An Impact of Cooperation and Altruism on Transmission

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Abstract. In wireless access, transmitter nodes need to make individual decisions for distributed operation and do not necessarily cooperate or bargaining with each other. We consider a single-receiver random access system of transmitters (users) with altruistic payoffs which are generalized weighted individual objectives of their throughput rewards, transmission energy costs and delay costs. We compare altruistical behaviour with selfish (Nash equilibrium), cooperative (Shapley vector) and bargaining behaviour (Nash bargaining solution). We produce criteria where altruistical behaviour is more profitable for a user than either selfish, or cooperative, or bargaining ones.

Keywords: ALOHA, Nash equilibrium, Shapley vector, altruistical behaviour, bargaining solution.

1 Payoffs without Altruism

Aloha [1] and slotted Aloha [2,4] have long been used as random distributed medium access protocols for radio channels. They are in use in both satellite as well as cellular telephone networks for the sporadic transfer of data packets. In these protocols, packets are transmitted sporadically by various users. If packets are sent simultaneously by more than one user then they collide.

In this paper we focus on the following Aloha protocol model formulated in [3]. It is assumed that multiple nodes randomly transmit packets with fixed probabilities to a common receiver. Each transmitter has a packet queue of infinite buffer capacity. We consider a synchronous slotted system, in which each packet transmission takes one time slot. We assume saturated queues with always availability of packets. Let p_i denote the transmission probability of node i from the set N of n transmitters. We assume multi-packet reception channels with possible packet captures. The packet transmission of node i is successfully received with probability $q_{i|J}$, if nodes in set J (including node i) transmit in the same time slot. Throughout the paper, we will specialize the results to the particular case of the classical collision channels with $q_{i|J} = 1$ for $J = \{i\}$ and $q_{i|J} = 0$ for $J \neq \{i\}$. Any selfish node i has the objective of choosing the transmission probability p_i to maximize the utility function u_i that reflects the difference between the throughput rewards and costs of transmission energy

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and delay per time slot, i.e the utility of node i is defined as $u_i = r_i\lambda_i - e_i - d_i$, where λ_i is the throughput rate, namely the average number of successful packet transmissions per time slot, r_i is the reward for any successful packet transmission, e_i is the average transmission energy cost per time slot and d_i is the average delay-type cost per time slot. We will focus on two users game. Node (user) i transmits a packet with probability p_i and it is successfully received with probability $q_{i|i}$, if the other node decides not to transmit, or captured with probability $q_{i|12}$, if the other node also transmits in the same time slot. We assume $0 \le q_{i|12} \le q_{i|i} \le 1$ for $i = 1, 2$. We also assume that throughput rate for user i is given as follows $\lambda_i = p_i(p_{\hat{i}}q_{i|12} + (1 - p_{\hat{i}})q_{i|i})$, the average transmission energy cost e_i per time slot is permanent and equals E_i, and the average delay-type cost per time slot d_i is proportional of failed transmission with coefficient D_i. Thus, the payoff to users are given as follows:

$$\begin{aligned}
u_1(p_1, p_2) &= r_1p_1(p_2q_{1|12} + (1 - p_2)q_{1|1}) \\
&\quad - D_1(1 - p_1 + p_1(1 - p_2q_{1|12} - (1 - p_2)q_{1|1})) \\
&\quad - E_1p_1, \\
u_2(p_1, p_2) &= r_2p_2(p_1q_{2|12} + (1 - p_1)q_{2|2}) \\
&\quad - D_2(1 - p_2 + p_2(1 - p_1q_{2|12} - (1 - p_1)q_{2|2})) \\
&\quad - E_2p_2.
\end{aligned}$$

