Efficient Resource Allocation Algorithm for Spatial Multiuser Access in MISO OFDMA Systems

Vasileios D. Papoutsis, Ioannis G. Fraimis, and Stavros A. Kotsopoulos

Wireless Telecommunications Laboratory,
Department of Electrical and Computer Engineering,
University of Patras, Rio, Greece 265 00
{vpapoutsis,ifraimhs,kotsop}@ece.upatras.gr

Abstract. The problem of user selection and resource allocation for the downlink of wireless systems operating over a frequency-selective channel is investigated. It is assumed that the Base Station (BS) uses many antennas, whereas a single antenna is available to each user and Orthogonal Frequency Division Multiple Access (OFDMA) is used as a multiple access scheme. The general mathematical formulation is provided but achieving the optimal solution has a high computational cost. For practical implementation, a suboptimal, but efficient algorithm is devised that is based both on Zero Forcing (ZF) beamforming and on spatial correlation and is less complex than other approaches. The algorithm maximizes the sum of the users' data rates subject to constraints on total available power and proportional fairness among users' data rates. Simulation results are provided to indicate that the algorithm can satisfy the fairness criterion. Thus, the algorithm can be applied to latest-generation wireless systems that provide Quality-of-Service (QoS) guarantees.

Keywords: MISO, OFDMA, resource allocation, Zero-Forcing, proportional fairness.

1 Introduction

OFDMA [1] is a multi-user version of the popular Orthogonal Frequency Division Multiplexing (OFDM) [2] digital modulation scheme. In OFDMA, multiple access is achieved by first dividing the spectrum of interest into a number of subcarriers and then assigning subsets of the subcarriers to individual users. OFDMA helps exploit multiuser diversity in frequency-selective channels, since it is very likely that some subcarriers that are "bad" for a user are "good" for at least one of the other users [3]. Because of its superior performance in frequency-selective fading wireless channels, OFDMA is the modulation and multiple access scheme used in latest wireless systems such as IEEE 802.16e (Mobile WiMAX).

In recent years, many dynamic resource allocation algorithms have been developed for the Single Input Single Output (SISO)-OFDMA systems. In [4] [5], the system throughput is maximized with a total power constraint and in [6] [7],

I. Tomkos et al. (Eds.): BROADNETS 2010, LNICST 66, pp. 198–212, 2012.

the total power consumption is minimized with constraints on the users' data rates. In [8]-[11], proportional fairness among the users' data rates and in [12] [13], the fulfillment of every user's data rate constraints are guaranteed in order to maximize the sum of the users' data rates. In [14], the sum throughput is maximized with long term access proportional fairness. Finally, in [15], system throughput is maximized but the resource allocation unit is not the subcarrier, as in previous algorithms [4]-[14], but a time/frequency unit (slot), in accordance with WiMAX systems.

An additional major advance in recent wireless systems is the use of Multiple Input Multiple Output (MIMO) transmission to improve communication performance. The capacity of the broadcast channel (downlink) has been studied extensively in [16] [17]. In fading environments MIMO technology offers significant increase in the data throughput and the link range without additional bandwidth or transmit power requirements by opening up multiple data pipes in the same frequency band of operation [3]. Because of these properties, MIMO systems have received increasing attention in the past decade.

When Channel State Information (CSI) is available to both the transmitter and the receiver, in general, in order to achieve transmission on the boundary of the capacity region, the BS needs to transmit to multiple users simultaneously in each subchannel, and needs to employ Dirty Paper Coding (DPC) [18]. An iterative algorithm for computing the sum data rate is presented in [19]. In [20] [21], DPC is combined with QR decomposition to completely eliminate the interference among transmitting users. However, DPC-based techniques have large implementation complexity. For this reason, suboptimal transmission methods with smaller complexity have been proposed.

Space Division Multiple Access (SDMA) using transmit beamforming has been proposed as a promising solution for resource allocation that retains the benefits of MIMO and is less complex than DPC-based techniques [22]. In SDMA, a group of compatible users share the common resources improving the efficiency of the system but transmitting to one user affects the transmission of all others. In order to separate users in downlink transmission, ZF transmit beamforming is used [23]. In [24], a suboptimal user selection and beamforming algorithm for the Multiple Input Single Output (MISO) case is proposed and the idea is extended to the MIMO case in [25] [26]. A more realistic scenario is described in [27], where per-antenna power constraints are applied. Moreover, in [28], a low complexity space-time-frequency scheduling is introduced and in [29], multiuser diversity in MIMO systems with antenna selection is analysed. Finally, in [30], a new user selection algorithm is proposed that offers considerable complexity savings.

MIMO related algorithms can be implemented in each subcarrier and by combining OFDMA with MIMO transmission, wireless systems can offer larger system capacities and improved reliability. The data rate maximization problem was first considered in [31], but SDMA was not enabled. In [32], the performance of [31] is challenged and in [33], the total power consumption is minimized. In [34], the system capacity is maximized but certain users may be completely shut off during a scheduling period. In addition, in [35], the priority of users is dynamically adjusted frame by frame and the aim in [36], is to maximize the MISO-OFDMA system throughput with the constraints of total available power and Bit Error Rate (BER) while supporting a kind of fairness among users.

