On the Scheduling and Multiplexing Throughput Trade-Off in MIMO Networks

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Abstract. In this paper we explore the cross-layer MIMO-MAC resource allocation problem in interference-limited wireless networks. This is primarily motivated by the trade-off between maximizing the throughput of individual non-interfering links, using spatial multiplexing, and maximizing the spatial reuse of lower rate interfering links, using spatial multiplexing in conjunction with nulling. First, we formulate a cross-layer optimization problem that jointly decides the scheduling and MIMO stream allocation in order to maximize the average sum rate of a given set of single-hop links, subject to signal-to-interference-and-noise-ratio (SINR) constraints. Second, we characterize the problem as a non-convex integer programming problem which is quite challenging to solve. However, we show that under low SINR regimes, an approximate problem can be cast into a geometric programming formulation which is convex. Finally, we characterize the optimal solution for the case of two links and utilize the developed decision rules as a basis for a distributed iterative MIMO link scheduling (IMLS) algorithm that achieves significant gains for arbitrary number of links. Numerical results show that, for plausible scenarios, IMLS achieves more than 2-fold improvement over one-link-per-slot utilizing full spatial multiplexing gain.

Keywords: MIMO networks, convex optimization, scheduling, spatial multiplexing, interference nulling.

1 Introduction

Multiple-input multiple-output (MIMO) [1] is a major breakthrough in wireless communications that has received considerable attention in the point-to-point literature due to its substantial spectral efficiency and reliability advantages for the same power and bandwidth. Exploring the multiple access trade-offs of different MIMO schemes in multi-user settings, namely interference mitigation, spatial multiplexing (SM), and diversity, has received less attention. Thus, we focus on the MIMO-MAC resource allocation problem over the interference channel.

The problem of networking MIMO radios has started to receive recent attention in the literature [3, 4, 5, 6, 7, 8, 9, 10]. Exploiting the interference reduction advantages of smart antennas and reducing the MAC overhead constitute major thrusts. However, optimally allocating MIMO spatial streams in network settings has not received sufficient attention. This problem is motivated by a fundamental trade-off between scheduling and spatial multiplexing. In this paper, we analyze

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this trade-off which reveals SINR-based decision rules that constitute a foundation for future MAC protocols. It constitutes a first step towards understanding the more general diversity-multiplexing-scheduling trade-off. Hence, our focus in this paper is on the trade-off, problem formulation, complexity and solution approach. Protocol design and performance comparison to other protocols lie out of the scope of this work and is a subject of future research.

Our contribution in this paper is three-fold: i) Formulating the cross-layer MIMO-MAC resource allocation problem, ii) Investigating convexity and casting an approximate problem into convex geometric programming, under low SINR and iii) Introducing Iterative MIMO Link Scheduling (IMLS) that demonstrates significant improvement over scheduling non-interfering links with full Spatial multiplexing gain.

First, we characterize the MIMO-MAC resource allocation problem as a non-convex integer programming problem which is quite challenging. Hence, we cast an approximate problem as convex geometric programming under low SINR. Next, we characterize the optimal policy for two links and employ the decision rules as a foundation for scheduling arbitrary number of MIMO links using IMLS. Finally, we present numerical results for plausible scenarios that not only confirm the trade-off at hand but also show the IMLS throughput gains.

The paper is organized as follows: In section 2, we discuss related work in the literature. We introduce the assumptions and formulate the problem in section 3. In section 4, we analyze the complexity of the problem and formulate approximate problems. Next, we develop SINR-based decision rules that characterize the optimal solution for two links and constitute the basis for IMLS in section 5. In section 6, we show performance results for a number of interference scenarios. Finally, conclusions are drawn in section 7.

2 Related Work

Recent work has focused on the design of MAC protocols that exploit the unique capabilities offered by networking MIMO nodes [3, 4, 5, 6, 7, 8, 9, 10]. [4] focuses on handling the non-negligible encoding and decoding delays caused by Lucent's V-BLAST [18]. It introduces mechanisms for reducing the MAC overhead (e.g. RTS/CTS) as well as parallel stop-and-wait ARQ scheme to remedy the per packet ACK. [5] explores the role of spatial diversity schemes (e.g. space-time coding (STC)) to combat fading and achieve robustness in MIMO-enabled ad hoc networks. [6] introduces distributed scheduling for MIMO ad hoc networks (DSMA) within the CSMA/CA framework where SM stream allocation depends on the transmitter-receiver distance. In [7], SM with antenna subset selection for data packet transmission is proposed. In [8], the authors compare the asymptotic network spectral efficiency in the presence and absence of channel state information (CSI) at the transmitters. In fact, the theoretical study in [8] motivated us to investigate cross-layer multiplexing-scheduling schemes that balance this trade-off. In [9], three MIMO MAC protocols are introduced, namely SRP, SMP and SRMP, however, the multiplexing-scheduling trade-off is not analyzed.

Unlike our approach, the interference model is not SINR-based and there are no insights as to which protocol is the best under what conditions.

This work extends upon our earlier work [11] in which we limited our attention to the simple case of two MIMO links without solving or analyzing the complexity of the general number of links case. In [12], we studied the problem of cross-layer diversity and scheduling optimization for reliable communications. Despite the fundamental differences between [12] and the problem at hand, which focuses on multiplexing and scheduling optimization for maximizing the average sum rate, both problems lend themselves to a somewhat similar solution approach which is quite interesting. This, in turn, calls for a unified framework for both problems which constitutes an entry point to the generalized scheduling-multiplexing-diversity problem in future work.

In [3], the authors present stream controlled multiple access (SCMA) for MIMO ad hoc networks. It focuses on SM and explores the gains of stream control and partial interference suppression. However, this work differs from [3] with respect to the following: i) Formulating a cross-layer optimization problem that formally captures the scheduling-spatial multiplexing trade-off, ii) Investigating complexity and formulating an approximate geometric programming problem and iii) Developing distributed SINR-based decision rules that are inspired by the optimal solution for two links and serve as the basis for iteratively scheduling the MIMO links using IMLS.

3 Joint MIMO-MAC Resource Allocation

3.1 Assumptions

We focus on the interference channel with K MIMO links involving $2K$ distinct stationary nodes. Two types of interference may arise; *Primary interference*, e.g. common receiver and self-interference. *Secondary interference* arises when a receiver, Rx, receiving from a particular transmitter, Tx, overhears other transmissions intended elsewhere. In this paper, we target the more challenging secondary interference while handling primary interference is considered complementary to this work and lies out of its scope.