This game is equivalent to the following bimatrix game $M = (A, B)$ with two pure strategies: to transmit (T) and do not transmit (N), where

$$A := \begin{array}{c} \\ \text{T} \\ \text{N} \end{array} \begin{array}{c} \begin{array}{cc} \quad\text{T} & \qquad\qquad\text{N} \end{array} \\ \begin{pmatrix} R_1q_{1|12} - E_1 - D_1 & R_1q_{1|1} - E_1 - D_1 \\ -D_1 & -D_1 \end{pmatrix} \end{array}$$

and

$$B := \begin{array}{c} \\ \text{T} \\ \text{N} \end{array} \begin{array}{c} \begin{array}{cc} \qquad\qquad\text{T} & \qquad\text{N} \end{array} \\ \begin{pmatrix} R_2q_{2|12} - E_2 - D_2 & -D_2 \\ R_2q_{2|2} - E_2 - D_2 & -D_2 \end{pmatrix} \end{array}$$

with

$$R_1 = r_1 + D_1, \quad R_2 = r_2 + D_2.$$

This game always has Nash equilibrium in pure strategies and its form is determined by the fact whether the transmission by both users or each of them separately is too expensive or accessible for them, namely, as it is given in Table 1.

Thus, in particular the case where it is too expensive to transmit for both users simultaneously, but transmission is preferable for each of them separately turns out to be very competitive since two pure Nash equilibrium (T, N) and (N, T) arise simultaneously. This situation takes place under the following conditions:

$$q_{1|12} < E_1/R_1 < q_{1|1} \text{ and } q_{2|12} < E_2/R_2 < q_{2|2}. \tag{1}$$

Table 1. Nash equilibrium

E_1/E_2	$\le R_2 q_{2\|12}$	$\in [R_2 q_{2\|12}, R_2 q_{2\|2}]$	$\ge R_2 q_{2\|2}$
$\le R_1 q_{1\|12}$	(T,T)	(T,N)	(T,N)
$\in [R_1 q_{1\|12}, R_1 q_{1\|1}]$	(N,T)	(N,T), (T,N)	(T,N)
$\ge R_1 q_{1\|1}$	(N,T)	(N,T)	(N,N)

Besides for the case (1) a mixed Nash equilibrium $((p_1, 1-p_1), (p_2, 1-p_2))$ exists where

$$p_1 = \frac{1}{q_{2|2} - q_{2|12}} \left(q_{2|2} - \frac{E_2}{R_2} \right),$$
$$p_2 = \frac{1}{q_{1|1} - q_{1|12}} \left(q_{1|1} - \frac{E_1}{R_1} \right)$$

with payoffs

$$(v_1^s, v_2^s) = (-D_1, -D_2).$$

In spite of quite competitive situation the payoffs for mixed strategies coincide with payoffs for pure strategies where both users have just chosen do not to transmit at all which is quite senseless since, of course, they have a chance to improve their payoff.

2 Comparing Selfish, Cooperative and Bargaining Solutions

In the strong competitive situation (1) with two equilibrium cooperative and bargaining approach can be applied to improve user's outcome.

First we consider the Shapley solution of the bargaining problem. To do so, we note that our bimatrix game can be present as $M = (A, B)$ where

$$A = \begin{array}{c} \\ \text{T} \\ \text{N} \end{array} \begin{array}{c} \begin{array}{cc} \text{T} & \text{N} \end{array} \\ \begin{pmatrix} T_{11} - D_1 & T_1 - D_1 \\ -D_1 & -D_1 \end{pmatrix} \end{array},$$

$$B = \begin{array}{c} \\ \text{T} \\ \text{N} \end{array} \begin{array}{c} \begin{array}{cc} \text{T} & \text{N} \end{array} \\ \begin{pmatrix} T_{22} - D_2 & -D_2 \\ T_2 - D_2 & -D_2 \end{pmatrix} \end{array}$$

with

$$T_i = q_{i|i} R_i - E_i,$$
$$T_{ii} = q_{i|12} R_i - E_i, i = 1, 2.$$

Then, in the new notation the conditions (1) are equivalent to the following ones:

$$T_{11} < 0 < T_1 \text{ and } T_{22} < 0 < T_2. \tag{2}$$

Also, note that the Pareto optimal boundary is just the line segment L joining two points: $(T_1 - D_1, -D_2)$ and $(-D_1, T_2 - D_2)$.