In this paper, a user selection and resource allocation algorithm for multiuser downlink MISO-OFDMA systems is developed that is less complex than other approaches and incorporates fairness by imposing proportional constraints [8]-[11] among the users' data rates. The beamforming scheme of [24] [30] is applied in each subcarrier, where each user experiences flat fading [2], but the user selection procedure takes fairness into account. Simulation results sustain its effectiveness in distributing the sum data rate fairly and flexibly among users.

The remainder of the paper is organized as follows. A description of the MISO-OFDMA system model is introduced in Section 2, whereas the problem of sum data rate maximization using proportional data rate constraints is formulated in Section 3. The proposed algorithm is introduced in Section 4 and Section 5 contains the complexity analysis of the proposed algorithm and a complexity comparison with other algorithms. Simulation results, analysis and a comparison between the proposed algorithm and previous resource allocation schemes are provided in Section 6. Finally, Section 7 contains concluding remarks.

In the following, $(\cdot)^T$ denotes transpose, whereas $(\cdot)^*$ denotes conjugate transpose. $\mathbf{x}$ denotes a column vector, $\mathbf{A}$ denotes a matrix, and $\|\cdot\|$ represents the Euclidean norm. Finally, $[x]_+ = \max\{0, x\}$.

2 System Model

Consider an OFDMA downlink transmission with N subcarriers, T transmit antennas at the BS and K active users, each equipped with a single receive antenna. Also, let B be the overall available bandwidth, and $\mathbf{h}_{k,n} = [h_{k,n}^1 \ \dots \ h_{k,n}^T]^T$ be the $T \times 1$ baseband equivalent gain vector of the channel between the BS and user k in subcarrier n. Thus, for each subcarrier n, the baseband equivalent model for the system can be written as

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{x}_n + \mathbf{z}_n, \tag{1}$$

where

$$\mathbf{H}_n = [\mathbf{h}_{1,n} \ \mathbf{h}_{2,n} \ \dots \ \mathbf{h}_{K,n}]^T$$

is a $K \times T$ matrix with complex entries, $\mathbf{x}_n = [x_{1,n} \dots x_{T,n}]^T$ is the $T \times 1$ transmitted signal vector in subcarrier n, $\mathbf{y}_n = [y_{1,n} \dots y_{K,n}]^T$ is a $K \times 1$ vector containing the received signal of each user, and $\mathbf{z}_n = [z_{1,n} \dots z_{K,n}]^T$ is a $K \times 1$ vector denoting the noise that is assumed to be independent identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian with covariance matrix $\sigma^2 \mathbf{I}_K$.

It is also assumed that the channel vectors are statistically independent and that their distribution is continuous. Hence, $\text{rank}(\mathbf{H}_n) = \min(T, K)$. Moreover, the practically important case where $K \geq T$ is considered. Hence, $\text{rank}(\mathbf{H}_n) = T$. The total transmitted power, in the entire OFDM symbol, is P_{tot} and equal power is allocated to each subcarrier. Hence, trace$[\mathbf{C}_n] \leq \frac{P_{tot}}{N}$, where $\mathbf{C}_n = \mathbb{E}[\mathbf{x}_n (\mathbf{x}_n)^*]$ is the covariance matrix of the transmitted signal $\mathbf{x}_n$.

Using only transmit beamforming, which is a suboptimal strategy, the following model is obtained. Let $\mathbf{w}_{k,n} = [w_{k,n}^1 \; w_{k,n}^2 \; \ldots \; w_{k,n}^T]^T$ be the $T \times 1$ beamforming weight vector for user k in subcarrier n. Then, the baseband model (1) can be written as

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{W}_n \mathbf{D}_n \mathbf{s}_n + \mathbf{z}_n, \tag{2}$$

where

$$\mathbf{W}_n = [\mathbf{w}_{1,n} \; \mathbf{w}_{2,n} \; \ldots \; \mathbf{w}_{K,n}]$$

is the $T \times K$ beamforming weight matrix, $\mathbf{s}_n = [s_{1,n} \ldots s_{K,n}]^T$ is a $K \times 1$ vector containing the signals destined to each user, and

$$\mathbf{D}_n = \begin{pmatrix} \sqrt{p_{1,n}} & 0 & \ldots & 0 \\ 0 & \sqrt{p_{2,n}} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sqrt{p_{K,n}} \end{pmatrix}$$

accounts for the distribution of the power allocated to subcarrier n among the K users. According to (2), the resulting received signal vector for user k in subcarrier n, is given by

$$\begin{aligned} y_{k,n} &= \sum_{i=1}^{K} \mathbf{h}_{k,n} \mathbf{w}_{i,n} \sqrt{p_{i,n}} s_{i,n} + z_{k,n} = \\ &= \mathbf{h}_{k,n} \mathbf{w}_{k,n} \sqrt{p_{k,n}} s_{k,n} + \\ &+ \sum_{i=1, i \neq k}^{K} \mathbf{h}_{k,n} \mathbf{w}_{i,n} \sqrt{p_{i,n}} s_{i,n} + z_{k,n}, \end{aligned} \tag{3}$$

where the term in third line in (3) represents the multi-user interference caused by the simultaneous transmission of data to other users in subcarrier n. Concerning (3), a graphic representation of the MISO downlink beamforming block diagram is shown in Fig. 1.