Each node is supported by M transmit antennas and N receive antennas. We assume that the channel state information (CSI) is known only at the receiver, not at the transmitter. Hence, we focus on open-loop (OL), as opposed to closed-loop (CL), MIMO systems due to their practical relevance. We assume a pessimistic interference model which accounts for interference contributed by any transmitter at any receiver, no matter how small this interference is. This is justified by our focus on single-hop links in a neighborhood for the purposes of MAC analysis.

All nodes share a single frequency channel, time is divided into slots and the channel is assumed to be constant across the K slots under investigation. We assume fixed power (P) and modulation for all nodes. Accordingly, we focus on optimizing a single PHY variable, namely the number of spatial streams dedicated to SM, denoted X. In order to support SM in conjunction with interferer nulling, we assume a receiver structure that combines both SM receiver algorithms, which typically rely on multi-user detection (MUD), along with adaptive spatial nulling algorithms.

We adopt a Gaussian MIMO channel model where the channel matrix H is perfectly known at the receiver and is deterministic [1]. It is assumed to be an uncorrelated full-rank channel where $r(H) = min(M, N)$. The path loss follows exponential decay with distance, with a path loss exponent α. We model the receiver thermal noise as additive white Gaussian noise (AWGN), with power σ_n^2 dBm. The results of this paper can be extended to frequency-flat independent and identically distributed (*iid*) Rayleigh fading MIMO channels under high SINR since the open-loop capacity is given by $min(M, N)\ log\ SNR + O(1)$ which is similar to the capacity of the Gaussian MIMO channel in (1) below.

It has been shown in [13] that the open-loop capacity (or link spectral efficiency in bps/Hz) of a point-to-point link with M transmit, N receive antennas, a deterministic Gaussian channel with full rank matrix and a SM signaling scheme that attains full spatial multiplexing gain (SMG) (e.g. V-BLAST [2]) grows linearly with the channel rank,

$$C(SNR) = min(M, N)\ log(1 + SNR) \tag{1}$$

Thus, if we model interference as AWGN using the Gaussian approximation, then the link spectral efficiency in a multi-user setting can be approximated as,

$$R(SINR) \approx min(M, N)\ log(1 + SINR)$$

3.2 Problem Formulation

In this section, we formulate a cross-layer optimization problem that strikes a balance between activating high rate non-interfering links and simultaneously activating interfering lower rate links to maximize the average sum link rate. Our formulation studies K links over K slots to be able to compare to the baseline policy, namely one link with full SMG per slot.

Given K links and K slots, we define F as the optimization objective function that is given by the average sum link rate,

$$F \ = \ \frac{1}{K} \sum_{i=1}^{K} \sum_{j=1}^{K} R_{ij} \tag{2}$$

where i is the link index and j is the slot index. R_{ij} is the bit rate supported by link i in slot j and is approximated by,

$$R_{ij} = X_{ij}\ log(1 + SINR_{ij}) \tag{3}$$

where X_{ij} is the number of SM streams of link i in slot j and $SINR_{ij}$ is the SINR of link i in slot j and is given by,

$$SINR_{ij} = \frac{P_{ij}\ G_i^i\ G_r}{\sigma_n^2 + \sum_{k=1,k\neq i}^{K-[\frac{N}{X_{ij}}-2]} P_{kj} G_k^i} \tag{4}$$

Notice that $SINR_{ij}$ given in (4) is per SM stream, where a SM stream may include more than one data stream in case $X_{ij} < min(M, N)$. G_u^v is the path loss gain between the transmitter of link u and receiver of link v. $G_r = \frac{N}{X_{ij}}$ is the receiver array gain attributed to exploiting the CSI available at the receiver to null using the $\frac{N}{X_{ij}}$ streams within a single SM stream. Finally, the summation term in the denominator (interference) assumes interferers are spatially separated from the signal of interest to perfectly null the $(\frac{N}{X_{ij}} - 2)$ strongest interferers. This is a reasonable assumption in light of state-of-the-art beamforming algorithms [17]. For instance, the optimal beam former with L antennas can null up to $(L - 2)$ interferers using its $(L - 1)$ degrees of freedom (DoF) where a single DoF is utilized to detect the signal of interest.

The problem is formulated as a constrained optimization problem that maximizes F subject to SINR among other range constraints,

$$\mathbf{P1}: \quad \max_{\underline{X}, \underline{P}} F \tag{5}$$

$$\text{s.t.} \quad \begin{array}{ll} SINR_{ij} \geq \beta & \forall i, j \\ P_{ij} = \{0, P\} & \forall i, j \\ 1 \leq X_{ij} \leq min(M, N) & \forall i, j \\ X_{ij} \quad Integer & \forall i, j \end{array}$$

where the optimization variables are $\underline{X} = [X_{ij}]$, vector of number of SM streams for link i in slot j and $\underline{P} = [P_{ij}]$, vector of binary variables representing the link-slot assignment for link i in slot j, such that $P_{ij} = P$ when link i is activated in slot j, otherwise $P_{ij} = 0$. β is a minimum requirement on the SINR that is necessary for successful reception.

4 Problem Complexity

4.1 Non-convexity of P1

Motivated by the recent advances in convex optimization and its applications to wireless communications [14], we investigate the convexity of P1. We examine three complexity aspects of the problem, namely the integer optimization variables X_{ij} and P_{ij}, concavity of objective function F and convexity of the SINR constraints. Afterwards, we introduce approximate formulations to show how efficiently the MIMO-MAC resource allocation problem can be solved.

The main challenges towards solving P1 stem from: i) The non-concavity of F attributed to the non-concavity of R_{ij} in the presence of interference [14], ii) The Bilinear Matrix Inequality (BMI) nature of the SINR constraints and iii) The integer optimization variables X_{ij} and P_{ij}. Hence, P1 is characterized as a *non-convex integer programming* problem which is quite challenging to solve.

4.2 Approximate Problems

First, we tackle the integer programming challenge. We relax variables X_{ij} and P_{ij} to be real and denote the continuous variable optimization problem subject to the same constraints as **P2**. This relaxation is typically used for solving integer programming problems and, hence, can be justified for our problem where a continuous optimization problem is solved in each iteration of the branch and bound algorithm.

