Network Regulations and Market Entry*

Galina Schwartz[1], John Musacchio[2], Mark Felegyhazi[3], and Jean C. Walrand[1]

[1] University of California, Berkeley, Cory Hall, Berkeley, CA 94720
{schwartz,wlr}@eecs.berkeley.edu
[2] University of California, Santa Cruz, 1156 High Street, Santa Cruz, CA 95064
johnm@soe.ucsc.edu
[3] Budapest Univ. of Tech. and Econ.
mfelegyhazi@crysys.hu

Abstract. This paper uses a two-sided market model to study if last-mile access providers (ISPs), should charge content providers (CPs), who derive revenue from advertisers, for the right to access ISP's end-users. We compare two-sided pricing (ISPs could charge CPs for content delivery) with one-sided pricing (neutrality regulations prohibit such charges). Our analysis indicates that number of CPs is lower, and the number of ISPs often higher, with two- rather than one-sided pricing. From our results the superiority of one regime over the other depends on parameters of advertising rates, end-user demand, CPs' and ISPs' costs, and relative importance of their investments. Thus, caution should be taken in designing neutrality regulations.

Keywords: network neutrality, two-sided markets, market entry.

1 Introduction

Today, an Internet Service Provider (ISP) charges only end-users, who subscribe to that ISP for Internet access, and content providers (CPs) connected to the Internet directly via that ISP. That is, each ISP generally charges only CPs who buy access from it. One of the focal questions in the network neutrality policy debate is whether current ISPs' charging practices should continue and be mandated by law, or ISPs ought to be allowed charging CPs for the delivery of content to the end-users. This question is part of the larger debate on network neutrality, which includes diverse issues such as whether service differentiation should be allowed, or whether charges for content constitute an impingement of freedom of speech [9,2].

In our past work [6] as well as here, we use a two-sided market model of interactions between ISPs, end-users, and CPs, with the ISPs playing the role of a "platform" intermediating the two sides: CPs and end-users. We model a "non-neutral" network as a market with two-sided pricing, meaning that each ISP charges his end-users and also charges CPs for delivering traffic to his end-users.

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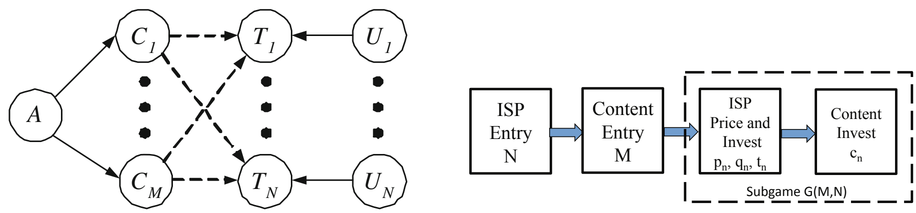


Fig. 1. Left panel: The direction of payments. Dotted lines reflect additional payments of the two-sided case. T_i and U_i are ISPs (transit providers) and each ISP's group of users, respectively. Right panel: The timing of the game G. The subgame $G(M, N)$ starts after M and N are chosen.

Conversely, a "neutral" network is modeled as a one-sided pricing regime in which an ISP is allowed to charge only CPs that buy their Internet access from it. We normalize this charge to zero, which allows us to model a neutral network as a one-sided regime in which each platform (ISP) charges his end-users only. After providers of both types choose to enter the market and sink irreversible entry costs, the ISPs simultaneously and independently choose their investments, end-user prices, and content-provider prices. Then, CPs choose their investments, simultaneously and independently. Our end-users demand for content variety has a flavor of the classical monopolistic competition model [3].

We explore how CP and ISP entry and investments differ with pricing regime. In [6], the numbers of ISPs and CPs are fixed, and we compare the social welfare in two-sided and one-sided pricing cases. In [6], we find that superior welfare regime depends on two key parameters: (i) the ratio of advertising rate to an end-user price sensitivity parameter, and (ii) the number of ISPs. Roughly, when (i) is extreme (large or small), a two-sided (non-neutral) pricing is welfare superior, but when (i) is mid-range, a one-sided (neutral) pricing is superior. An intuitive explanation for this is that when either the ISP or CPs have a much stronger ability to obtain revenue, a two-sided pricing which essentially allows more flexible revenue sharing between provider types, which allows to achieve a higher equilibrium welfare. Also, the parameter range for which a one-sided market is superior increases with the number of ISPs. An explanation for this is that with two-sided pricing, each ISP charges the CPs, and collectively the ISPs may "over-charge". This effect grows with the number of ISPs.

