

# A Novel Transmission Scheme Based on Sine/Chirp Hybrid Carriers

Qiming Zhao<sup>1,2</sup>, Qinyu Zhang<sup>1</sup>, and Naitong Zhang<sup>1,2</sup>

<sup>1</sup> Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen 518055, China

<sup>2</sup> Communication Research Center, Harbin Institute of Technology, Harbin 150001, China  
zqmhit@gmail.com, {zqy, ntzhang}@hit.edu.cn

**Abstract.** In this paper, a novel hybrid carrier transmission scheme is presented, which adds a group of chirp-carrier users to tradition systems sine-carrier based, sharing same frequency and time resources. The issues of interferences between users adopting different type carriers are investigated, and interference suppression approaches based on the fractional Fourier transform (FRFT) are proposed.

**Keywords:** hybrid carriers, chirp signals, fractional Fourier transform, interference suppression.

## 1 Introduction

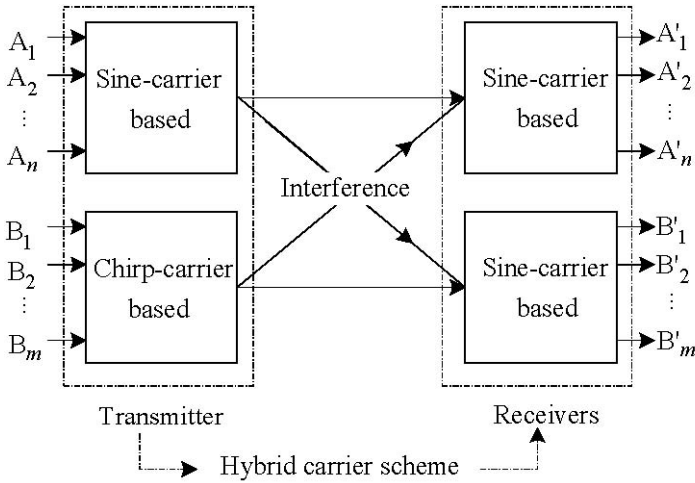
Traditional wireless communications systems sine-carrier based are challenged sometimes, such as applications in dense users [1] and some other emergency communications scenes. When frequency bands available are insufficient, it's difficult to increase capacities and data rates of the systems based on traditional schemes. In this work, we propose a novel scheme which adds a group of chirp-carrier users into a tradition system. Chirp signals are typical time-varying signals, which have large time-bandwidth product properties [2] and flexible time-frequency distribution. With signal processing methods such as matched filtering, finite length chirp signals can be compressed into a narrow pulse [3], so called pulse compression. Due to the properties of pulse compression, chirp signals can be employed as a type of spread spectrum waves. Transmission schemes based on the chirp spread spectrum (CSS) have been verified to be robust over multi-path propagation and fast fading channels [4, 5]. Besides, chirp signals have different fractional Fourier transform (FRFT) [6] properties from sine-type signals. Therefore, when chirp signals are regarded as carriers, it's possible to separate them from sine carriers by processing in the FRFT domain, and the interferences between users chirp and sine based users would be suppressed into an acceptable level.

In the remaining part of this paper, the principle of the hybrid carriers transmission scheme is discussed, and the interference between subsystems sine and chirp carriers based is the main issue concerned.

## 2 System Modeling

### 2.1 System Structure

A system structure considered is shown in Figure 1, in which two group of users are modulated by sine-type and chirp carriers respectively. The subsystem sine-carrier based can be regarded as a traditional system adopting typical multiple access schemes. In our research, it is considered to be a primary a FDMA (or OFDMA) system for discussion. Two key problems remained are schemes of the chirp-carrier subsystem and issues of the interference between the subsystems.