3 Problem Formulation

ZF beamforming is a spatial signal processing by which the multiple antenna transmitter can null multiuser interference signals in wireless communications. It inverts the channel matrix at the transmitter in order to create orthogonal channels between the transmitter and the receiver. The beamforming vectors are selected such that $\mathbf{h}_{i,n} \cdot \mathbf{w}_{j,n} = 0$, for $i \neq j$, and (3) becomes

$$y_{k,n} = \mathbf{h}_{k,n} \mathbf{w}_{k,n} \sqrt{p_{k,n}} s_{k,n} + z_{k,n}.$$

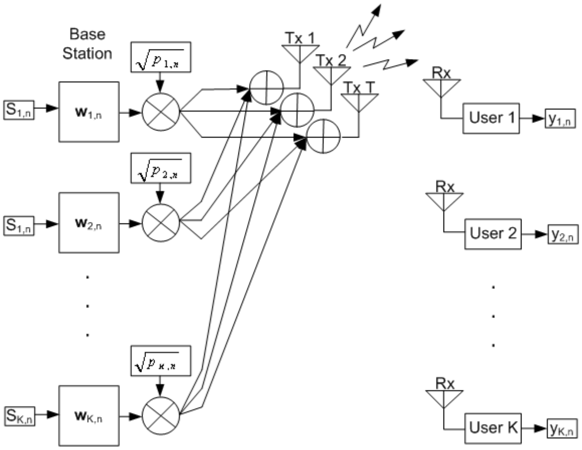

Fig. 1. MISO-OFDMA block diagram

It is then possible to encode users individually, and with smaller complexity compared to DPC. ZF at the transmitter incurs an excess transmission power penalty relative to ZF-DPC and the (optimal) MMSE-DPC transmission scheme. If $K \leq T$ and $\text{rank}(\mathbf{H}_n) = K$, the ZF beamforming matrix is the pseudo-inverse of $\mathbf{H}_n$, namely

$$\mathbf{W}_n = \mathbf{H}_n^* (\mathbf{H}_n \mathbf{H}_n^*)^{-1}. \tag{4}$$

However, if $K > T$, it is not possible to use (4) because $\mathbf{H}_n \mathbf{H}_n^*$ is singular and low complexity SDMA approaches are required. In that case, it is necessary to select $t \leq T$ out of K users in each subcarrier. Hence, there are I possible combinations of users transmitting in the same subcarrier, denoted as A_i, where $A_i \subset \{1, 2, \ldots, K\}$, $0 < |A_i| \leq T$, where $|A_i|$ denotes the cardinality of set A_i, and $I = \sum_{l=1}^{T} \begin{pmatrix} K \\ l \end{pmatrix}$.

Let a set of users $A_i = \{s_1, \ldots, s_t\}$, that produce the row-reduced channel matrix

$$\mathbf{H}_n(A_i) = [\mathbf{h}_{s_1,n} \ \mathbf{h}_{s_2,n} \ \ldots \ \mathbf{h}_{s_t,n}]^T$$

in each subcarrier. When ZF is used, the data rate of user $k \in A_i$, in subcarrier n, is given by [18]

$$r_{k,i,n} = \log_2(\mu_n c_{k,n}(A_i)), \tag{5}$$

where

$$c_{k,n}(A_i) = \{[(\mathbf{H}_n(A_i)\mathbf{H}_n(A_i)^*)^{-1}]_{k,k}\}^{-1} \tag{6}$$

and μ_n is obtained by solving the water-filling equation [37]

$$\sum_{k \in A_i} \left[\mu_n - \frac{1}{c_{k,n}(A_i)} \right]_+ = \frac{P_{tot}}{N}.$$

The power loading then yields

$$p_{k,i,n} = c_{k,n}(A_i)\left[\mu_n - \frac{1}{c_{k,n}(A_i)}\right]_+, \forall k \in A_i.$$

By applying the conclusions above, the linear beamforming optimization problem, that performs user selection in each subcarrier and resource allocation in the entire OFDM symbol, can be formulated as

$$\max_{\rho_{k,i,n}, p_{k,i,n}} \frac{B}{N} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{i=1}^{I} \rho_{k,i,n} r_{k,i,n} \tag{7}$$

subject to

$$\rho_{k,i,n} \in \{0,1\}, \forall k, i, n$$

$$p_{k,i,n} \geq 0, \forall k, i, n$$

$$\sum_{k=1}^{K} p_{k,i,n} \leq \frac{P_{tot}}{N}, \forall n, i$$

$$\sum_{k=1}^{K} \rho_{k,i,n} \leq T, \forall n, i$$

$$R_1 : R_2 : \ldots : R_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K$$

where $\rho_{k,i,n}$ is the subcarrier allocation indicator such that $\rho_{k,i,n} = 1$ if user $k \in A_i$ and A_i is selected in subcarrier n; otherwise $\rho_{k,i,n} = 0$, $k = 1, 2, \ldots, K$, $i = 1, 2, \ldots, I$ and $n = 1, 2, \ldots, N$. The total data rate for user k, denoted as R_k, is defined as

$$R_k = \frac{B}{N} \sum_{n=1}^{N} \sum_{i=1}^{I} \rho_{k,i,n} r_{k,i,n} \tag{8}$$

and $\{\gamma_k\}_{k=1}^{K}$ are the proportional data rate constraints.