In this paper, we consider the entry decisions of potential ISPs and CPs, thus endogenizing the number of providers of each type. Since the results of [6] depend roughly on whether the ISP or CP market is more profitable in some sense, allowing more providers to enter a market that is highly profitable could alter the situation. Here we find that the two-sided (non-neutral) pricing is indeed welfare superior for parameters similar (and likely wider) than in [6] (industry structure is fixed). In contrast with [6], where Nash equilibrium symmetry is assumed, in this paper we prove the symmetry. We establish the existence of an equilibrium in the entry game in which potential ISPs and CPs make their irreversible entry decisions.

Our model is based on the ideas of two-sided markets, and there is a considerable literature on the subject. (See surveys [10] and [1].) Other researchers applied the ideas of two-sided markets to study network neutrality. Hermalin and Katz [4] model network neutrality as a restriction on the product space, and consider whether ISPs should be allowed to offer more than one grade of service. Hogendorn [5] studies two-sided markets where intermediaries sit between "conduits" and CPs. In his context, net-neutrality means that content has open access to conduits where an "open access" regime affords open access to the intermediaries. Njoroge at al. [7] and [8], consider two-sided market model with heterogeneous CPs and end-users, and the ISPs play the role of a platform. In [8], they find that social welfare is higher in the non-neutral regime. Work [12] discusses policy issues related to two-sided markets.

The paper is organized as follows. Section 2 presents a model that permits us to quantify the effects of network regime on player incentives to enter the network industry and invest. In Section 3, we analyze the two-sided pricing (non-neutral) one-sided pricing (neutral) regimes. In section 4 we discuss our findings and conclude. To ease the exposition, the proofs are relegated to the appendix, and to save space, not all the proofs are included; the complete technical details available in our technical report, see [11].

2 Model

The internet consists of local ISPs (i.e, residential ISPs who provide last-mile access for end users), and transit ISPs that mainly serve the internet backbone. We assume that the CPs connect to the internet via a transit ISP, whereas end-users are attached to a local ISP. Our assumption reflects that under current practices, the local ISPs do not charge CPs. Thus, in further analysis we abandon the distinction between local ISPs and transit ISPs, and focus on local ISPs only, which from now on we simply call ISPs.

First we consider the subgame $G(M, N)$ after M CPs and N ISPs after they entered the market. Later, we consider the game G which includes the stages in which providers decide to enter the market.

Fig. 1 illustrates our setting. Each ISP T_n charges his attached end-users U_n $(n = 1, \dots, N)$ an access price p_n per click; each ISP has a monopoly over its end-user base U_n. Thus, to reflect the market power of local ISPs, the end-users are divided between the ISPs, with each ISP having $1/N$ of the entire market. Each ISP T_n charges each CP C_m an amount equal to q_n per click. CP C_m invests c_m and ISP T_n invests t_n.

We normalize the CPs' access payment (for network attachment) to zero due to the ISPs' competition. To reflect the actual payment, we assume that the CP's access payment is subtracted from their charges to advertisers. Let B_n denote demand of end-users U_n; the B_n depends on p_n and provider investments:

$$B_n = \{\mu\eta(c_1^v + \cdots + c_M^v)t_n^w\}\, e^{-p_n/\theta}, \text{where } \mu = \frac{(1 - e^{-kM})}{M^{1-v}}, \eta = \frac{1}{N^{1-w}}. \quad (1)$$

Here, $\theta > 0$, and $v, w \geq 0$ with $v + w < 1$, and $k \in (0, \infty)$. For a given network quality (the expression in the curly brackets), the end-user demand exponentially decreases with p_n; see [6] for the intuition about the normalization factor $\eta = 1/N^{1-w}$. The term μ is similar except that the factor $(1 - e^{-kM})$ reflects that the end-users prefer a higher M, i.e. more variety in content. As M increases the "love for variety" diminishes. Let R_{mn} denote the end-user demand U_n to C_m, and D_m – the total demand for C_m:

$$R_{mn} = \frac{c_m^v}{c_1^v + \cdots + c_M^v} B_n, \text{ and } D_m = \sum_n R_{mn}. \tag{2}$$