**Fig. 1.** A structure of hybrid carrier transmission scheme

### 2.2 Single-Link Transmission Based on Chirp Carriers

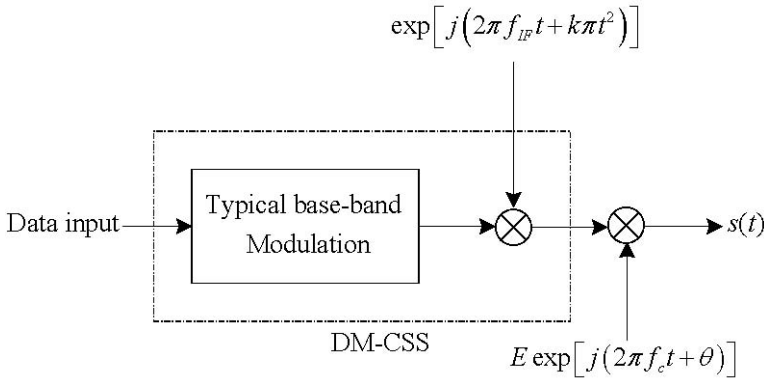
A modulated chirp-carrier signal can be represented as

$$s(t) = Eb(t)\exp\left[j\left(2\pi f_0 t + k\pi t^2 + \theta\right)\right] \quad (1)$$

where  $b(t)$  is a base-band modulated signal, adopting a typical modulation scheme such as MPSK or MQAM, etc; parameters  $E$ ,  $\theta$ ,  $f_0$  and  $k$  are the amplitude, initial phase, central frequency at  $t=0$  and chirp-rate respectively of the chirp carrier signal. Instantaneous frequency of a chirp signal is time-varying, the speed and direction of which decided by the chirp-rate parameter. The  $s(t)$  can be regarded as a common modulation signal using chirp signal instead of the sine carrier. Since (1) can be also represented as

$$s(t) = Eb(t) \exp \left[ j \left( 2\pi f_{IF} t + k\pi t^2 \right) \right] \exp \left[ j \left( 2\pi f_c t + \theta \right) \right] \quad (2)$$

Modulation of a chirp carrier can be implemented with a structure shown in Figure 2, which is a conventional sine-carrier transmission scheme added a CCS block at the intermediate frequency (IF) part. The modulation of CSS by  $b(t)$  is so-called the direction modulation (DM) method [7].



**Fig. 2.** A structure of chirp carrier modulation block

The demodulation of  $s(t)$  is based on matched filtering. For each symbol, when the symbol duration is  $T_s$  and if  $|k|T_s \gg 1$ , the frequency spectrum of  $s(t)$  at IF is represented as [8]

$$S_{IF}(f) \approx \frac{EB_i}{\sqrt{|k|}} \exp \left\{ j\pi \left[ -\frac{(f - f_{IF})^2}{k} + \frac{1}{4} \right] \right\}, \quad |f - f_{IF}| \leq \frac{|k|T_s}{2} \quad (3)$$

where  $B_i$  is the bandwidth of base-band modulation. After chirp modulation, the transmission bandwidth of each symbol spreads into  $B_i + |k|T_s$ .

A matched filter of  $S_{IF}(f)$  is

$$H(f) = \exp \left\{ j\pi \left[ \frac{(f - f_{IF})^2}{k} - \frac{1}{4} \right] \right\}, \quad |f - f_{IF}| \leq \frac{|k|T_s}{2} \quad (4)$$

The output signal filtered by  $H(f)$  represented in time domain is

$$s_o(t) = \text{IFT} [S_{IF}(f)H(f)] = ET_s \sqrt{|k|} b_i(t) \exp(j2\pi f_{IF} t) \frac{\sin[\pi t |k| T_s]}{\pi t |k| T_s} \quad (5)$$