The user selection and resource allocation under the fairness criterion (7) is an NP-hard combinatorial optimization problem [38] with non-linear constraints. The optimal solution can be obtained by exhaustive search of all possible user assignment sets in all subcarriers but the complexity is given by I^N, which is extremely complicated even for moderate K and N. The complexity becomes larger by not performing equal power allocation among subcarriers and by not restricting power allocation in each subcarrier to water-filling that, in contrast to the sum data rate case, is not necessarily optimal for the case of proportional fair data rates.

4 The Proposed User Selection and Resource Allocation Algorithm

In the following, a suboptimal, low-complexity user selection and resource allocation algorithm is proposed, that selects users independently in each subcarrier. The proposed algorithm is based on ZF beamforming and on spatial correlation between different users, denoting

$$\eta_{l,m} = \frac{|(\mathbf{h}_{l,n})^{*}\mathbf{h}_{m,n}|}{\| \mathbf{h}_{l,n} \| \| \mathbf{h}_{m,n} \|},\ 0 \leq \eta_{l,m} \leq 1, \tag{9}$$

the spatial correlation between user l and user m in subcarrier n. The larger the $\eta_{l,m}$ is, the more power is required to eliminate the interference between users l, m, and the less sum data rate is achieved.

Let $\mathcal{U} = \{1, 2, \ldots K\}$ denote the set of indices of all K users and $A_i = \{s_1, \ldots, s_t\} \subset \mathcal{U}$ denote the set of t selected users ($|A_i| = t,\ t \leq T$) that share a subcarrier. The proposed algorithm iteratively selects users based on the spatial correlation (9) between the users who have already been selected in subcarrier n, for $n = 1, 2, \ldots, N$, and the remaining ones. In each iteration, it forms a set of candidate users, $\mathcal{Q}$, of size L drawn from the set $\mathcal{U} - A_i$. The members of $\mathcal{Q}$ have the smallest spatial correlation with the users of set A_i. The proposed algorithm comprises the following steps

1. **Initialization:**
    - Set $\mathcal{S} = \{1 \ldots N\}$, $R_k = 0, \forall k \in \mathcal{U}$, $\rho_{k,i,n} = 0, \forall k \in \mathcal{U}$, $i = 1, 2, \ldots, I$ and $n \in \mathcal{S}$.
    - Set $\{\gamma_k\}_{k=1}^{K}$, the proportional data rate constraints.
2. **While** $|\mathcal{S}| \neq \emptyset$:
    - Set $\mathcal{U} = \{1, \ldots, K\}$, $|A_i| = \emptyset$.
    - Find user k satisfying $\frac{R_k}{\gamma_k} \leq \frac{R_i}{\gamma_i}$ $\forall i$, $1 \leq i \leq K$.
    - Find subcarrier $n = \arg\max_{j \in \mathcal{S}} \| \mathbf{h}_{k,j} \|$.
    - Set $t = 1$, $\rho_{k,i,n} = 1$, $A_i(t) = \{k\}$, and $\mathcal{U} = \mathcal{U} - \{k\}$.
    - Compute R_k, according to (5), (8).
    - For $t = 2$ to T:
        - For each $l \in A_i(t-1)$ and $m \in \mathcal{U}$, compute $\eta_{l,m}$, according to (9).
        - Compute the average correlation between already selected users $A_i(t-1)$ and each candidate user $m \in \mathcal{U}$, according to equation $\overline{C}_m = \frac{\sum_{l \in A_i(t-1)} \eta_{l,m}}{|A_i(t-1)|}$.
        - Form the group, $\mathcal{Q}$, of candidates that contains the L users with the lowest values of $\overline{C}_m$, $m \in \mathcal{U}$.
        - Find a user, $s_t \in \mathcal{Q}$, such that

$$\sum_{k \in A_i(t-1) \cup \{s_t\}} r_{k,i,n} > \sum_{k \in A_i(t-1)} r_{k,i,n} \text{ and}$$

$$\left| \frac{R_{s_t} + r_{s_t,i,n}}{\gamma_{s_t}} - \frac{R_k}{\gamma_k} \right| \leq D,\ \forall k \in A_i(t-1)$$

where D is a system parameter that indicates the relation between proportional fairness among the users' data rates and sum of the users' data rates.

- If user s_t is found, set $\rho_{s_t,i,n} = 1$, $A_i(t) = A_i(t-1) \cup s_t$, and $\mathcal{U} = \mathcal{U} - \{s_t\}$.
- Compute R_k, $\forall k \in A_i(t)$, according to (5), (8).

– Set $\mathcal{S} = \mathcal{S} - \{n\}$.

3. **Output:**
   – The users' data rates: R_k, $\forall k = 1, \ldots, K$.
   – The subcarrier allocation indicator: $\rho_{k,i,n}$, for $k = 1, 2, \ldots, K$, $i = 1, 2, \ldots, I$, and $n = 1, 2, \ldots, N$.

