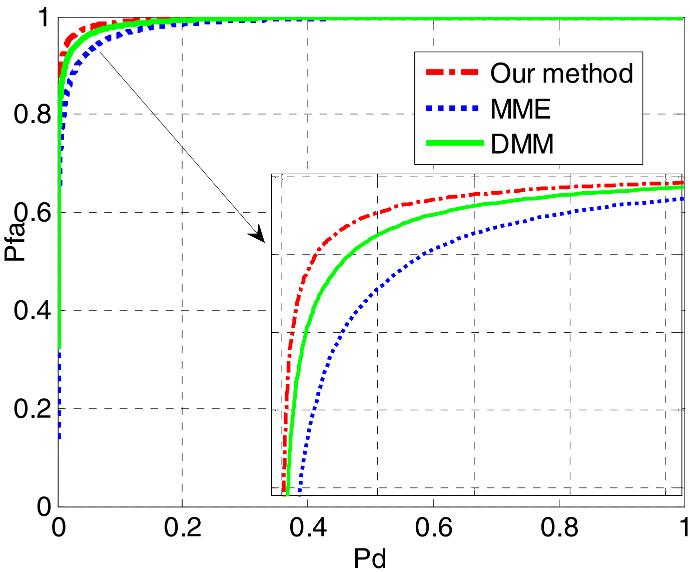
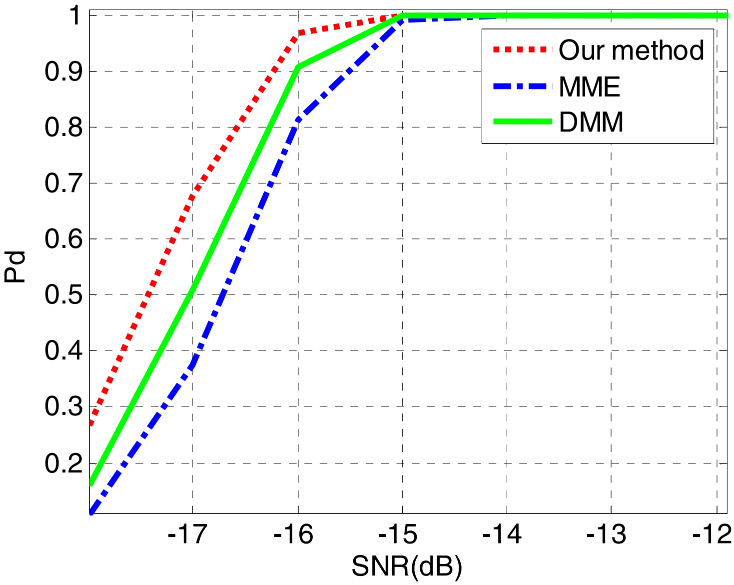
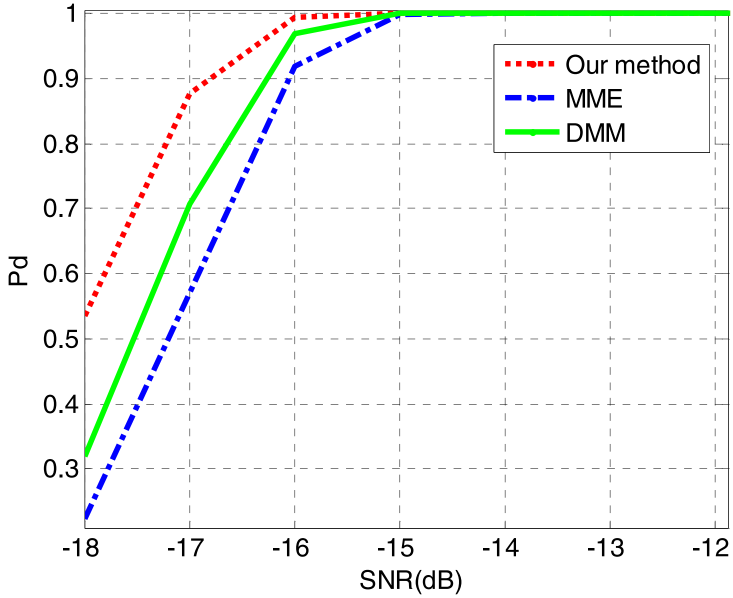


Fig. 2. ROC performance comparison (N=2048)



(a) Pfa=0.001



(b) Pfa=0.01

Fig. 3. Performance comparison of our method, MME and DMM method (N=2048)

Table 1. The threshold of three detectors

Pfa	Our Method	DMM	MME
0.001	**1.1766**	**0.2723**	**1.3098**
0.01	**1.1597**	**0.2581**	**1.2936**

Fig.3 compares the performance of three methods. The given P_{fa} is 10^{-3} and 10^{-2} respectively. One can see that, indeed, our method, DMM and CMME methods are well done at low SNR. For example, when SNR=-16dB, $P_{fa}=10^{-3}$, P_d of our method, DMM method, and MME method are about 96.83%, 90.86%, 81.35% respectively. It is clear that our method is the best compared with the other two methods, and also the computation complexity of our method is greatly reduced.

Table3 gives the detection threshold of three methods, which are computed according to the equation (24) and the threshold equations in [2], [6].

4 Conclusion

This paper proposes a spectrum sensing method based on MSWF technology. The employed test statistic requires no information of the noise power and without any eign-decomposition (or SVD). The simulation results demonstrate its effectiveness and robustness.

Acknowledgments. This work was supported in part by the Natural Science Research Program of Shaanxi's Provincial Education Department (11JK0925), the Nature Science Foundation of China (60602053), Program for New Century Excellent Talents in University (NCET-08-0891), the Natural Science Foundation of Shaanxi Province (2010JQ80241),the Natural Science Foundation of Hubei Province (2009CDB308) and the Fund from Education Department of Shaanxi Government.

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