When  $s_o(t)$  is sampled at  $t=0$  (central of the symbol duration), the primary symbol  $b_i$  is obtained with processing gain  $G_p$

$$G_p = T_s \sqrt{|k|} \tag{6}$$

### 2.3 Multiple Access Scheme of the Chirp-Carrier Subsystem

In this subsection, we propose a multiple access scheme based on FRFT. According to definition [9], the FRFT of function  $f(t)$  is represented as

$$F_p(u) = \int_{-\infty}^{\infty} f(t)K_p(u, t)dt \tag{7}$$

where  $p$  is the transform order. When  $p=2n+1$  ( $n \in Z$ ), the FRFT is just the Fourier transform, and when  $p \notin Z$  the kernel  $K_p(u, t)$  is defined as

$$K_p(u, t) = \sqrt{1 - j \cot \alpha_p} \exp \left[ j \left( \cot \alpha_p u^2 + \cot \alpha_p t^2 - 2ut \csc \alpha_p \right) \right] \tag{8}$$

where  $\alpha_p = \pi p/2$  is considered to be a rotational angle between time axis and the transform domain axis in the time-frequency plane [10].

Therefore, the FRFT of a finite chirp signal of  $T$  duration is

$$C_p(u) = E \sqrt{1 - j \cot \alpha_p} \exp(j\theta) \cdot \int_{-T/2}^{T/2} \exp \left\{ j\pi \left[ (k + \cot \alpha_p) t^2 + 2(f_0 - u \csc \alpha_p) t + u^2 \cot \alpha_p \right] \right\} dt \tag{9}$$

For  $p \in [-1, 1]$ , only if  $\cot \alpha_p = -k$ , in other words  $p = 2 \operatorname{arccot}(-k)/\pi$

$$C_p(u) = ET \sqrt{1 - j \cot \alpha_p} \exp \left[ j \left( \pi \cot \alpha_p u^2 + \theta \right) \right] \frac{\sin \left[ \pi (f_0 - u \csc \alpha_p) T \right]}{\pi (f_0 - u \csc \alpha_p) T} \tag{10}$$

The  $|C_p(u)|$  in (10) has a Sinc function envelope whose peak appears at  $u_0 = f_0 \sin \alpha_p$ , and most of the energy is concentrated in interval  $|u - u_0| < 2|\sin \alpha_p|/T$ . It means that if two chirp signals have a same chirp-rate  $k$  and the central frequency interval  $\Delta f$  satisfying  $\Delta f > 2/T$ , they can be separated by filtering in the FRFT domain of  $2 \operatorname{arccot}(-k)/\pi$  order. Besides, for  $\Delta f = k \Delta t$ , if the interval  $\Delta t$  between the beginning time of each chirp signal satisfies  $\Delta t > 2/|k|T$ , these chirp signals can also be separated.

The principle of chirp signals separation discussed above is regard as the multiple access scheme of the chirp-carrier subsystem, which is a generalization of FDMA or TDMA.

### 3 Analysis of the Interference between Subsystems

First, a hybrid system including a FDMA subsystem and only one chirp-carrier user is considered. The FDMA subsystem has  $N$  carriers and each carrier channel has  $B_0$  transmission bandwidth. For the total transmission bandwidth available is limited, time-varying of the chirp-carrier's instantaneous frequency must be periodic. Assuming that chirp-rate and period of frequency time-varying of the chirp carrier are  $NB_0/T$  and  $T$  respectively, under this circumstance, each user in FDMA subsystem suffers from the interference from the chirp-carrier user periodically. Denoting  $T_s$  as the duration of symbols FDMA users transmitted, when  $T$  is much larger than  $T_s$ , the BER performances of the FDMA users would not be degraded seriously. In contrast, the chirp-carrier user suffers from the interference from the FDMA users all the time. In order to achieve an acceptable BER performance, extending the symbol durations to increase processing gain of matched filtering may be necessary, according to (6). The time-frequency relationship between FDMA and the chirp-carrier users is shown in Figure 3, which gives an explanation of the interference issue discussed above.

