Outage-Based Optimal Transmit Antenna Location for Distributed Antenna Systems with Selection Transmission

Liang Han, Tian Liu, Shihai Shao, Chaojin Qing, and Youxi Tang

National Key Lab of Science and Technology on Communications, University of Electronic Science and Technology of China, Chengdu 611731, China {hanliang,liutian,ssh,qingchj,tangyx}@uestc.edu.cn

Abstract. In this paper, we investigate the optimal transmit antenna location for distributed antenna systems (DAS) with selection transmission in the downlink. Considering the effects of path loss, shadow fading, rayleigh fading and white Gaussian noise, we first derive the approximate cell averaged outage probability at high signal-to-noise ratio (SNR). Then, we obtain the transmit antenna location by minimizing the approximate cell averaged outage probability. Simulation results validate the analytical results and show that DAS with optimized antenna locations offers smaller outage probability over the traditional co-located antenna system (CAS).

Keywords: antenna location, distributed antenna systems, outage probability, selection transmission.

1 Introduction

Distributed antenna system (DAS) was originally introduced by Saleh for coverage improvement in indoor wireless communications[1]. In recent years, DAS has attracted worldwide research interests as a promising technique [2]–[4]. Compared with co-located antenna system (CAS), DAS has many advantages: more independent channel fading, larger system capacity [5][6], shorter access distance, and lower transmission power [7].

In [8], the spectral efficiency of random layout DAS is analyzed and it has been found that the system performance is influenced by the antenna location. In [9], the optimal antenna location for a linear cell is obtained by minimizing the area averaged bit error probability for STBC-OFDM systems. The authors of [10] studied the antenna positioning problem for the uplink of DAS and propose the squared distance criterion. However, there has been little research on the antenna location optimization from the perspective of outage probability.

Our contribution in this paper can be briefly described as follows. First, we derive the approximate outage probability in a circular cell for distributed antenna systems with selection transmission. Second, we obtain the optimal transmit antenna location by minimizing the approximate cell averaged outage probability.

P. Ren et al. (Eds.): WICON 2011, LNICST 98, pp. 125-132, 2012.

[©] Institute for Computer Sciences, Social Informatics and Telecommunications Engineering 2012

The rest of the paper is organized as follows. In section 2, the system model is introduced. In section 3, the expression of approximate cell averaged outage probability is derived and the antenna location is then obtained by minimizing the approximate cell averaged outage probability. Afterward, in section 4, some simulation results are given to demonstrate the analytical results. Finally, section 5 concludes this paper.

The notation used in this paper follows the usual convention. Matrices and vectors are denoted by symbols in boldface. $(\cdot)^T$ and $(\cdot)^H$ are transpose and conjugate transpose of (\cdot) , respectively. $E(\cdot)$ is the mathematical expectation of (\cdot) .

2 System Model

2.1 Circular Cell Architecture

The circular cell architecture is shown in Fig. 1. In a circular cell of radius R, M_T distributed transmit antennas are connected to a single base station by fibers or coax cables. The polar coordinate for the *n*th transmit antenna is denoted by (r_n, θ_n) , $n = 1, \dots, M_T$. In order to make the analysis tractable, we assume that the transmit antennas are placed uniformly on a circle of radius ρ , i.e.,

$$\begin{cases} r_1 = r_2 = \dots = r_{M_T} = \rho \\ \theta_n = \frac{2\pi(n-1)}{M_T}, \quad n = 1, 2, \dots M_T \end{cases}$$
(1)

A mobile station (MS) is assumed to be uniformly distributed in the cell [11], and its polar coordinate is denoted by (r, θ) . Thus, the probability density function (PDF) of (r, θ) is given by

$$f(r,\theta) = \frac{r}{\pi R^2}, \quad 0 \le r \le R, 0 \le \theta \le 2\pi.$$
(2)



Fig. 1. A circular cell layout of distributed antenna system(DAS)

The distance d_n from the MS to the *n*th transmit antenna can be calculated as [12]

$$d_n = \sqrt{r^2 + \rho^2 - 2r\rho\cos\left(\theta - \theta_n\right)}.$$
(3)

2.2 Transmission Strategy

There are many possible strategies to use the distributed transmit antennas for signal transmission. In this paper, we considered one of the most commonly used: selection transmission [5][6]. In selection transmission scheme, only a single distributed antenna module is selected out of all the antennas for transmission by the criterion of minimizing propagation pathloss. When MS is located in the shadow region as shown in Fig. 2, the base station selects the 1st antenna to transmit signals. Selection transmission has gained increasing interest in DAS because it reduces the other-cell interference (OCI) and retains the effects of the spatial diversity.