The proposed algorithm allocates users and power separately in each subcarrier, but the sequence with which the subcarriers are considered is determined based on the data rates after each iteration. More specifically, the algorithm determines which of the K users is treated most "unfairly" after the last allocation step by calculating $\frac{R_k}{\gamma_k}$, $\forall k \in \mathcal{U}$. The only exception is the first iteration, when all users have zero data rates and any user can be chosen. After determining the user k, all available subcarriers are searched and n is chosen that maximizes the data rate of k if that user were to transmit alone in that subcarrier. Additional users are admitted to the subcarrier based on two criteria: 1) similar to [24] [30], the sum data rate in the subcarrier should increase, and 2) the newly admitted user s_t can achieve "fair" sum, OFDM-symbol, data rate compared to the sum data rate of the other users that have already been admitted to the subcarrier, according to system parameter D. The size of the set $\mathcal{Q}$ is set heuristically equal to $L = \min\{\mathcal{U}, T\}$, because it was shown to lead to good performance in most simulated cases. $A_i(t)$ means the allocation result of the t step. Therefore, at least one, and at most T users can be transmitting in the same subcarrier. Regardless of the number of users, the total power in each subcarrier is equal to $\frac{P_{tot}}{N}$.

5 Complexity Analysis

In order to analyze the computational complexity of the proposed algorithm, recall that K refers to the total number of users in the system and T refers to the number of transmit antennas at the BS. N on the other hand refers to the number of subcarriers, which is much larger than both K and T.

Initialization step of the proposed algorithm requires constant time. In the while loop, which runs for every subcarrier of set $\mathcal{S}$, the best user k among K users for N subcarriers is found, in the worst case scenario. Thus requires $O(KN)$ operations. After determining the best user k for subcarrier n, at most $T-1$ other users are found for subcarrier n. In each one of the $t = 2$ to T steps, the average correlation $\overline{C}_m$ between each candidate user ($m \in \mathcal{U}$) and already selected users ($A_i(t-1)$) must be computed. The computation of spatial correlation $\eta_{l,m}$ for each pair (l, m) can be done within time $O(T)$, as it mainly requires an inner product and a division. Additionally, the computation of $\overline{C}_m$ needs time $O(KT)$ in the worst case scenario. Moreover, the evaluation of at most T data rates $r_{k,i,n}$ is required. In order to evaluate $r_{k,i,n}$, inversion of $\mathbf{H}_n(A_i(t-1) \cup s_t)\mathbf{H}_n(A_i(t-1) \cup s_t)^*$ (6) is required which can be done in time $O(T^2)$, for the worst case when T users are admitted to each subcarrier, using the matrix inversion lemma as described in [24]. Repeating this over at most L users ($s_t \in Q$) in each one of the $t = 2$ to T steps, and over all subcarriers of set $\mathcal{S}$, the complexity is obtained to be $O([KT + LT^2]TN)$. Given that $LT < K$, the complexity becomes $O(KNT^2)$. Thus the overall computational complexity of the proposed algorithm is $O(KNT^2)$.

As was mentioned before the complexity of exhaustive search for the optimal solution of the original problem is given by I^N, where $I = \sum_{l=1}^{T} \binom{K}{l}$. Alternatively, the complexity is $O(K^{NT})$ and is prohibitive even for moderate values of K, N, and T. The complexity of the algorithm described in [34] is $O(K^T N)$ while the complexity of the algorithm that is described in [24] for flat fading channels is $O(KT^3)$. Implementing it in each subcarrier, in order to compare it with our approach, its complexity becomes $O(KNT^3)$. In addition, the complexity of the algorithm proposed in [36] is $\sum_{n=1}^{N} \binom{K}{s_n}$, where $s_n = 1, 2, \ldots, T$. In the worst case scenario, where there are T users transmitting on each subcarrier, the complexity becomes $\sum_{n=1}^{N} \binom{K}{T}$ which is of order $O(K^T N)$.

It is easily observed that the proposed algorithm has a very dramatic reduction in complexity compared to $O(K^{NT})$ required by the exhaustive search. In addition it has similar complexity to [24] and smaller than [34] [36]. Table 1 summarizes the complexity of the algorithms mentioned above.

Table 1. Computational Complexity

Exhaustive Search	Proposed Algorithm	Algorithm in [24]	Algorithm in [34]	Algorithm in [36]
$O(K^{NT})$	$O(KNT^2)$	$O(KNT^3)$	$O(K^T N)$	$O(K^T N)$

6 Simulation Results

In this section, the performance of the algorithm is evaluated using simulation. The proposed algorithm is compared with the algorithm proposed in [24], applying it in each subcarrier, the existing greedy capacity maximization with prescreening [34] algorithm, the fairness based resource allocation [36] algorithm, Round Robin (RR) algorithm and the method that employs transmission, using Maximal Ratio Combining (MRC), only to the user with the strongest channel. In [24], it was suggested, via numerical examples, that the sum data rate of sub-optimal, ZF beamforming schemes can approach that of DPC even for a moderate number of users. However, an inherent drawback of the maximum sum data rate criterion is the lack of fairness, because certain users may be completely shut off during a scheduling period. This is dealt with by imposing