The security levels (maxmin) for users are given as solution of two zero-sum games with matrix A and B. Clearly, under condition (1) (N, T) is the saddle point for zero-sum game with matrix A and (T, N) is the saddle point for zero-sum game with matrix B. Thus, the security level is $(-D_1, -D_2)$ which presents status quo point (x_*, y_*). Thus, to find Shapley solution we have to find the point (x, y) maximizing

$$T := (x - x_*)(y - y_*) = (x + D_1)(y + D_2)$$

with

$$y = -\frac{T_2}{T_1}x + \frac{T_1T_2 - D_1T_2 - D_2T_1}{T_1}$$

for $x \in [-D_1, T_2 - D_2]$. Then we have the following result.

Theorem 1. *The Shapley solution for the bargaining problem* (v_1^b, v_2^b) *is given by*

$$v_1^b = \frac{T_1}{2} - D_1 = \frac{1}{2}(R_1 q_{1|1} - D_1) - D_1,$$
$$v_2^b = \frac{T_2}{2} - D_2 = \frac{1}{2}(R_2 q_{2|2} - D_2) - D_2.$$

Besides, bargaining solution a cooperative approach can be applied to deal with competitive situation (1). Namely, the users have cooperatively to maximize the joint payoff

$$v = v_1 + v_2,$$

so, to solve the following optimization problem:

$$\max_{p_1, p_2} v.$$

It is clear that the the cooperative problem has the following optimal strategies $(p_1^c, 1 - p_1^c)$, $(p_2^c, 1 - p_2^c)$ where

$$p_1^c = \frac{T_2}{T_1 - T_{11} + T_2 - T_{22}},$$
$$p_2^c = \frac{T_1}{T_1 - T_{11} + T_2 - T_{22}}$$

with joint payoff

$$v^c = \frac{T_1T_2}{T_1 - T_{11} + T_2 - T_{22}} - D_1 - D_2.$$

This payoff they can share, for example, according to Shapley vector which is given in the following theorem.

Theorem 2. *The Shapley vector for the cooperative solution $\varphi = (\varphi_1, \varphi_2)$ is given as follows:*

$$\varphi_1 = \frac{T_1 T_2}{2(T_1 - T_{11} + T_2 - T_{22})} - D_1,$$
$$\varphi_2 = \frac{T_1 T_2}{2(T_1 - T_{11} + T_2 - T_{22})} - D_2.$$

We can compare the bargaining and cooperative solution estimating the difference of the corresponding total payoffs, i.e. $v = v_1^b + v_2^b - v^c$. It is clear that

$$\begin{aligned} & v_1^b + v_2^b - v^c \\ & \quad = \frac{T_1 + T_2}{2} - \frac{T_1 T_2}{T_1 - T_{11} + T_2 - T_{22}} \\ & \quad = \frac{T_1^2 + T_2^2 - (T_1 + T_2)(T_{11} + T_{22})}{T_1 - T_{11} + T_2 - T_{22}} \\ & \quad > (\text{by } (2)) > 0. \end{aligned}$$

Thus, the bargaining approach can essentially increase the quality of the network as a whole. To estimate what it can bring to a user we have to investigate the difference of corresponding payoffs, so values $v_1^b - \varphi_1$ and $v_1^b - \varphi_1$. It is clear that

$$v_1^b - \varphi_1 = \frac{T_1 - T_2 - T_{11} - T_{22}}{T_1 - T_{11} + T_2 - T_{22}} T_1,$$
$$v_2^b - \varphi_2 = \frac{T_2 - T_1 - T_{11} - T_{22}}{T_1 - T_{11} + T_2 - T_{22}} T_2.$$

Thus, by (2) we have the following result.

Theorem 3. *(a) If*

$$T_{11} + T_{22} \le T_1 - T_2 \le -T_{11} - T_{22}$$

then both users benefits from bargaining approach compare to cooperative one,
(b) if

$$T_1 - T_2 > -T_{11} - T_{22}$$

then only user 1 benefits from bargaining approach compare to cooperative one,,
(c) if

$$T_1 - T_2 < T_{11} + T_{22}$$

then only user 2 benefits from bargaining approach compare to cooperative one,.

3 Payoffs with Altruism

In this section we consider the other way of user's cooperation where in behavior of users some altruism presents, namely, in payoff each user takes into account payoff of the other one with some weights (say, normalized by 1), namely, the user payoffs are given as follows:

$$u_1^J = \alpha_1 u_1 + \alpha_2 u_2, \quad u_2^J = \beta_1 u_1 + \beta_2 u_2.$$

where $\alpha_i, \beta_i \in [0, 1]$.