Second, we examine the convexity of P2. Even though the SINR constraints are non-linear, as written, they can be re-written in a bilinear form as follows.

Lemma 1. *The SINR constraints are non-convex, bilinear with respect to the* P_{ij} *and* X_{ij} *optimization variables.*

Proof. The SINR constraints can be written as follows,

$$P_{ij}\ G_i^i\ N\ \geq\ X_{ij}\ \beta\ (\sigma_n^2 + \sum_{k=1,k\neq i}^{K-[\frac{N}{X_{ij}}-2]} P_{kj}G_k^i) \qquad \forall i,j \tag{6}$$

Re-writing it in the standard form ($f(x)\ \leq\ 0$) yields,

$$g(P_{11}, P_{12}, ...P_{KK}, X_{ij})\ =$$

$$\beta\ \sigma_n^2\ X_{ij} +\ \beta\ X_{ij} \sum_{k=1,k\neq i}^{K-[\frac{N}{X_{ij}}-2]} P_{kj}G_k^i\ -\ G_i^i\ N\ P_{ij}\ \leq\ 0\ \ \forall i,j \tag{7}$$

The first term of g is linear in X_{ij} and the last term is linear in P_{ij}. However, the summation in the middle term includes the product of P_{kj} and X_{ij} and, hence, is bilinear. In fact, the Hessian of g does not satisfy the second derivative condition of convexity $\bigtriangledown^2 g(\underline{P}, \underline{X})\ \geq\ 0$ since it has a negative eigenvalue. Hence, we conclude that $g(\underline{P}, \underline{X})$ is non-convex, bilinear.

Unfortunately, BMI problems are known to be non-convex [19] and, moreover, there are no systematic procedures in the literature for solving BMIs. This adds even more complexity to the problem. Notice that the above result differs from prior results studying the convexity/linearlity of SINR constraints due to the following reasons. SINR convexity has been examined under different problems, e.g. [14] studied SINR convexity with respect to antenna weights in the transmitter beamforming problem. For power control problems, the SINR constraints are simply linear in the powers [15]. Another reason is due to our focus on the MIMO-MAC problem where optimization is with respect to two sets of variables, namely X_{ij} and P_{ij} $\forall i, j$. This gives rise to bilinear terms in $g(\underline{P}, \underline{X})$ as seen above which immediately suggests that g is no longer linear and examining convexity is in order.

In the rest of this section, we examine low and high SINR regimes in an attempt to circumvent the above hurdles.

We show that, under low SINR, P2 can be approximated to a convex geometric programming problem. The MIMO rate function becomes $R_{ij} \approx X_{ij}\ SINR_{ij}$, which yields problem **P3** maximizing $F = \frac{1}{K}\sum_{i=1}^{K}\sum_{j=1}^{K} X_{ij}SINR_{ij}$ subject to the same constraints of the continuous variable version of P1, namely P2.

Lemma 2. *Under low SINR, the objective function F in P3 is non-linear and non-concave.*

Proof. Under low SINR, F is approximated by,

$$F = \frac{1}{K} \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{P_{ij}\ G_i^i\ N}{\sigma_n^2 + \sum_{k=1,k\neq i}^{K-[\frac{N}{X_{ij}}-2]} P_{kj} G_k^i} \tag{8}$$

Notice that F is non-linear in P_{ij} and X_{ij} $\forall i, j$ due to their contributions in the denominator of each term in (8). Hence, examining concavity is in order. With respect to P_{ij}, individual terms as stated in (8) have the same structure as the SINR in classical power control problems, known to be non-concave [16]. Moreover, the terms in (8) monotonically decrease as X_{ij} increases. Hence, F in P3 is non-concave.

This, in turn, yields the following complexity result for P3.

Theorem 1. *Under low SINR, the approximate problem P3 is not convex.*

Proof. The result follows directly from the non-convexity of the SINR constraints shown in Lemma 1 and non-concavity of F shown in Lemma 2.

It should be noted that the low SINR approximation does not reduce P3 in its current form to a convex problem. This is in agreement with [16] which has to cast the SINR maximization problem into a geometric programming formulation to establish convexity. On the contrary, the low SINR approximation directly renders the problem of minimizing the sum of powers subject to rate constraints linear as shown in [15].

Even though P3 is not convex in its current form, an approximate problem can be cast into a geometric programming formulation similar to [16]. This is attributed to the fact that $\frac{1}{X_{ij}\ SINR_{ij}}$ is a posynomial function, where a function $f(x)$ is said to be posynomial if it takes the following form:

$f(x) = \sum_k c_k\ x_1^{a_{1k}}\ x_2^{a_{2k}} ... x_n^{a_{nk}}$ where $c_k \geq 0$ and a_{ik} is real. This yields a new objective function to minimize: $U = K \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{1}{X_{ij}\ SINR_{ij}}$ that is posynomial.

However, this is an "approximate" problem since $U \neq F^{-1}$. Similarly, the SINR constraint can be re-formulated into a posynomial form in order to reach the approximate geometric programming problem P4 known to be convex,

$$\mathbf{P4}: \quad \min_{\underline{X}, \underline{P}} U \tag{9}$$

$$\text{s.t.} \quad \begin{array}{ll} SINR_{ij}^{-1} \leq \beta^{-1} & \forall i, j \\ 0 \leq P_{ij} \leq P & \forall i, j \\ 1 \leq X_{ij} \leq min(M, N) & \forall i, j \end{array}$$

Under high SINR, the problem turns out to be more challenging and cannot be approximated to convex geometric programming primarily because of the role of the stream allocation variables X_{ij} in breaking the posynomial structure of the objective function approximation. Accordingly, the MIMO rate function becomes logarithmic in the SINR, i.e. $R_{ij} \approx X_{ij}\ logSINR_{ij}$. We show next that this non-linear, non-convex problem (due to the BMI nature of the SINR constraints), denoted **P5**, cannot be approximated to geometric programming.