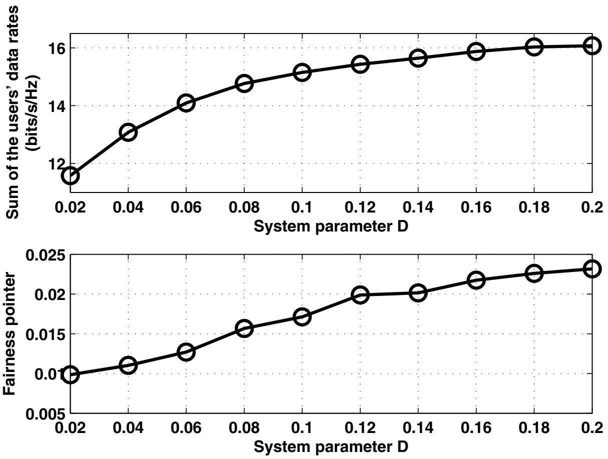

Fig. 2. Choice of system parameter

fairness criteria, such as the proportional data rate constraint [8]-[10] considered in this paper. In RR algorithm, each user is given a fair share of the channel resource regardless of the channel state. T users are selected in each subcarrier. Both equal power (EQ) allocation and water-filling (WF) power allocation over the parallel subchannels are considered.

In all simulations presented in this section, the frequency-selective channel consists of six independent Rayleigh multipath components (taps) for every downlink transmission path between any of the T transmit antennas and users. For each channel realization the proposed algorithm is used. As in [9], an exponentially decaying power delay profile is assumed, the ratio of the energy of the lth tap to the first tap being equal to e^{-2l}. A maximum delay spread of 5μs and maximum doppler of 30Hz is assumed. The channel information is sampled every 0.5ms to update the user selection and resource allocation. BS consists of $T = 4$ transmit antennas and the number of subcarriers is $N = 64$. The number of channel realizations is equal to 1000 and 100 time samples for each realization are used for each user.

For each channel realization, a set of proportional constants $\{\gamma_k\}_{k=1}^{K}$ are assigned to available users. It is assumed that these constants follow the probability mass function

$$p_{\gamma_k} = \begin{cases} 1 \text{ with probability } 0.5 \\ 2 \text{ with probability } 0.3 \\ 4 \text{ with probability } 0.2 \end{cases}$$

In Fig. 2, the performance of the proposed algorithm is shown for different values of system parameter D, when there are $K = 16$ available users in the system and $SNR = 15$. Fairness pointer indicates the maximum difference between users' data rates and respective fairness constraints. It is shown that as the system

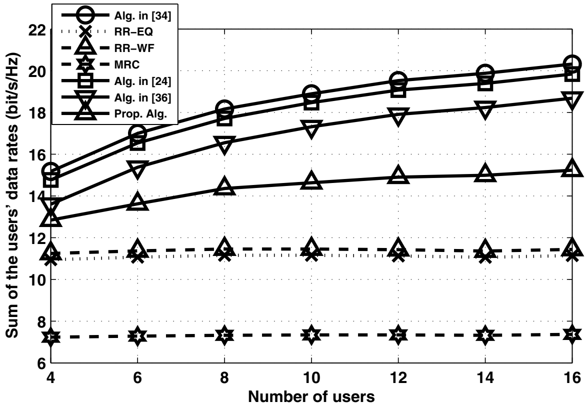

Fig. 3. Sum of the users' data rates vs number of users

parameter becomes larger, the sum of the users' data rates becomes larger too, but the fairness criterion is more relaxed. Thus, the system parameter indicates a tradeoff between sum of the users' data rates and accomplishing fairness between users' data rates.

Figs. 3 - 5, are shown for a fixed system parameter $D = 0.1$, which is chosen heuristically to ensure a reasonable trade off between sum of the users' data rates and accomplishing fairness between users' data rates. In Fig. 3, the number of users varies from 4-16 in increment of 2, while figs. 4, 5 are shown when all, $K = 16$, users are present in the system. In figs. 3, 5, $SNR = 15$.

In Fig. 3 it can be seen, the reasonable price being paid in order to guarantee fairness by using the proposed algorithm. As the number of users increases, the difference in sum data rates also increases because additional multiuser diversity is available to [24] [34] that only target sum data rate maximization. On the other hand, more users put more constraints to the proposed algorithm, because new users need to share the same resources. In addition, sum data rate of the proposed algorithm is significantly enhanced over both RR-WF and RR-EQ algorithm. MRC algorithm is the lower bound of the proposed algorithm as in MRC each subcarrier is allocated to only one user. Sum data rate of [36] is degraded compared with [24] [34] and enhanced over the other algorithms. This is because it imposes a kind of fairness between users' data rates.

The same conclusions can be drawn from Fig. 4, where the sum of the users' data rates is shown as a function of SNR. It can also been seen that MRC is a viable choice only in the low SNR regime.

In Fig. 5, fairness index F_p is a modified version of the one introduced in [9], and is defined as

$$F_p = \frac{(\sum_{k=1}^{K} \frac{R_k}{\gamma_k})^2}{K \sum_{k=1}^{K} (\frac{R_k}{\gamma_k})^2},$$

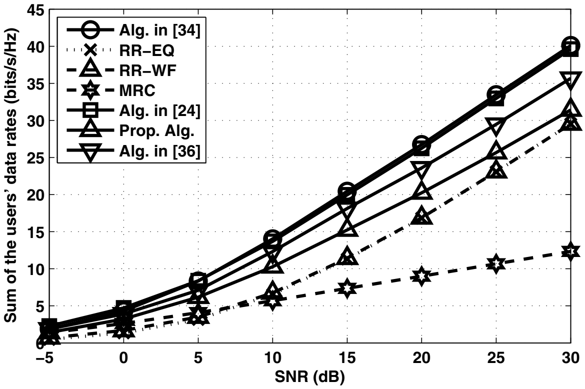

Fig. 4. Sum of the users' data rates vs SNR(dB)