Then the game has the following bimatrix form $M = (A, B)$ with

$$A = \begin{matrix} \\ \mathrm{T} \\ \mathrm{N} \end{matrix} \begin{matrix} \mathrm{T} & \mathrm{N} \\ \left(\begin{matrix} \alpha_1(T_{11} - D_1) + \alpha_2(T_{22} - D_2) \\ -\alpha_1 D_1 + \alpha_2(T_2 - D_2) \end{matrix} \right. & \left. \begin{matrix} -\alpha_2 D_2 + \alpha_1(T_1 - D_1) \\ -\alpha_1 D_1 - \alpha_2 D_2 \end{matrix} \right) \end{matrix}$$

$$B = \begin{matrix} \\ \mathrm{T} \\ \mathrm{N} \end{matrix} \begin{matrix} \mathrm{T} & \mathrm{N} \\ \left(\begin{matrix} \beta_1(T_{11} - D_1) + \beta_2(T_{22} - D_2) \\ -\beta_1 D_1 + \beta_2(T_2 - D_2) \end{matrix} \right. & \left. \begin{matrix} -\beta_2 D_2 + \beta_1(T_1 - D_1) \\ -\beta_1 D_1 - \beta_2 D_2 \end{matrix} \right) \end{matrix}.$$

For this game we have that

(a) (N, N) is a Nash equilibrium if $T_1 \leq 0$ and $T_2 \leq 0$,
(b) (T, T) is a Nash equilibrium, if $\alpha_1 T_{11} + \alpha_2 T_{22} \geq \alpha_2 T_2$ and $\beta_1 T_{11} + \beta_2 T_{22} \geq \beta_1 T_1$,
(c) (N, T) is a Nash equilibrium, if $\alpha_1 T_{11} + \alpha_2 T_{22} \leq \alpha_2 T_2$ and $T_2 \geq 0$,
(d) (T, N) is a Nash equilibrium, if $\beta_1 T_{11} + \beta_2 T_{22} \leq \beta_1 T_1$ and $T_1 \geq 0$.

Also, if

$$T_1 \geq 0, T_2 \geq 0, \beta_1 T_{11} + \beta_2 T_{22} \leq \beta_1 T_1 \text{ and } \alpha_1 T_{11} + \alpha_2 T_{22} \leq \alpha_2 T_2 \qquad (3)$$

besides two pure Nash equilibrium (T, N) and (N, T), the game has a mixed Nash equilibrium $((p_1, 1 - p_1), (p_2, 1 - p_2))$ where

$$p_1 = \frac{\beta_2 T_2}{\beta_1(T_1 - T_{11}) + \beta_2(T_2 - T_{22})},$$
$$p_2 = \frac{\alpha_1 T_1}{\alpha_1(T_1 - T_{11}) + \alpha_2(T_2 - T_{22})}$$

with the corresponding payoffs

$$v_1^J = \frac{\alpha_1 \alpha_2 T_1 T_2}{\alpha_1(T_1 - T_{11}) + \alpha_2(T_2 - T_{22})} - \alpha_1 D_1 - \alpha_2 D_2,$$
$$v_2^J = \frac{\beta_1 \beta_2 T_1 T_2}{\beta_1(T_1 - T_{11}) + \beta_2(T_2 - T_{22})} - \beta_1 D_1 - \beta_2 D_2.$$

So, in the competitive situation the payoffs for mixed strategies is greater than the payoffs for pure strategies where both users have just chosen do not to transmit at all which tells that taking into account interest of the opponent can improve the work of network even in selfish scenario of user's behaviour. Also. it is interesting that in the competitive cases (2) and (3) the domain (T_{11}, T_{22}) for altruistical payoff contains the corresponding domain for the selfish payoffs, that produces some extra advantage.

The following results allow to tell when altruistical behavior even is more profitable for users than selfish one.