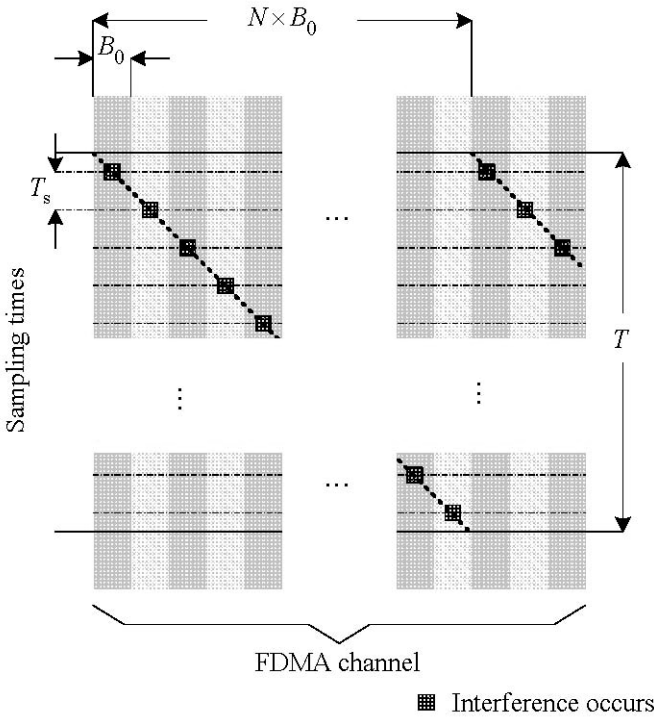


Fig. 3. Issue of interference explained by time-frequency relationship

BER performances of FDMA users and the chirp-carrier user are examined, as shown in Figure 4. The parameters in simulation are set as follows: for the FDMA subsystem, there are 10 carriers (denoted as  $c_0 \sim c_9$ ) with interval 2 KHz and symbols

are QPSK modulated, with 1ms duration; for the chirp carrier, the chirp-rate is 2 KHz/ms, frequency time-varying period is 10ms, and symbols are also QPSK modulated with 1Kbps or 0.5Kbps bit rate; powers of all carriers are same (SIR=0dB). ( $E_b$ : bit energy;  $n_0$ : noise power spectrum density)