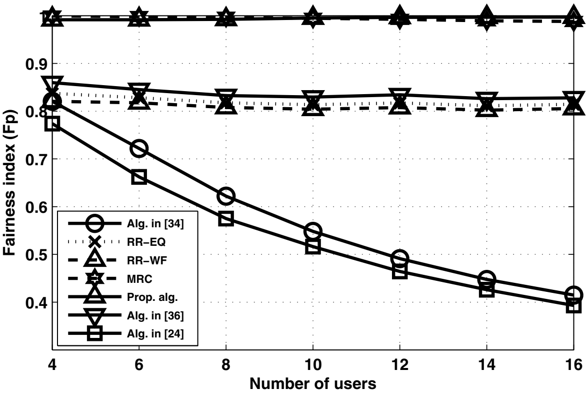

Fig. 5. Fairness index vs number of users

where F_p is a real number in the interval $(0, 1]$ with the maximum value of 1 for the case that the achieved data rate proportions among the users are the same as the predetermined set $\{\gamma_k\}_{k=1}^{K}$. Employing [24] [34], no guarantees are provided for the fairness between user data rates. In addition, as the number of users increases, fairness index degrades. RR-WF, RR-EQ and [36] experience almost the same fairness index regardless of the number of users. This is because these algorithms achieve approximately equal data rates among users. Both the proposed algorithm and MRC distribute the sum data rate very well among users, very close to the defined ideal data rate constraints which is the main point of this paper. However, MRC does not exploit the $T = 4$ degrees of freedom that are available in each subcarrier.

7 Conclusion

A fairness-aware user selection and resource allocation algorithm for the MISO downlink over frequency-selective channels was introduced. The algorithm is based on ZF beamforming and on spatial correlation (9) between users transmitting in the same subcarrier. It maximizes the sum of the users' data rates subject to constraints on total available power and proportional fairness among users' data rates. Although it is suboptimal, as was shown by simulations, the loss with respect to the unconstrained case where the only target is the maximization of the sum data rate is reasonable in order to achieve proportionality among users' data rates. Finally, the proposed algorithm is less complex than other approaches.

References

1. Andrews, J.G., Ghosh, A., Muhamed, R.: Fundamentals of WiMAX: Understanding Broadband Wireless Networking. Prentice-Hall (2007)
2. Nee, R.V., Prasad, R.: OFDM for Wireless Multimedia Communications. Artech House (2000)
3. Tse, D., Viswanath, P.: Fundamentals of Wireless Communication. Cambridge University Press (2005)
4. Jang, J., Lee, K.B.: Transmit power adaptation for multiuser OFDM systems. IEEE J. Sel. Areas Commun. 21(2), 171–178 (2003)
5. Li, G., Liu, H.: On the Optimality of the OFDMA Network. IEEE Commun. Lett. 9(5), 438–440 (2005)
6. Wong, C.Y., Cheng, R.S., Letaief, K.B., Murch, R.D.: Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation. IEEE J. Sel. Areas Commun. 17(10), 1747–1758 (1999)
7. Kivanc, D., Li, G., Liu, H.: Computationally Efficient Bandwidth Allocation and Power Control for OFDMA. IEEE Trans. Wireless Commun. 2(6), 1150–1158 (2003)
8. Mohanram, C., Bhashyam, S.: A Sub-optimal Joint Subcarrier and Power Allocation Algorithm for Multiuser OFDM. IEEE Commun. Lett. 9(8), 685–687 (2005)
9. Shen, Z., Andrews, J.G., Evans, B.L.: Adaptive Resource Allocation in Multiuser OFDM Systems With Proportional Rate Constraints. IEEE Trans. Wireless Commun. 4(6), 2726–2737 (2005)
10. Sadr, S., Anpalagan, A., Raahemifar, K.: Suboptimal Rate Adaptive Resource Allocation for Downlink OFDMA Systems. Intern. J. Veh. Techn. (2009)
11. Papoutsis, V.D., Fraimis, I.G., Kotsopoulos, S.A.: Fairness-aware resource allocation for the SISO downink over frequency-selective channels. In: IEEE Wireless Commun. and Networking Conf., Sydney, NSW, Australia (2010)
12. Zhang, Y.J., Letaief, K.B.: Multiuser Adaptive Subcarrier-and-Bit Allocation With Adaptive Cell Selection for OFDM Systems. IEEE Trans. Wireless Commun. 3(5), 1566–1575 (2004)
13. Mao, Z., Wang, X.: Efficient Optimal and Suboptimal Radio Resource Allocation in OFDMA System. IEEE Trans. Wireless Commun. 7(2), 440–445 (2008)
14. Ma, Y.: Rate-Maximization for Downlink OFDMA with Proportional Fairness. IEEE Trans. Veh. Techn. 57(5), 3267–3274 (2008)