Now we consider the game G, which includes the entry decisions of potential providers. The order of play is as follows. First, potential ISPs decide whether to enter; if they enter, the entry cost is t_e. Second, content providers observe the number of ISP entrants, and then, sink their entry costs c_e. Third, the ISPs announce end-user prices p_n (in the one-sided pricing case) and also content provider charges q_n in the two-sided case. Fourth, content providers choose their investments c_m, and ISPs – their investments t_n. Each CP's objective is to maximize its profit Π_{Cm} which is equal to the revenues net of its investment.

$$\Pi_{Cm} = \sum_{n=1}^{N} (a - q_n) R_{mn} - \beta c_m - c_e. \tag{3}$$

Here a is the amount that advertisers pay to CPs per unit of end-user demand; $\beta > 1$ is the outside option (alternative use of funds), and c_e – the CP's entry cost. Each ISP objective is to maximize profit Π_{Tn}:

$$\Pi_{Tn} = (p_n + q_n) B_n - \alpha t_n - t_e. \tag{4}$$

Where $\alpha > 1$ and t_e are respectively the ISP's outside option and entry cost.

3 Analysis

3.1 Analysis of the Subgame $G(M, N)$

Let $\Pi_C(M, N)$ and $\Pi_T(M, N)$ denote profits for each CP and ISP in the game $G(M, N)$. To compare one-sided and two-sided pricing (neutral and non-neutral networks), we make the following assumptions.

(a) One-sided pricing (neutral network): First, each T_n chooses (t_n, p_n). Here $q_n = 0$. Then, each C_m chooses c_m.
(b) Two-sided pricing (non-neutral network): First, each T_n chooses (t_n, p_n, q_n). Then, each C_m chooses c_m.

Two-Sided Pricing. In $G(M, N)$, in a network with two-sided pricing, each ISP chooses (t_n, p_n, q_n) and each CP chooses c_m. For a given (t_n, p_n, q_n), the optimal c_m maximizes (3). From the first order conditions,

$$\beta c_m^{1-v} = v\mu\eta \sum_n (a - q_n) t_n^w e^{-p_n/\theta} =: \beta c^{1-v}. \tag{5}$$

For that value of c_m, we find that:

$$\Pi_{Tn}(M,N) = M\mu\eta(q_n + p_n)t_n^w e^{-p_n/\theta}\left[\frac{v\mu\eta}{\beta}\sum_{k=1}^{N}(a-q_k)e^{-p_k/\theta}t_k^w\right]^{\frac{v}{(1-v)}} - \alpha t_n. \tag{6}$$

The n-th ISP chooses (t_n, p_n, q_n) that maximize (6). By analyzing IPSs best response functions we find:

Proposition 1. *With the two-sided pricing, in all $G(M,N)$ equilibria $t_n = t, p_n = p, q_n = q$ and $c_m = c$.*

The proof of Proposition 1 works in the following way. First, from the ISPs' FOCs wrt p_n reveal that $p_n = \theta - a$ for any n in equilibrium – thus equilibrium user prices are identical. Next, from the ISPs' FOCs wrt q_n one can infer that if $q_i \geq q_j$ it must be that $t_i \leq t_j$. But from the ISPs' FOCs wrt t_n we infer that if $q_i \geq q_j$ we must have $t_i \geq t_j$. This is possible only if $q_i = q_j$ and $t_i = t_j$. Thus, only a symmetric equilibrium could exist, and we demonstrate its existence by construction. We find the symmetric equilibrium by construction. It has the following form:

$$p_n = p^\ddagger = \theta - a, \text{ and } q_n = q^\ddagger = a - \theta\pi; \tag{7}$$

$$t_n = t^\ddagger = \left[(x^\ddagger)^{1-v}\cdot(y^\ddagger)^v\cdot e^{-(\theta-a)/\theta}\right]^{\frac{1}{(1-w-v)}}; \tag{8}$$