Theorem 2. *Under high SINR, problem P5 cannot be approximated to a geometric programming problem.*

Proof. The objective function F of P5 can be written as,

$$F = \frac{1}{K} \sum_{i=1}^{K} \sum_{j=1}^{K} X_{ij}\ logSINR_{ij} \tag{10}$$

$$= \frac{1}{K}\ log \prod_{i=1}^{K} \prod_{j=1}^{K} SINR_{ij}^{X_{ij}} \tag{11}$$

We propose to minimize a related function V, even though it is not exactly equivalent to maximizing F since $V \neq F^{-1}$.

$$V = \prod_{i=1}^{K} \prod_{j=1}^{K} \frac{1}{SINR_{ij}^{X_{ij}}} \tag{12}$$

The objective is to reach a posynomial function which facilitates geometric problem formulation. However, the role of X_{ij} as $SINR_{ij}$ exponent does not yield the posynomial structure. This confirms that, unlike P4, problem P5 cannot be cast into a geometric programming formulation and, hence, cannot be solved using convex optimization techniques.

In conclusion, approximate problems (P3 and P5) are not only non-convex but also require costly integer programming solvers. At best, P3 can be approximated to a geometric programming problem as shown in P4. It is evident by now that P1 cannot be solved in closed form and is quite challenging to solve numerically. Hence, in the next section, we take a drastically different approach towards this rather challenging problem. First, we focus on $K = 2$ links and characterize the optimal policy for problem P1 (based on [11]). In the rest of the paper, we shift our attention to introducing a low-complexity distributed algorithm, for scheduling arbitrary number of MIMO links, that is founded on the optimal decision rules for two links.

5 Iterative MIMO Link Scheduling (IMLS)

In this section, we first present SINR-based decision rules that characterize the optimal MIMO stream allocation and link scheduling for $K = 2$ links. These rules serve as the basis for IMLS which iteratively partitions the links over successive slots depending on the levels of interference.

5.1 Optimal Spatial Multiplexing and Scheduling for Two Links

In this section, we solve the problem for the simple case of two links which constitutes the basis for solving arbitrary number of links using IMLS in the next section. This greatly simplifies the scheduling sub-problem and, hence, facilitates solving the MIMO stream allocation sub-problem.

The decision rules derived in this section stem from the optimality of P1 and the SINR constraints. Next, we focus on these two conditions and their direct impact on optimizing the MIMO stream allocation (number of SM streams, X) and link scheduling (transmit or not with fixed power P).

For $K = 2$ links, the scheduling policy space collapses to two simple policies, namely Policy A: 1 link per slot and Policy B: 2 links per slot. Scheduling one of the links exclusively in the two slots violates the requirement that each link should be activated at least once over the K slots. Hence, the combinatorial complexity of scheduling vanishes. Furthermore, MIMO stream allocation for, typically, small M and N in state-of-the-art radios is solved using search which yields upper and lower bounds on X.

5.1.1 Lower Bound on *X*

In this section, we address the question which establishes a lower bound on X: When does policy B outperform policy A with respect to F? This can be written formally as,

$$F_B > F_A \tag{13}$$

where F_B denotes the average sum rate in the presence of interference from the other link and F_A denotes the average sum rate in the absence of interference. Next, we show how to quantify F_A. In this case, all MIMO spatial streams (i.e. SMG $= min(M, N)$) are utilized for improving the link rate through SM (e.g. V-BLAST [2] which achieves full SMG) and no nulling is needed. Based on the definition of C in (1) and the fact that under policy A each link transmits only in 1 slot, then it can be easily shown that,

$$F_A \;=\; \frac{1}{2}\sum_{i=1}^{2} C_i \;=\frac{1}{2}(C_1 + C_2) \tag{14}$$

Next, we quantify F_B where any link i is activated in any slot j among the two slots. Due to the presence of interference in each slot, a subset of MIMO streams (X) is dedicated to SM, and the remaining degrees of freedom at the receiver are dedicated to nulling the other interferer. Accordingly,

$$F_B \;=\; \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} R_{ij} =\; \frac{1}{2}(R_{11} + R_{12} + R_{21} + R_{22}) \tag{15}$$

Clearly, the interplay of interference and nulling and their effect on SINR under policy B (as opposed to SNR under policy A) and the associated smaller SMG X yields the outcome of which policy constitutes the optimal. The above formula could be simplified if we factor in the fact that interference under policy B, where all (two in this case) links are activated in each slot, is the same and does not vary from slot to slot. This inherent characteristic of policy B yields $R_{11} = R_{12} = R_1$, $R_{21} = R_{22} = R_2$ and, hence, we can drop the slot index and (13) can be re-written as,

$$R_1 + R_2 > \frac{1}{2}(C_1 + C_2) \tag{16}$$

Notice that different values of X_i may or may not satisfy the above condition. This is primarily attributed to the effect of X_i on the LHS since the RHS is independent of X_i. As X_i is increased from 1 to $min(M, N)$ (i.e. higher SMG), the LHS increases due to the role of the linear pre-log factor in (3). Our prime interest is to find the minimum value of X_i that satisfies (16) which constitutes the lower bound (LB) on X_i.

5.1.2 Upper Bound on *X*

In this section, we shift our attention to reception success which is governed by the SINR constraints in P1 and dictates the upper bound (UB) on X.

$$SINR_i \geq \beta \qquad \forall i \tag{17}$$

The RHS of (17) is independent of X_i whereas the LHS varies with X_i. It is straightforward to verify that as X_i decreases from $min(M, N)$ to 1 (i.e. more streams dedicated to nulling at the receiver), SINR increases. Our objective is to find maximum X_i that satisfies (17) and constitutes the UB on X_i.

The interplay of the upper bound and lower bound on X, yields the distributed SINR-based decision rules derived in the next section. Although these rules fully characterize the optimal solution for two links, they constitute only a sub-optimal solution for $K > 2$ due to comparing only two extreme scheduling policies: Policy A (1 link/slot) and Policy B (K links/slot) and leaving many other schedules unexamined. Nevertheless, we introduce in the next section a novel iterative MIMO link scheduling (IMLS) algorithm based on the SINR decision rules examined at individual receivers in a distributed fashion.

5.2 IMLS for Arbitrary Number of Links

5.2.1 Distributed Decision Rules

So far, we have focused on centralized optimization problems, namely P1 and its simplified two link version studied in section 5.1, that maximize a global objective function F.

In this section, we shift our attention to the distributed problem and extend the previous section to scenarios with arbitrary number of links. Given K slots, each link strives to maximize its aggregate rate over the K slots, i.e. the distributed objective function for each link is given by $F_i = R_i = \sum_{j=1}^{K} R_{ij}$.