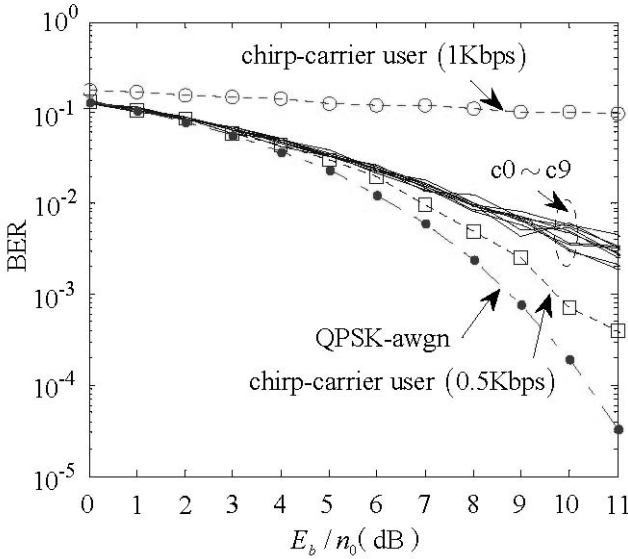


Fig. 4. BER performance of different users

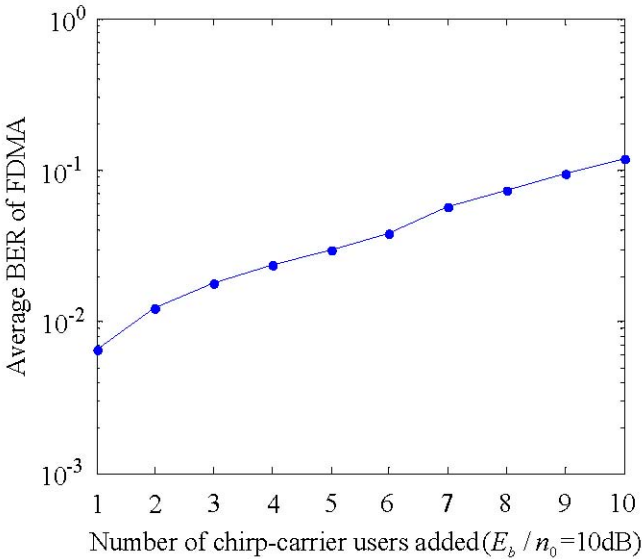
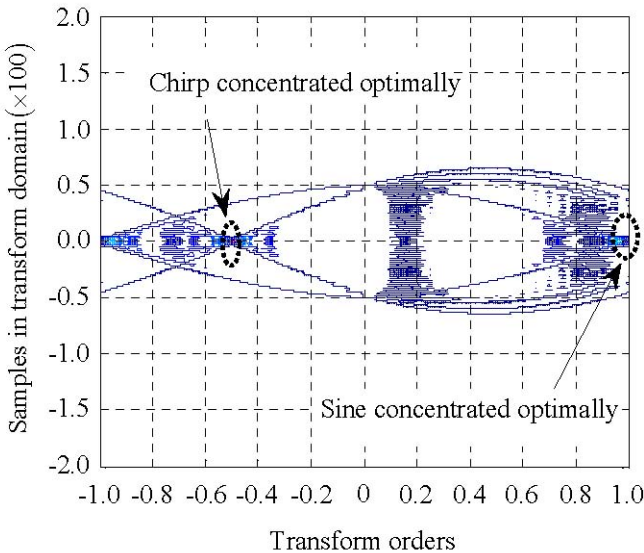


Fig. 5. BER performance of FDMA degrades as chirp-carrier users added

When more chirp-carrier users are added, interference to the FDMA users appears more frequently. Just keep the parameters in simulations above, the average BER performance of the FDMA users are examined according to different number of chirp-carrier users added, at  $E_p/n_0=10\text{dB}$ , as shown in Figure 5. It means that a traditional FDMA system would fail when too many chirp-carrier users are added without efficient approaches of interference suppression employed. For the chirp-carrier users, since the interferences from FDMA users are unchanged, the BER performances will not be degraded.

### 4 Interference Suppression Based on FRFT

For interference to users chirp-carrier based can be suppressed by matched filtering, suppressing the interference to FDMA users becomes the main problem. For transmission bandwidth of each FDMA user is covered by chirp signals. Band-pass filtering in the frequency domain is invalid. Considering FRFT properties of chirp signals, approaches of interference suppression based on FRFT are practicable. Comparing with chirp signals' FRFT properties discussed in Section 2, energy of a sine signal can not be concentrated by FRFT except at  $\pm 1$  orders, as shown in Figure 6. Therefore, when a chirp signal and a sine signal are transmitted in a same band synchronously, employing a band-stop filtering (BSF) in an optimal FRFT domain can eliminate a chirp signal almost completely and remain part of the sine signal. The energy concentrated interval  $\Delta u=2|\sin\alpha_p|/T_s'$  of a chirp signal is regarded as the stop band of BSF, where  $T_s'$  is the duration of symbols the chirp signal modulated.



**Fig. 6.** An example of sine and chirp signals' different FRFT properties

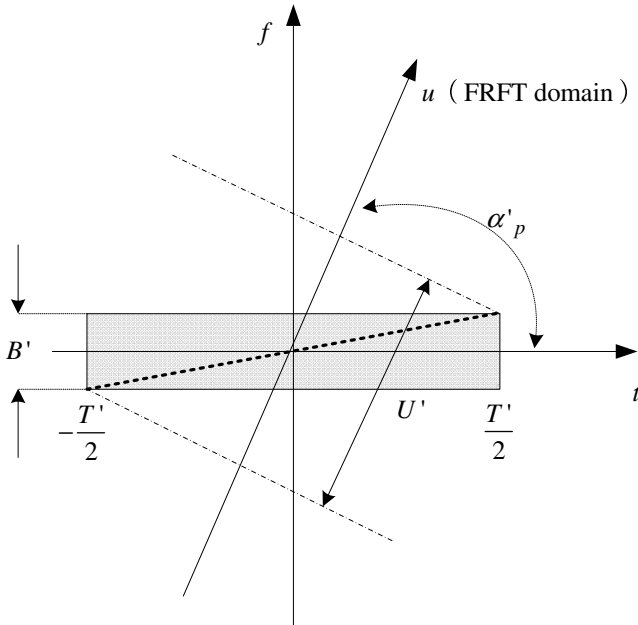
Certainly, BSF in the FRFT domain causes energy loss of the sine signal concerned inevitable. In order to estimate the degree of energy loss, calculation of a modulated sine signal's general bandwidth in the FRFT domain, denoted as  $U$ , is required. Although an accurate value of  $U$  is non-available, a geometrical method can be used for approximate calculation, as shown in Figure 7, and  $U$  can be represented as