$$c_m = c^\ddagger = \left[(x^\ddagger)^w\cdot(y^\ddagger)^{1-w}\cdot e^{-(\theta-a)/\theta}\right]^{\frac{1}{(1-v-w)}} \times [\mu\eta N]^{\frac{1}{1-v}}; \tag{9}$$

$$\text{where } x^\ddagger = M\,(\mu\eta)^{\frac{1}{1-v}}\cdot\left(\frac{\theta w}{\alpha}\right)N^{\frac{v}{1-v}},\ y^\ddagger = \frac{\theta v}{\beta}\pi, \text{ and } \pi = \frac{v}{N(1-v)+v}. \tag{10}$$

From (7) - (10) and Proposition 1, the equilibrium uniqueness follows immediately. Proposition 1 is proved in the appendix.

One-Sided Pricing. The game $G(M,N)$ with one-sided pricing is similar to one with two-sided pricing, except that $q_n = 0$ for all n. Given a $\{q_n = 0, p_n, t_n\}$, the one finds that the CPs' best responses are identical and satisfy:

$$\beta c_m^{1-v} = v\mu\eta[\sum_k a t_k^w e^{-p_k/\theta}] =: \beta c^{1-v}.$$

Substituting into (4) we find that

$$\Pi_{Tn}(M,N) = M\mu\eta p_n t_n^w e^{-p_n/\theta}\left[\frac{v\mu\eta a}{\beta}\times\sum_{k=1}^{N}e^{-p_k/\theta}t_k^w\right]^{\frac{v}{(1-v)}} - \alpha t_n. \tag{11}$$

The ISP T_n chooses (t_n, p_n) that maximizes the above. Analysis of the above payoff function leads to the following result.

Proposition 2. *With one-sided pricing, in all $G(M,N)$ equilibria $t_n = t, p_n = p, q_n = 0$ and $c_m = c$.*

The intuition of the proof of Proposition 2 is similar to that of Proposition 1. We use Proposition 2 to construct a unique symmetric equilibrium from the FOCs of (11). We find:

$$p_n = p^\dagger = \theta(1-\pi), \text{ and } q_n = q^\dagger = 0; \tag{12}$$

$$t_n = t^\dagger = \left[(x^\dagger)^{1-v}(y^\dagger)^v e^{-p^\dagger/\theta}\right]^{\frac{1}{1-v-w}}; \tag{13}$$

$$c_m = c^\dagger = \left[(x^\dagger)^{w}(y^\dagger)^{1-w} e^{-p^\dagger/\theta}\right]^{\frac{1}{1-v-w}} \cdot [\mu\eta N]^{\frac{1}{1-v}}; \tag{14}$$

$$\text{where } x^\dagger := x^\ddagger \text{ and } y^\dagger := \frac{av}{\beta}. \tag{15}$$

From (12) - (15) and Proposition 2, the equilibrium is unique. The proof of Proposition 2 is omitted because of space limitations, but it is similar to the proof of Proposition 1 provided in the appendix.

3.2 The Entry Game *G*

Since the equilibrium of $G(M,N)$ exists and is unique and symmetric, in any equilibrium in which (M,N) CPs and ISPs respectively enter, a necessary condition for equilibrium is that

$$\Pi_{Cm}(M,N) \geq c_e, \text{ and } \Pi_{Cm}(M+1,N) < c_e.$$

if $M > 0$ otherwise $\Pi_{Cm}(1,N) < c_e$ if $M = 0$. Suppose that there is a unique $M(N)$ that satisfies the above for each N. (We show this is indeed true in our proof of Proposition 3 below.) Since the potential CPs get to observe the number N of ISPs that enter, another necessary condition[1] for equilibrium is that

$$\Pi_{Tn}(M(N),N) \geq t_e, \text{ and } \Pi_{Tn}(M(N+1),N+1) < t_e$$

if $N > 0$ otherwise $\Pi_{Tn}(M(1),1) < t_e$ if $N = 0$. Together these conditions are necessary and sufficient for (M,N) to be an equilibrium of the game G. These conditions lead to the following propositions.