This enables each receiver to autonomously find a solution, i.e. number of SM streams (X_i) and link activation (P_i), for its link and feed it back to its transmitter. Hence, the IMLS algorithm is distributed since it involves communication between the transmitter-receiver parties of each link with absolutely no inter-link communication. The knowledge of number of contending links K can be obtained through higher layers, e.g. topology control and routing mechanisms.

Next, we show how to develop the SINR-based decision rules for arbitrary number of links based on insights distilled from the two links case studied in section 5.1. For each link, we compare only two extreme scheduling policies to get a solution: Policy A: 1 link per slot and Policy B: K links per slot. The rationale behind this is two-fold. First, it greatly simplifies the problem as it eliminates the combinatorial complexity of the scheduling portion of the problem. Second, it opens room for distributed IMLS which iteratively packs links into different slots depending on the levels of interference. Along the same lines of section 5.1, we compare the performance of policies A and B for link i (i.e. $F_{iB} > F_{iA}$) as follows,

$$R_i > C_i \tag{18}$$

Since link i experiences same interference in all slots under policy B, i.e. $R_{i1} = R_{i2} = ... = R_{iK}$, (18) reduces to,

$$R_{i1} > \frac{C_i}{K} \tag{19}$$

This condition yields the LB_i on X_i.

The SINR constraint determines the UB_i on X_i which, along with LB_i, yield the following distributed decision rules:

Distributed Decision Rules executed at the receiver of link i:

- If $LB_i \leq UB_i$:
  Scheduling: Activate the K links in each of the K slots
  MIMO: SM with $X_i = UB_i$ streams
  Nulling with $\frac{N}{X_i}$ streams
- Else:
  Scheduling: Activate 1 link in each slot (TDMA)
  MIMO: SM with $X_i = min(M, N)$ streams

5.2.2 IMLS Description

Based on the distributed decision rules, we introduce an iterative MIMO link scheduling (IMLS) algorithm that achieves near-maximal link packing per slot. This is attributed to examining the feasibility of extreme policy B, which packs all K links to one slot and, if not, iteratively partitions it to smaller link subsets which are likely to be feasible under reduced interference conditions. This process proceeds in a *distributed manner* as described next.

Fig. 1 shows the flowchart of IMLS, executed at the receiver (Rx) of link i, to schedule K links over minimal number of slots, denoted s. The variable s denotes also the number of iterations until IMLS finds a solution starting from iteration $s = 0$. As indicated before, IMLS commences with K links which get partitioned over successive iterations to subsets denoted $K_s < K$ where $K_0 = K$. Accordingly, the time axis over which IMLS operates is partitioned to s slots. Each slot is preceded by a short *probing interval* where the K_s links examined in the s^{th} iteration probe the wireless medium with short probing packets in order

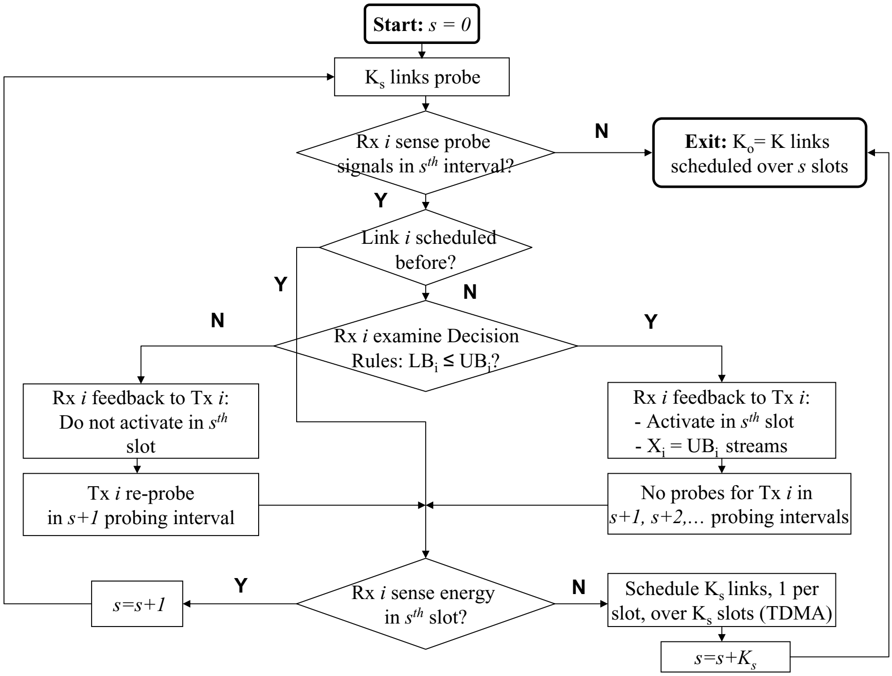

Fig. 1. IMLS Flowchart at Rx of link i to schedule K links over minimal number of slots, s

for receivers to individually examine the distributed decision rules. Hence, the s^{th} IMLS iteration operates over the s^{th} probing interval and s^{th} slot.

In essence, IMLS iteratively partitions a given set of links to find maximal subsets which can be activated simultaneously in consecutive slots. As shown in Fig. 1, it involves four conditional statements where the first and last ones are responsible for exit conditions, as illustrated later, whereas the third condition constitutes the core of IMLS. Given K_s links that probe the medium in iteration s, Rx i computes the LB and UB of X_i and compares them to decide whether it can be activated in the next slot or not. If $LB_i \leq UB_i$, then link i can survive the $(K_s - 1)$ interferers, i.e. achieve $SINR \geq \beta$ with $X_i = UB_i$ and $\frac{N}{X_i}$ streams for nulling. Hence, it can be activated in the s^{th} slot and does not need to probe anymore. These Rx-based decisions are fed back to the transmitter (Tx) side for execution in future slots and iterations. If $LB_i > UB_i$, then no solution exists for link i at present interference levels and, hence, it should not transmit in the s^{th} slot and should re-probe again in the $s + 1$ probing interval.