15. Biagioni, A., Fantacci, R., Marabissi, D., Tarchi, D.: Adaptive Subcarrier Allocation Schemes for Wireless OFDMA Systems in WiMAX Networks. IEEE J. Sel. Areas Commun. 27(2), 217–225 (2009)
16. Goldsmith, A., Jafar, S.A., Jindal, N., Vishwanath, S.: Capacity Limits of MIMO Channels. IEEE J. Sel. Areas Commun. 21(5), 684–702 (2003)
17. Rhee, W., Cioffi, J.M.: On the Capacity of Multiuser Wireless Channels With Multiple Antennas. IEEE Trans. Inf. Theory 49(10), 2580–2595 (2003)
18. Weingarten, H., Steinberg, Y., Shamai, S.: The capacity region of the Gaussian MIMO broadcast channel. IEEE Trans. Inf. Theory 52(9), 3936–3964 (2006)
19. Jindal, N., Rhee, W., Viswanath, S., Jafar, S.A., Goldsmith, A.: Sum Power Iterative Water-Filling for Multi-Antenna Gaussian Broadcast Channels. IEEE Trans. Inf. Theory 51(4), 1570–1580 (2005)
20. Caire, G., Shamai, S.: On the Achievable Throughput of a Multiantenna Gaussian Broadcast Channel. IEEE Trans. Inf. Theory 49(7), 1691–1706 (2003)
21. Tu, Z., Blum, R.S.: Multiuser Diversity for a Dirty Paper Approach. IEEE Commun. Lett. 7(8), 370–372 (2003)
22. Sharif, M., Hassibi, B.: A Comparison of Time-Sharing, DPC, and Beamforming for MIMO Broadcast Channels with Many Users. IEEE Trans. Commun. 55(1), 11–15 (2007)
23. Spencer, Q.H., Swindlehurst, A.L., Haardt, M.: Zero-Forcing Methods for Downlink Spatial Multiplexing in Multiuser MIMO Channels. IEEE Trans. Signal Processing 52(2), 461–471 (2004)
24. Dimic, G., Sidiropoulos, N.D.: On Downlink Beamforming With Greedy User Selection: Performance Analysis and a Simple New Algorithm. IEEE Trans. Signal Processing 53(10), 3857–3868 (2005)
25. Yoo, T., Goldsmith, A.: On the Optimality of Multiantenna Broadcast Scheduling Using Zero-Forcing Beamforming. IEEE J. Sel. Areas Commun. 24(3), 528–541 (2006)
26. Shen, Z., Chen, R., Andrews, J., Health Jr., R., Evans, B.L.: Low Complexity User Selection Algorithms for Multiuser MIMO Systems with Block Diagonalization. IEEE Trans. Signal Processing 54(9), 3658–3663 (2006)
27. Yu, W., Lan, T.: Transmitter Optimization for the Multi-Antenna Downlink With Per-Antenna Power Constraints. IEEE Trans. Signal Processing 55(6), 2646–2660 (2007)
28. Fuchs, M., Galdo, G.D., Haardt, M.: Low-Complexity Space-Time-Frequency Scheduling for MIMO Systems With SDMA. IEEE Trans. Veh. Techn. 56(5), 2775–2784 (2007)
29. Zhang, X., Lv, Z., Wang, W.: Performance Analysis of Multiuser Diversity in MIMO Systems with Antenna Selection. IEEE Trans. Wireless Commun. 7(1), 15–21 (2008)
30. Karachontzitis, S., Toumpakaris, D.: Efficient and Low-Complexity User Selection for the Multiuser MISO Downlink. In: IEEE Personal, Indoor and Mobile Radio Commun. Symposium, Tokyo, Japan (2009)
31. Koutsopoulos, I., Tassiulas, L.: Adaptive Resource Allocation in SDMA-based Wireless Broadband Networks with OFDM Signaling. In: IEEE INFOCOM, New York, USA, vol. 3, pp. 1376–1385 (2002)
32. Tsang, Y.M., Cheng, R.S.: Optimal Resource Allocation in SDMA/Multi-Input-Single-Output/OFDM Systems under QoS and Power Constraints. In: IEEE Wireless Commun. and Networking Conf., Georgia, USA, vol. 3, pp. 1595–1600 (2004)

33. Zhang, Y.J., Letaief, K.B.: An Efficient Resource-Allocation Scheme for Spatial Multiuser Access in MIMO/OFDM Systems. IEEE Trans. on Commun. 53(1), 107–116 (2005)
34. Chan, P.W.C., Cheng, R.S.: Capacity Maximization for Zero-Forcing MIMO-OFDMA Downlink Systems with Multiuser Diversity. IEEE Trans. Wireless Commun. 6(5), 1880–1889 (2007)
35. Tsai, C.F., Chang, C.J., Ren, F.C., Yen, C.M.: Adaptive Radio Resource Allocation for Downlink OFDMA/SDMA Systems with Multimedia Traffic. IEEE Trans. Wireless Commun. 7(5), 1734–1743 (2008)
36. Kai, S., Ying, W., Zi-xiong, C., Ping, Z.: Fairness based resource allocation for multiuser MISO-OFDMA systems with beamforming. J. China Univ. of Posts and Telec. 16(1), 38–43 (2009)
37. Cover, T.M., Thomas, J.A.: Elements of Information theory. John Wiley & Sons, Inc., Hoboken (2006)
38. Ibaraki, T., Katoh, N.: Resource Allocation Problems-Algorithmic Approaches, M. Garey, Ed. MIT Press (1988)