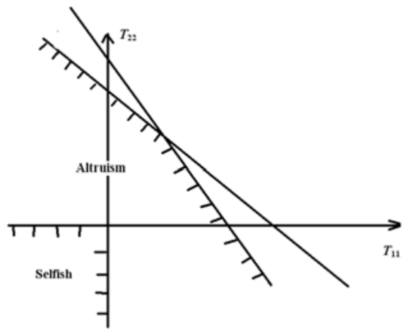

Fig. 1. Areas of competitive solution for selfish and altruistical payoffs

Theorem 4. *Let (2) hold. Then*

$$v_1^J \geq v_1^s, \quad v_2^J \geq v_2^s.$$

if and only if

$$\frac{\alpha_1\alpha_2 T_1 T_2}{\alpha_1(T_1 - T_{11}) + \alpha_2(T_2 - T_{22})} \geq (\alpha_1 - 1)D_1 + \alpha_2 D_2,$$
$$\frac{\beta_1\beta_2 T_1 T_2}{\beta_1(T_1 - T_{11}) + \beta_2(T_2 - T_{22})} \geq \beta_1 D_1 + (\beta_2 - 1)D_2,$$

In particular, for small delay costs D_1 and D_2 or where there are no these costs at all (so, $D_1 = D_2 = 0$), the altruistical behavior is more profitable for users than selfish one.

The following theorem compares selfish altruistical behavior with cooperative one (note, that to escape bulky formulas here we consider only the case where there are no delay costs at all).

Theorem 5. *Let $D_1 = D_2 = 0$. Then*

$$v_1^J \geq v_1^c, \quad v_2^J \geq v_2^c.$$

if and only if

$$\alpha_1(1 - 2\alpha_2)(T_1 - T_{11}) + \alpha_2(1 - 2\alpha_1)(T_2 - T_{22}) \geq 0,$$
$$\beta_1(1 - 2\beta_2)(T_1 - T_{11}) + \beta_2(1 - 2\beta_1)(T_2 - T_{22}) \geq 0.$$

In particular,
(a) if

$$\alpha_1, \alpha_2, \beta_1, \beta_2 \geq 1/2$$

then selfish altruistical behavior always more profitable than cooperative one.
(b) if

$$\alpha_1, \alpha_2, \beta_1, \beta_2 < 1/2$$

then cooperative one is more profitable than selfish altruistical behavior.

Similarly we can compare altruistical behavior with cooperative solution.

Theorem 6. *Let* $D_1 = D_2 = 0$*. Then*

$$v_1^J \geq v_1^b, \quad v_2^J \geq v_2^b.$$

if and only if

$$2\alpha_1\alpha_2 T_2 \geq \alpha_1(T_1 - T_{11}) + \alpha_2(T_2 - T_{22}),$$
$$2\beta_1\beta_2 T_1 \geq \beta_1(T_1 - T_{11}) + \beta_2(T_2 - T_{22}).$$

In particular, if $\alpha_1, \beta_2 < 1/2$ *then bargaining one is more profitable than selfish altruistical behavior.*

The Pareto optimal boundary is just the line segment L joining two points: $(-\alpha_1 D_1+\alpha_2(T_2-D_2), -\beta_1 D_1+\beta_2(T_2-D_2))$ and $(-\alpha_2 D_2+\alpha_1(T_1-D_1), -\beta_2 D_2+\beta_1(T_1-D_1))$. The security level is $(-\alpha_1 D_1-\alpha_2 D_2, -\beta_1 D_1-\beta_2 D_2)$ which presents status quo point (x_*, y_*). Thus, to find Shapley solution for the bargaining problem we have to find the point $(x, y) \in L$ maximizing

$$T := (x - x_*)(y - y_*) = (x + \alpha_1 D_1 + \alpha_2 D_2)(y + \beta_1 D_1 + \beta_2 D_2)$$

where

$$y = -\frac{\beta_1 T_1 - \beta_2 T_2}{\alpha_1 T_1 - \alpha_2 T_2} x + \frac{(\beta_1\alpha_2 - \beta_2\alpha_1)(D_1 T_2 + D_2 T_1 - T_1 T_2)}{\alpha_1 T_1 - \alpha_2 T_2}$$