So far, we have described the fundamental operation of a single iteration. However, transition from iteration to another in a distributed manner is a major contributor to IMLS performance gains. Consider an arbitrary iteration with K_s probing links, the third condition (decision rules) would partition K_s into two subsets: i) Feasible Subset: which share slot s and do not need to re-probe the medium and ii) Infeasible Subset: which do not activate in slot s and need to re-probe in the next iteration. In fact, this latter group constitutes $K_{s+1} < K_s$ probing links of the next iteration. This yields our first key observation: *IMLS goes through another iteration iff the Infeasible Subset is non-empty.* In essence, if all links are feasible in iteration s, then no transmitters will re-probe the medium in iteration $s+1$. This defines our exit condition in the first conditional statement, i.e. if an arbitrary *Rx i* does not sense a probing signal in the s^{th} probing interval, it exits with its own scheduling and MIMO stream allocation solution along with the information that the K links are scheduled over s slots.

The second key observation stems from the other extreme, i.e. What if the K_s links probing in iteration s are all infeasible? This implies that those links cannot share the same slot and, hence, there is no need for more iterations under same interference conditions since it will not change the infeasibility result. Introducing criteria for further partitioning infeasible links lie out of the scope of this work. Instead, IMLS simply falls back to 1 link per slot for the K_s links, as suggested by the decision rules. This defines our exit condition in the last conditional statement, i.e. if any *Rx i* does not sense any transmission in the s^{th} slot, it exits with its own scheduling and MIMO solution along with the s slots needed for the K links, where the last K_s slots are scheduled in a TDMA fashion.

Finally, the second conditional statement indicates that once Rx i finds a solution, say in iteration s, it does not need to re-probe or re-solve the problem any more, it just needs to keep track of the evolution of the algorithm for other links, via incrementing the iteration counter and examining the exit conditions. This is essential for all links to proceed synchronously over IMLS and exit with consistent results, irrespective of which iteration yields their individual solutions.

It is evident that IMLS is distributed since each link takes its link activation and MIMO stream allocation decisions independent of other links. The only communication needed is the feedback from each receiver to its respective transmitter. It should also be noted that if K is finite, the links are guaranteed to find a solution in a *finite* number of iterations. For the two extremes, namely K links are feasible and K links are infeasible, a solution is found in a single iteration. Under typical scenarios, interference decreases from iteration to another as we partition the links until iteration s has: i) $K_s = 1$ which is trivial, ii) $K_s > 1$ and feasible which activates the links in slot s and exits and iii) $K_s > 1$ and infeasible which yields a TDMA solution for the K_s links and exits. It can be shown that the worst case number of iterations for IMLS is $\frac{K}{2}$ which yields only two feasible links in each iteration, since two is the minimum number of links sharing a slot.

6 Performance Results

In this section, we present numerical results obtained using Matlab for: i) The Optimality regions for two MIMO links and ii) IMLS performance for arbitrary number of links.

6.1 Optimality Regions for K=2 Links

We consider two links where each link is supported with $M = N$ =8 antennas. For ease of exposition, we focus in this section on two symmetric links where the transmitter-receiver separation is 250m. In addition, the distances between each receiver and the other transmitter (interferer) are equal and denoted D. The parameter D is varied across different runs, from 500m to 5000m, in order to model varying levels of interference. The symmetry in this scenario gives rise to equal interference at both receivers and, hence, same solution for the MIMO-MAC problem. Accordingly, we focus our analysis on a single link and drop the link index i in X_i.

The transmit power per node, which can be split among different antennas, is fixed at $P = 20$ dBm. The minimum SINR requirement β is set to 5 dB. The path loss exponent is set to α=4 and σ_n^2 is set to -90 dBm.

Table 1. Optimal Policies for Two 8x8 Symmetric MIMO Links

D (km)	# links per slot	LB on X	UB on X	Optimal X	Max. F (bps/Hz)
5	2	4	8	8	9.972
3	2	4	8	8	9.965
2	2	4	8	8	9.391
1.5	2	4	4	4	6.895
1	2	4	4	4	6.67
0.75	2	4	4	4	6.148
0.65	2	4	4	4	5.661
0.6	2	4	4	4	5.315
0.55	1	8	2	8	4.98
0.5	1	8	2	8	4.98

First, we compute the lower bound through examining the throughput constraint in (16) with X growing from 1 to 8. The minimum value for X that satisfies this constraint constitutes the lower bound. Table 1 shows the lower bounds obtained under gradually increasing interference levels due to reducing the receiver-interferer distance D. It is evident that the LB increases as interference increases which agrees with intuition. This is primarily attributed to the fact that higher interference yields lower SINR which makes it impossible for policy B to outperform policy A with small values of X.

Second, we compute the upper bound through examining the SINR constraint in (17) with decreasing number of SM streams, from X=8 to X=1. The maximum value of X that satisfies this constraint determines the UB. Again, Table 1 shows the UB while gradually increasing interference. Unlike the LB, the UB decreases as interference becomes more intense since more degrees of freedom are needed for nulling which, in turn, implies smaller and smaller SMG, X.

The decision rules in section 5.2 decide the optimal X. The first 3 rows in Table 1 exhibit optimality with slot sharing and X=8 due to negligible interference. For the next 5 rows, slot sharing with X=4 turns out to be the optimal due to increasing interference. Finally, for the last two rows, TDMA with X=8 yields the maximum average sum link rate.

Fig. 2 shows the trends of F and associated optimality regions for five MIMO-MAC resource allocation policies. The objective function F is plotted against $\frac{1}{D}$. Policy 1 represents TDMA with X=8 (corresponds to scheduling policy A) whereas policies 2 through 5 represent slot sharing with different values of X (correspond to scheduling policy B).

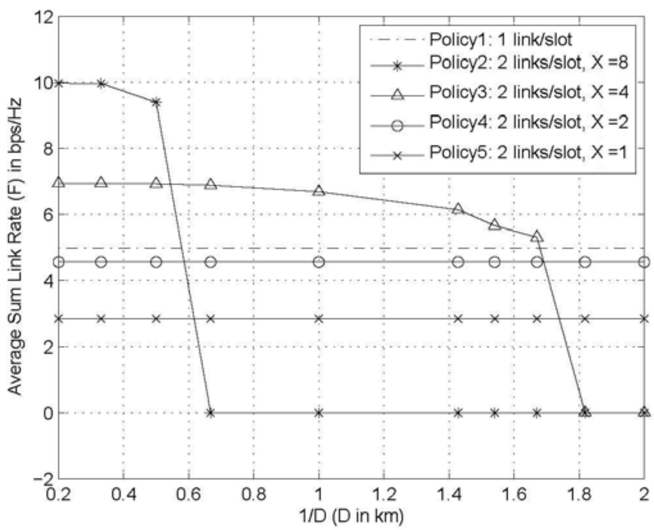

Fig. 2. Optimality Regions for Two Symmetric 8x8 Links

Notice that policy 1 performance does not vary with D since transmissions are interference-free. Policy 2 and 3 performance varies with D due to the impact of interference on the SINR and, hence, on the achievable link rate. Finally, policies 4 and 5 performance does not vary with D, despite the fact that these are slot sharing policies. The reason for this trend is attributed to the fact that these policies have small number of SM streams (X=1,2) which leaves sufficient spatial streams for the receiver to null the other interferer. Therefore, policies 4 and 5 completely null interference and, hence, experience no SINR variation with D.