Proposition 3. *The equilibrium of the game G exists and is unique.*

Proposition 4. *Consider a game $\tilde{G}$, in which CPs and ISPs enter simultaneously rather than sequentially. Then, a pure strategy Nash equilibrium in $\tilde{G}$, provided it exists, coincides with the equilibrium of G.*

From Proposition 4, when a pure strategy Nash equilibrium in the game $\tilde{G}$ exists, it is unique. The proofs of both propositions are found in the appendix.

[1] If $M(N)$ were instead a set valued function there would have to exist elements of the sets $M(N)$ and $M(N+1)$ satisfying the inequality relation to support an equilibrium with N ISPs.

3.3 User Welfare and Social Welfare

We compute the consumer surplus (aka the end-user welfare) by taking the integral of the demand function from the equilibrium price to infinity. This yields

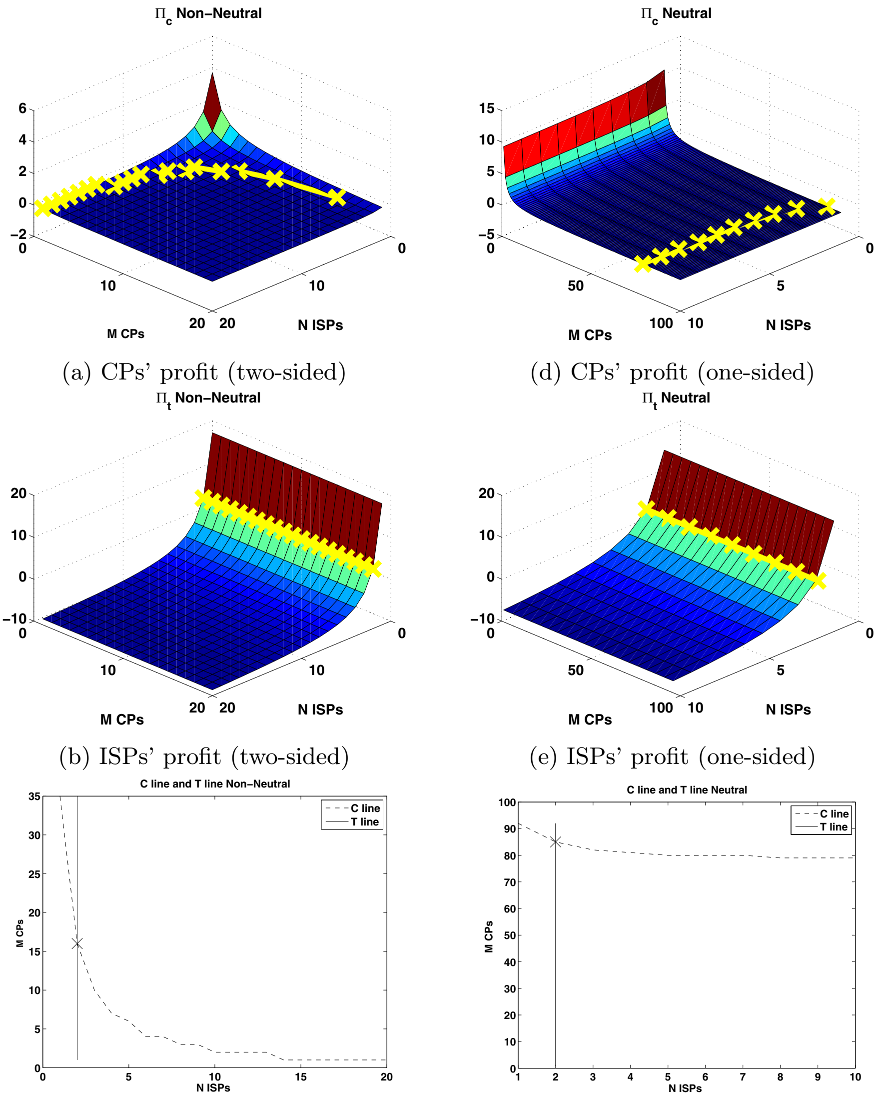
$$W_U(M,N) = NM^{=}\mu\eta\theta \cdot [(x)^w (y)^v]^{1/(1-w-v)}\, e^{-p/[\theta(1-w-v)]}.$$