Theorem 7. *The Shapley solution for the bargaining problem* (v_1^{Jb}, v_2^{Jb}) *is given by*

$$v_1^{Jb} = \frac{\alpha_1\beta_2 - \alpha_2\beta_1}{2(\beta_2 T_2 - \beta_1 T_1)} T_1 T_2 - \alpha_1 D_1 - \alpha_2 D_2,$$
$$v_2^{Jb} = \frac{\alpha_2\beta_1 - \alpha_1\beta_2}{2(\alpha_2 T_2 - \alpha_1 T_1)} T_1 T_2 - \beta_1 D_1 - \beta_2 D_2.$$

4 Discussion

In this paper we considered a single-receiver random access system of transmitters (users) with altruistic payoffs which are a generalized weighted individual objectives of their throughput rewards, transmission energy costs and delay costs. We compared altruistical behaviour with selfish (Nash equilibrium), cooperative (Shapley vector) and bargaining behaviour (Nash bargaining solution). We produced criteria where altruistical behaviour is more profitable for a user than either selfish, or cooperative, or bargaining ones.

Finally we note that as the other altruistical payoffs for users we could take weighted fairness utility ([5]), namely, the user altruistical payoffs could be given as follows:
for $\alpha \neq 1$

$$\begin{aligned} u_1^F &= \alpha_1 \frac{u_1^{1-\alpha}}{1-\alpha} + \alpha_2 \frac{u_2^{1-\alpha}}{1-\alpha}, \\ u_2^F &= \beta_1 \frac{u_1^{1-\alpha}}{1-\alpha} + \beta_2 \frac{u_2^{1-\alpha}}{1-\alpha} \end{aligned}$$

and for $\alpha = 1$

$$\begin{aligned} u_1^F &= \alpha_1 \ln(u_1) + \alpha_2 \ln(u_2), \\ u_2^F &= \beta_1 \ln(u_1) + \beta_2 \ln(u_2). \end{aligned}$$

Here we just briefly produce the mixed equilibrium strategies for the plot where there are no delay costs D_1 and D_2 at all (so, $D_1 = D_2 = 0$). Then the mixed Nash equilibrium $((p_1, 1-p_1), (p_2, 1-p_2))$ is given as follows:
for $\alpha = 1$

$$\begin{aligned} p_1 &= \frac{\alpha_1 T_2}{(\alpha_1 + \alpha_2)(T_2 - T_{22})}, \\ p_2 &= \frac{\beta_2 T_1}{(\beta_1 + \beta_2)(T_1 - T_{11})}, \end{aligned}$$

for $\alpha \neq 1$

$$\begin{aligned} p_1 &= \frac{T_2}{T_2 - T_{22}} \frac{T_1 - (T_1 - T_{11})p_2}{T_1 - \left(1 - \frac{\beta_1 \alpha_2}{\beta_2 \alpha_1}\right)(T_1 - T_{11})p_2}, \\ \frac{\beta_1 \alpha_2}{\beta_2 \alpha_1} \frac{(T_1 - T_{11})p_2}{T_1 - (T_1 - T_{11})p_2} &= \frac{1}{T_2 - T_{22}} \left(\frac{\alpha_2}{\alpha_1} \frac{(T_2 - T_{22})p_2^{1-\alpha}}{(T_1 - (T_1 - T_{11})p_2)^{1-\alpha}} \right)^{1/\alpha}. \end{aligned}$$

References

1. Abramson, N.: The Aloha system – another alternative for computer communications. In: AFIPS Conference Proceedings, vol. 36, pp. 295–298 (1970)
2. Altman, E., El Azouzi, R., Jimenez, T.: Slotted Aloha as a game with partial information. Computer Networks 45, 701–713 (2004)
3. Sagduyu, Y.E., Ephremides, A.: A Game-Theoretic Analysis of Denial of Service Attacks in Wireless Random Access. Journal of Wireless Networks 15, 651–666 (2009)
4. Roberts, L.G.: Aloha packet system with and without slots and capture, Tech. Rep. Note 8, Stanford Research Institute, Advance Research Projects Agency, Network Information Center (1972)
5. Altman, E., Avrachenkov, K., Garnaev, A.: Generalized Alpha-Fair Resource Allocation in Wireless Networks. In: Proc. of the 47th IEEE Conference on Decision and Control (CDC 2008), Cancun, Mexico, pp. 2414–2419 (2008)