In addition, the figure reveals different regions of optimality for different policies. For the leftmost region (D>1500 m), policy 2 achieves maximum F due to slot sharing while using the 8x8 MIMO for SM due to the negligible interference. For $600m < D < 1500m$, policy 2 fails to maintain the SINR constraint, due to interference buildup and, hence, its throughput falls sharply to zero. On the other hand, the less aggressive policy 3 assumes the optimal role for this region due to dedicating MIMO resources to nulling. Finally, as interference dominates for $D < 550m$, none of the slot sharing policies achieves the optimal and, interestingly, naive TDMA with $X = 8$ achieves maximum F. This suggests that contention-free TDMA is the only resort in case of high interference, where none of the links could guarantee their SINR minimum requirement β and still achieve high link rates.

Finally, it should be noted that policies 4 and 5 are not optimal in any region. This is attributed to the fact that these policies have small X (which reduces the SMG) and dedicate more resources than needed for nulling a lone interferer.

6.2 IMLS Performance for K>2 Links

In this section, we analyze the performance of IMLS. In particular, we compare three scheduling paradigms: i) Naive TDMA where 1 link is activated per slot, ii) Slot sharing and TDMA, denoted SS/TDMA, where only the first iteration of IMLS is executed for K links and the Feasible Subset is activated in a single slot whereas the Infeasible Subset is scheduled in a TDMA fashion without further iterations and iii) IMLS where multiple iterations activate different subsets of the K links over different slots.

We consider three scenarios, randomly generated, with K=10 8x8 MIMO links and simulation parameters similar to previous section. First, we analyze the scenario shown in Fig. 3 where the average Tx-Rx separation d is 248m and the average distance between any receiver and other transmitters (interferers) D is approximately 2393m. Link indices are written next to individual links in the figure. Large D yields low interference which permits receivers to suppress it via dedicating a subset of the spatial streams to nulling. In fact, this scenario turns out to be an extreme one where *all* receivers can share the same slot using the following stream allocations $\underline{X}$=[4 4 4 4 4 8 8 8 4 2], where X_i denotes stream allocation for link i. Hence, IMLS yields a solution after one iteration. Although this scenario does not represent the typical case in ad hoc networks, it reveals insights about the gains of cross-layer MIMO-MAC over scheduling high rate links with 8 SM streams in an interference-free manner. Using TDMA, the average sum link rate is given by $F_{TDMA} = 5.129$ bps/Hz whereas $F_{IMLS} =$ 42.047 bps/Hz. This confirms the profound impact of IMLS, that is almost 8-fold improvement over TDMA. This is attributed to low interference which not only permits activating the 10 links simultaneously but also using minimum number of spatial streams for nulling.

Second, scenario 2 in Fig. 4 exhibits higher interference since D is approximately 1739m and d is 258m. Therefore, interference cannot be resolved in a single iteration as in the previous example. Instead, IMLS takes four iterations. In the first iteration, only 3 out of 10 links (links 4, 8 and 10) are feasible using X = 8, 4, 2 streams, respectively. Next, IMLS attempts to solve the infeasible subset of $K_1 = 7$ links, namely 1, 2, 3, 5, 6, 7, 9 where $s = 1$. Reduced interference enables links 2, 3, 6 to become feasible using X = 2, 2, 2 streams respectively. For $s = 2$, IMLS attempts to solve the remaining infeasible $K_2 = 4$ links, namely 1, 5, 7, 9 where links 7 and 9 manage to share a slot under reduced interference.

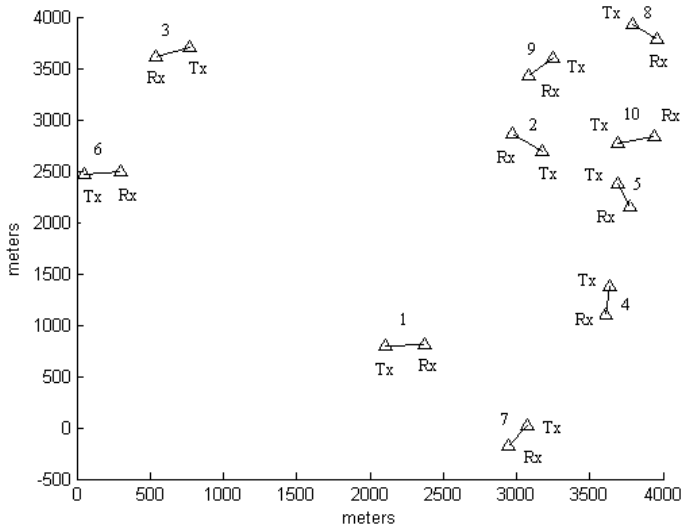

Fig. 3. Scenario1: 10 links with average receiver-interferer distance D=2393m and average Tx-Rx distance d=248m

Finally, links 1 and 5 are examined in the last iteration, however, they cannot share the same slot due to their high mutual interference, even in the absence of the other 8 links which have been already scheduled in previous iterations.

Next, we compare the performance of five policies, namely TDMA, SS/TDMA, IMLS and two variations of it. Under TDMA, the throughput performance $F_{TDMA} = 4.792\ bps/Hz$ whereas SS/TDMA achieves $F_{SS/TDMA} = 6.8632\ bps/Hz$, i.e. 43% improvement over TDMA. On the other hand, IMLS yields $F_{IMLS} = 8.32\ bps/Hz$, that is 73% improvement over TDMA and 21% improvement over SS/TDMA due to IMLS iterative nature which attempts to achieve near-maximal link packing per slot.