Fig. 2. Selection transmission

2.3 Channel Model

For practical reasons, the MS is assumed to have a single antenna. When MS is located in the shadow region, the received signals can be expressed as

$$r = h_1 x + n \tag{4}$$

where x is the transmitted signal from the 1st distributed antenna with $E[|x|^2] = E_b$, h_1 denotes the fading channel coefficient from the 1st transmit antenna to the MS, n denotes the additive white complex Gaussian noise with zero-mean and variance N_0 .

The fading channel coefficient h_1 is modeled as [13]

$$h_1 = \xi_1 \sqrt{L_1 S_1} \tag{5}$$

where ξ_1 is a zero-mean complex Gaussian random variable with unit variance that models the Rayleigh fading, S_1 represents the shadow fading between the 1st transmit antenna and the MS, and L_1 denotes the path loss from the 1st transmit antenna to the MS.

We model S_1 as a log-normal random variable, meaning that $10 \log_{10} (S_1)$ is assumed to be normally distributed with zero mean and standard deviation $\sigma_{s,1}$.

 L_1 is modeled as the following form

$$L_1 = \frac{c}{d_1^{\alpha}} \tag{6}$$

where c is a constant, α is the path loss exponent.

3 Performance Analysis

3.1 Approximate Cell Averaged Outage Probability

When MS is located in the shadow region, the instantaneous signal-to-noise ratio (SNR) received from the 1st transmit antenna is $\gamma = \frac{E_b S_1 L_1 |\xi_1|^2}{N_0}$ and the local mean SNR is $\bar{\gamma} = \frac{E_b S_1 L_1}{N_0}$. The outage probability for a fixed MS location conditioned on the local mean SNR $\bar{\gamma}$ can be given by

$$P_{out} = \Pr\left(\gamma < \gamma_{req} \left| \bar{\gamma} \right.\right) = 1 - \exp\left(-\frac{\gamma_{req}}{\bar{\gamma}}\right) \tag{7}$$

where γ_{req} is the SNR target for the service requested by MS. For large $\bar{\gamma}$, the outage probability can be approximately expressed as [14]

$$P_{out} \approx \frac{\gamma_{\rm req}}{\bar{\gamma}} \approx \frac{N_0 \gamma_{\rm req}}{E_b S_1 L_1}.$$
(8)

Under the assumption of log-normal distribution for S_1 , $\frac{1}{S_1}$ is also a log-normal random variable, i.e., $10 \log_{10} \frac{1}{S_1} \sim N(0, \sigma_{s,1}^2)$. Then equation (8) can be written as

$$P_{out} \approx \exp\left(\frac{\sigma_{s,1}^2 \ln^2 10}{200}\right) \frac{N_0 \gamma_{\text{req}}}{E_b L_1} \\ \approx \exp\left(\frac{\sigma_{s,1}^2 \ln^2 10}{200}\right) \frac{N_0 \gamma_{\text{req}}}{cE_b} d_1^{\alpha}.$$
(9)

Considering the symmetry of the transmit antenna, the average outage probability of the entire cell is equivalent to the the average outage probability of the shadow region. Then the approximate cell averaged outage probability can be obtained

$$\bar{P}_{out} = \mathcal{E}_{MS} \left(P_{out} \right) \\
= \mathcal{E}_{MS} \left[\exp \left(\frac{\sigma_1^2 \ln^2 10}{200} \right) \frac{N_0 \gamma_{req} d_1^{\alpha}}{cE_b} \right] . \tag{10}$$

$$= \exp \left(\frac{\sigma_1^2 \ln^2 10}{200} \right) \frac{N_0 \gamma_{req}}{cE_b} \mathcal{E}_{MS} \left[d_1^{\alpha} \right]$$

3.2 Optimization Problem and Solution

The mathematical model for the optimization problem can be written as

$$\rho^* = \underset{\rho}{\arg\min} \bar{P}_{out} = \underset{\rho}{\arg\min} \operatorname{E}_{\mathrm{MS}} \left[d_1^{\alpha} \right].$$
(11)

Form (11), we note that the optimal antenna location depends on the path loss exponent. Without loss of generality, we just consider $\alpha = 2$ and $\alpha = 4$ in this paper. Other path loss exponents can be obtained in the same way.