3.4 Numerical Analysis

In this section, we numerically analyze some examples to illustrate the behavior of the model. We begin by studying the profits of CPs and ISPs in the post-entry game $G(M,N)$ as a function of M and N.. Fig. 2 shows the profits of CPs and ISPs for the two-sided (non-neutral) and one-sided (neutral) case. In this example we have chosen k to be large so that $(1 - e^{-kM}) \approx 1$. Recall that this factor was included to model a preference among users for a larger number of CPs, so making k large is equivalent to removing this effect. Panel (a) of Fig. 2 shows that the CPs' profits decrease in both M and N in the two-sided case, while the lower left panel shows that the dependence of profits on N The "X" marks on the figure indicate the maximum number M that can enter for a given N and still generate positive profits for the CPs. Panels (b) and (e) of Fig. 2 shows that the transit provider profits decrease in N, but are independent of M, both for the one- and two-sided cases. Note this independence does not hold when k is not taken to be large. Panels (c) and (f) show the lines of the maximum M for CPs to be profitable given N ("C-line") and maximum N for ISPs to be profitable given M ("T-line"). The X on each of these graphs indicates the sequential equilibrium $(M^*(N^*), N^*)$. In this example the sequential equilibrium occurs at the intersection of the C-line and T-line. (In cases for which the "T-line" has M dependence – k not large – the sequential equilibrium need not occur at the intersection of the C-line and T-line.)

Fig. 3 studies how equilibrium consumer surplus and social welfare are effected by the transit entry cost t_e (upper plots) and advertising rate a (lower plots). The upper left plot shows that as t_e increases, the number of ISPs drops, though the drop is discontinuous because the number of providers is an integer. For the two-sided case, the social welfare decreases with the number of ISPs. Thus, when there are more ISPs trying to charge the CPs, they collectively "over-charge" the ISPs which in-turn discourages CP investment and reduces social welfare. In the one-sided case, we still observe that social welfare increases as t_e increases and the equilibrium number of ISPs decreases. An explanation for this is that the equilibrium consumer price p in the one-sided case increases as the ISP market becomes more fragmented – in other words there are more ISPs serving smaller and smaller subscriber bases. (See formula for π and $p^{\dagger}$). Thus as fewer ISPs enter, consumer prices go down, more usage (clicks) can occur, and thus more CPs enter.

In the lower plots of Fig. 3, we see that as a increases, the social welfare increases in the one-sided case. A higher a induces more CPs to enter the market. Generally, the social welfare in the two-sided case increases as well, except in



(a) CPs' profit (two-sided) (d) CPs' profit (one-sided)

(b) ISPs' profit (two-sided) (e) ISPs' profit (one-sided)

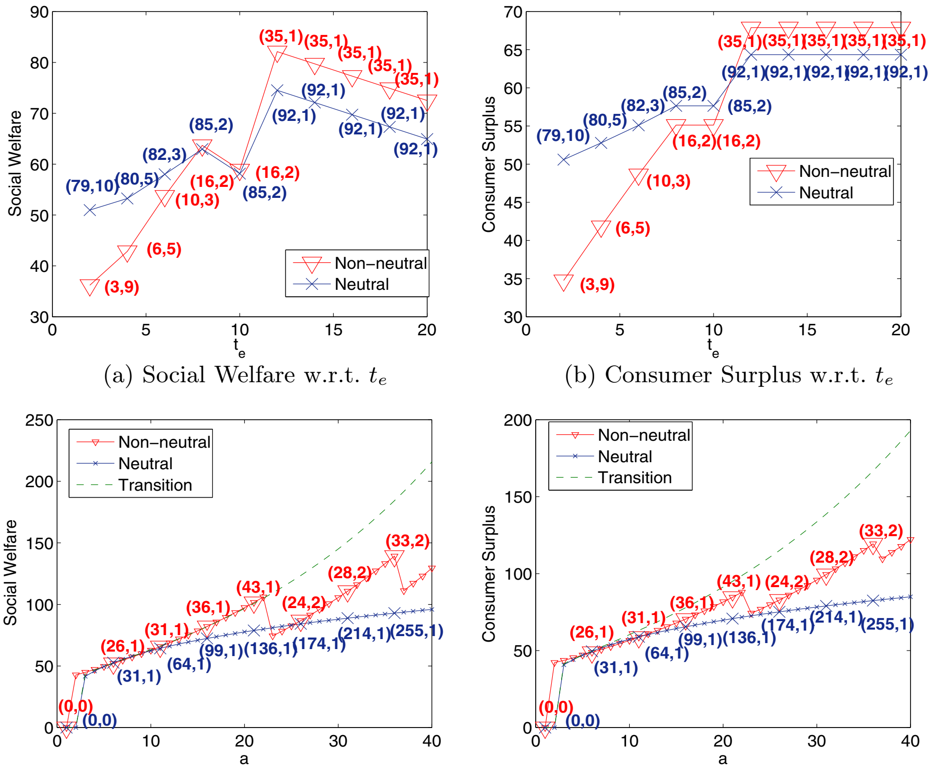
(c) Equilibrium (two-sided) (f) Equilibrium (one-sided)