The following IMLS variations optimize its performance and address fairness respectively. The first variation is inspired by the observation that stream allocation (X) can be further optimized for a set of feasible links (e.g. links 4, 8, 10 for $s = 0$) under reduced interference, i.e. after eliminating interference from infeasible links who could not share slot 0 with these three links anyway. For instance, plain IMLS yields $X = 8, 4, 2$ for links 4, 8, and 10 respectively when all 10 links were transmitting. On the other hand, optimized IMLS, denoted IMLS1, re-examines the decision rules with these 3 links alone, once it decides their feasibility in iteration $s = 0$. This yields $X = 8, 4, 4$ for links 4, 8, and 10 respectively (notice the improvement in the SMG for link 10 due to the reduced interference). This results in $F_{IMLS1} = 10.1\ bps/Hz$, i.e. 21% improvement over plain IMLS and more than 2-fold improvement over TDMA with full SMG.

The second IMLS variation trades throughput for fairness, depending on how the K slots are assigned. The IMLS performance reported so far has been computed over K slots, where K is always greater than the number of iterations s upon IMLS completion as discussed earlier. This implies allocating more than one slot to each feasible link subset identified over IMLS iterations. If m links can share a single slot in iteration s, we assign those links m out of the K slots. Clearly, this could lead to overall throughput improvement over the K slots due to favoring highly packed slots, however, it could lead to unfairness with respect to lightly packed slots (e.g. link 1 in scenario 2 cannot share a slot with any other link and, hence, it is assigned only 1 out of K slots). An intuitive measure of fairness in this context is the difference between the maximum number of slots assigned to a link and the minimum number of slots assigned to a link, i.e. $f = \max_i$(# slots out of K assigned to link i) - $\min_i$(# slots out of K assigned to link i). As f gets far from 0, the scheduling algorithm becomes less fair. For scenario 2 above, $f = 3 - 1 = 2$, and hence plain IMLS exhibits low fairness.

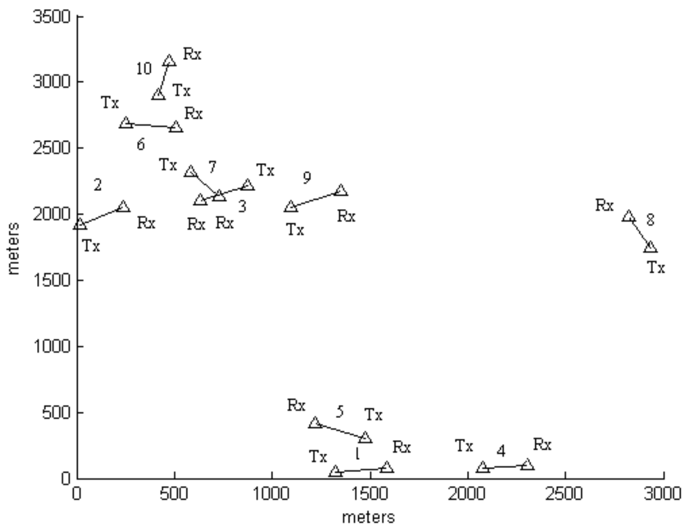

Fig. 4. Scenario2: 10 links with average receiver-interferer distance D=1739m and average Tx-Rx distance d=258m

An approach that achieves better fairness, at the expense of throughput loss, is to split the K slots equally among the different subsets of links identified in different iterations. For the example above, assigning two slots for each of the five link subsets identified in the four iterations yields $F_{IMLS2} = 7.14\ bps/Hz$, i.e. 14% loss compared to IMLS. However, this is compensated with improved fairness since $f = 2 - 2 = 0$.

Finally, the scenario shown in Fig. 5 exhibits highest interference due to the role of $D = 490m$ and $d = 250m$. This yields no feasible links in the first iteration

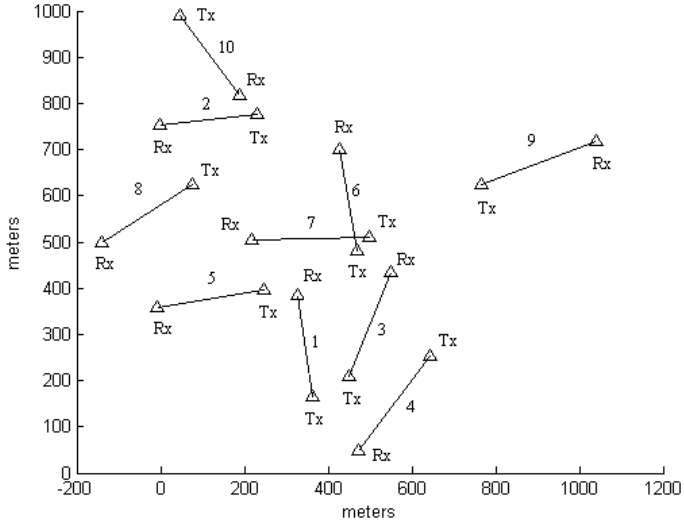

Fig. 5. Scenario3: 10 links with average receiver-interferer distance D=490m and average Tx-Rx distance d=250m

of IMLS. Accordingly, IMLS cannot proceed with partitioning the links based on their different slot sharing capabilities as illustrated earlier. Thus, TDMA yields modest performance of $F_{TDMA} = 5.11\ bps/Hz$. Extending IMLS to handle this scenario lies out of the scope of the paper and is a subject of future research.

7 Conclusions

We studied the problem of MIMO-MAC resource allocation for the MIMO interference channel. The prime motivation is to balance the trade-off between maximizing the throughput of individual non-interfering links, using spatial multiplexing, and maximizing the spatial reuse of lower rate interfering links, using spatial multiplexing in conjunction with nulling. We formulate a cross-layer optimization problem and characterize it as a non-convex integer programming problem which is quite challenging. However, we show that under low SINR regimes, an approximate problem can be cast into a convex geometric programming formulation. Finally, we characterize the optimal solution for two links and use the distributed decision rules as a basis for Iterative MIMO Link Scheduling (IMLS) that achieves significant gains for arbitrary number of links. Numerical results confirm the trade-off as well as show more than 2-fold improvement by IMLS over TDMA with maximum spatial multiplexing gain. This work can be extended along the following directions: i) Extend the formulation to the generalized diversity-multiplexing-scheduling trade-off and ii) Develop MAC protocols based on the decision rules and iterative MIMO link scheduling.

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