$$U = \sqrt{B^2 + \frac{1}{T_s^2}} \cos\left(\alpha'_p - \arctan \frac{B}{T_s}\right) \tag{11}$$

where  $\alpha'_p = \text{sgn}(\sin\alpha_p)\text{arccot}(-k)$ , and  $B$  and  $T_s$  are the bandwidth and symbol duration of the modulated sine signal respectively. Therefore, the degree of energy loss can be estimated by the ratio

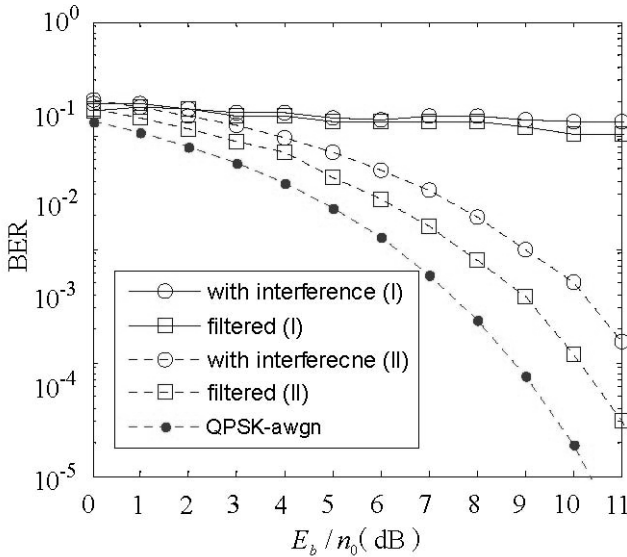
$$\eta \approx \frac{2 \sin \alpha'_p}{T'_s \sqrt{B^2 + \frac{1}{T_s^2}} \cos\left(\alpha'_p - \arctan \frac{B}{T_s}\right)} \tag{12}$$

Performances of interference suppression by BSF in the FRFT domain are examined, as shown in Figure 8. The parameters of the FDMA users and chirp carriers are kept as ones in Section 3, and 10 chirp-carrier users are added this time. Assuming the bit rate of each chirp-carrier user is 1 or 0.5Kbps, denoted as interference type I or II respectively, performances of interference suppression via BSF in the FRFT domain are examined, with about 2.6 or 1.3 dB loss of  $E_b/n_0$  accordingly.



**Fig. 7.** Approximate calculation of a sine signal's general bandwidth in a FRFT domain





**Fig. 8.** Performances of interference suppression via BSF in the FRFT domain (the bit rate of chirp users in I and II are 1 and 0.5Kbps)

## 5 Conclusions

In this paper, a novel hybrid carrier transmission scheme has been presented. First, principles including system structure, modulation methods and multiple access schemes have been discussed. Then the issues of interference between subsystems sine and chirp carriers based have been analyzed. Respecting interference from chirp-carrier users to sine-carrier users is a key problem, an approach of interference suppression based on FRFT has been proposed, which can eliminate chirp signals interference almost completely with a certain energy loss of sine-carrier users. Increasing capacity of a traditional system in some emergency senses is regarded as a potential application of the scheme proposed.

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