When $\alpha = 2$, the cell averaged outage probability can be obtained as

$$\bar{P}_{out} = \exp\left(\frac{\sigma_{s,1}^2 \ln^2 10}{200}\right) \frac{N_0 \gamma_{\text{reg}}}{cE_b} E_{\text{MS}} \left[d_1^2\right] \\
= \exp\left(\frac{\sigma_{s,1}^2 \ln^2 10}{200}\right) \frac{N_0 \gamma_{\text{reg}}}{cE_b} \frac{N}{\pi R^2} \int_{-\frac{\pi}{M_T}}^{\frac{\pi}{M_T}} \int_0^R r \left(r^2 + \rho^2 - 2r\rho\cos\left(\theta - \theta_n\right)\right) dr d\theta . \\
= \exp\left(\frac{\sigma_{s,1}^2 \ln^2 10}{200}\right) \frac{N_0 \gamma_{\text{reg}}}{cE_b} \left[\frac{R^2}{2} + \rho^2 - \frac{4\rho RM_T}{3\pi}\sin\left(\frac{\pi}{M_T}\right)\right]$$
(12)

We set the first order derivative of (12) equal to 0, and obtain the optimal antenna location

$$\rho^* = \frac{2RM_T}{3\pi} \sin\left(\frac{\pi}{M_T}\right). \tag{13}$$

When $\alpha = 4$, the cell averaged outage probability can be obtained as

$$\begin{split} \bar{P}_{out} &= \exp\left(\frac{\sigma_{s,1}^{2}\ln^{2}10}{200}\right) \frac{N_{0}\gamma_{\text{reg}}}{cE_{b}} \text{E}_{\text{MS}}\left[d_{1}^{4}\right] \\ &= \exp\left(\frac{\sigma_{s,1}^{2}\ln^{2}10}{200}\right) \frac{N_{0}\gamma_{\text{reg}}}{cE_{b}} \frac{M_{T}}{\pi R^{2}} \int_{-\frac{\pi}{M_{T}}}^{\frac{\pi}{M_{T}}} \int_{0}^{R} r\left(r^{2} + \rho^{2} - 2r\rho\cos\left(\theta - \theta_{n}\right)\right)^{2} dr d\theta \\ &= \exp\left(\frac{\sigma_{s,1}^{2}\ln^{2}10}{200}\right) \frac{N_{0}\gamma_{\text{reg}}}{cE_{b}} \frac{M_{T}}{\pi R^{2}} \\ &\left\{\frac{R^{4}}{3} + \rho^{4} - \frac{8M_{T}\sin\left(\frac{\pi}{M_{T}}\right)R}{3\pi}\rho^{3} + \left[\frac{R^{2}M_{T}\sin\left(\frac{2\pi}{M_{T}}\right)}{2\pi} + 2R^{2}\right]\rho^{2} - \frac{8\rho R^{3}M_{T}\sin\left(\frac{\pi}{M_{T}}\right)}{5\pi}\right\} \end{split}$$
(14)

We set the first order derivative of (14) equal to 0, and get

$$\rho^{3} - \frac{2RM_{T}\sin\left(\frac{\pi}{M_{T}}\right)}{\pi}\rho^{2} + \frac{R^{2}M_{T}\sin\left(\frac{2\pi}{M_{T}}\right) + 4\pi R^{2}}{4\pi}\rho - \frac{2R^{3}M_{T}\sin\left(\frac{\pi}{M_{T}}\right)}{5\pi} = 0.$$
(15)

According to the formula for finding roots of cubic equations, the real root in the interval [0, R] is just the solution to our optimization problem.

4 Numerical and Simulation Results

In this section, we validate the analytical results by simulations. Some basic simulation parameters are given as follows: R = 1000m, $\sigma_{s,1} = 6$ dB, $\gamma_{req} = 10$ dB, $N_0 = -100$ dBm.



Fig. 3. Cell averaged outage probability versus transmit antenna location when path loss exponent $\alpha = 2$. R = 1000m, $\sigma_{s,1} = 6$ dB, $\gamma_{req} = 10$ dB, $N_0 = -100$ dBm, $E_b = -10$ dBm.

Fig. 3 illustrates the cell averaged outage probability versus transmit antenna location when path loss exponent $\alpha = 2$. As shown in Fig. 3, the optimal transmit antenna location is close to 0.6R and the simulation results approximately agree with the analytical results.

Fig. 4 shows the cell averaged outage probability versus transmit antenna location when path loss exponent $\alpha = 4$. From Fig. 4, we can observe that the optimal transmit antenna location is close to 0.65R. It is further proved from Fig. 4 that the simulation results approximately agree with the analytical results.

Both Fig. 3 and Fig. 4 validate the analytical results and show that DAS with optimized antenna locations offers smaller outage probability over the traditional co-located antenna system.



Fig. 4. Cell averaged outage probability versus transmit antenna location when path loss exponent $\alpha = 4$. R = 1000m, $\sigma_{s,1} = 6$ dB, $\gamma_{req} = 10$ dB, $N_0 = -100$ dBm, $E_b = 40$ dBm.

5 Conclusion

In this paper, we have derived the approximate cell averaged outage probability for distributed antennas systems with selection transmission in the downlink and obtained the optimal transmit antenna location by minimizing approximate cell averaged outage probability. Simulation results validate the analytical results and show that DAS with optimized antenna locations offers smaller outage probability over the traditional CAS. The research results provide a scheme to determine the location of transmit antennas for the next generation wireless communication networks based on DAS.

Acknowledgment. This work was supported in part by the National major projects (No.2010ZX03003-002,2011ZX03001-006-01), National Natural Science Foundation of China (No.60902027, No.60832007 No.U1035002/L05 and No.60901018), and the Fundamental Research Funds for the Central Universities under Grant number ZYGX2009J008, ZYGX2009J010 of China.

References

- Saleh, A., Rustako, A., Roman, R.: Distributed antennas for indoor radio communication. IEEE Trans. Commun. 35(12), 1245–1251 (1987)
- Zhou, S., Zhao, M., Xu, X., Yao, Y.: Distributed wireless communication system: a new architecture for future public wireless access. IEEE Commun. Mag. 41(3), 108–113 (2003)

- 3. Roh, W., Paulraj, A.: MIMO channel capacity for the distributed antenna systems. In: Proc. IEEE VTC, Vancouver, Canada, pp. 706–709 (February 2002)
- He, X., Luo, T., Yue, G.: Optimized distributed MIMO for cooperative relay networks. IEEE Commun. Lett. 14(1), 9–11 (2010)
- Choi, W., Andrews, J.G.: Downlink performance and capacity of distributed antenna systems in a multicell environment. IEEE Trans. Wireless Commun. 6(1), 69–73 (2007)
- Zhong, C., Wong, K., Jin, S.: Capacity bounds for MIMO Nakagami-m fading channels. IEEE Trans. Sig. Proc. 57(9), 3613–3623 (2009)
- Zhang, J., Andrews, J.G.: Distributed antenna systems with randomness. IEEE Commun. Lett. 7(9), 3636–3646 (2008)
- Zhuang, H., Dai, L., Xiao, L., Yao, Y.: Spectral efficiency of distributed antenna system with random antenna layout. Electron. Lett. 39(6), 495–496 (2003)
- Shen, Y., Tang, Y., Kong, T., Shao, S.: Optimal antenna location for STBC-OFDM downlink with distributed transmit antennas in linear cells. IEEE Commun. Lett. 11(5), 387–389 (2007)
- Wang, X., Zhu, P., Chen, M.: Antenna location design for generalized distributed antenna systems. IEEE Commun. Lett. 13(5), 315–317 (2009)
- Mukherjee, S., Avidor, D.: Effect of microdiversity and correlated macrodiversity on outages in a cellular system. IEEE Trans. Wireless Commun. 2(1), 50–58 (2003)
- 12. Coxeter, H.S.M.: Introduction to Geometry, 2nd edn. Wiley, New York (1969)
- 13. Stüber, G.L.: Principles of Mobile Communications. Kluwer, New York (2002)
- Simon, M.K., Alouini, M.-S.: Digital Communications over Fading Channels: A Unified Approach to Performance Analysis. Wiley, New York (2000)