Fig. 2. Profits in the two-sided market as a function of M and N. Parameters are as shown in Table 1.

"steps" where the number of equilibrium ISPs increases by 1. We also see that, roughly, the two-sided regime becomes social welfare superior to the one-sided regime when a is high. An explanation is that a high a creates more revenue potential and a two-sided market permits some of that revenue to be shared with ISPs so that they see an incentive to increase investment.

Table 1. Baseline parameters; k is large, thus $(1 - e^{-kM}) \approx 1$ for any $M \geq 1$

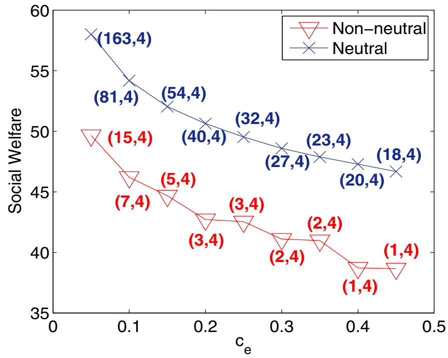
Parameter	v	w	a	θ	t_e	c_e	α	β	k
Value	0.1	0.3	15	50	8	0.1	1.2	1.2	"large"



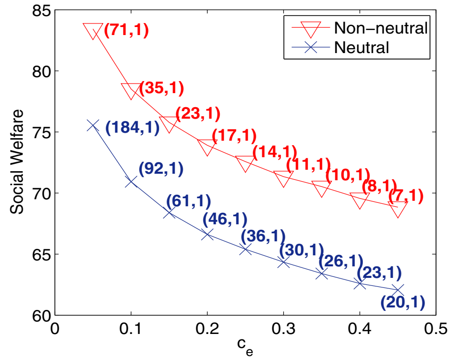
(a) Social Welfare w.r.t. t_e
(b) Consumer Surplus w.r.t. t_e
(c) Social Welfare w.r.t. a
(d) Consumer Surplus w.r.t. a

Fig. 3. Upper Row: The social welfare and consumer surplus as a function of t_e. Lower Row: The consumer surplus as a function of a. Non-neutral (two-sided) and neutral (one-sided) regimes are shown in both cases. A "transition" regime for which the equilibrium numbers of providers in a neutral (one-sided) regime are introduced to a (two-sided) regime before choosing prices and investments is also shown in the plots. Parameters other than the parameter being varied in each plot are as shown in Table 1. The number pairs in figure show the equilibrium number of CPs and ISPs.

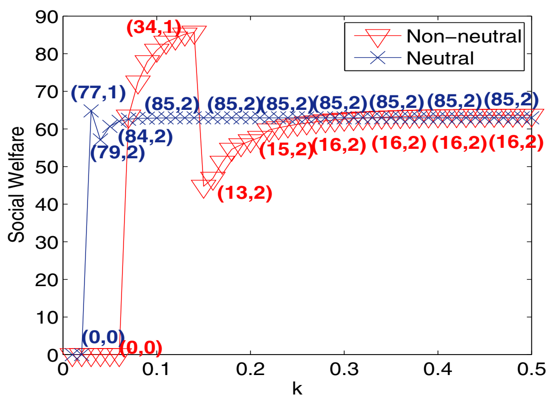
Panels (a) and (b) of Fig. 4 depict the social welfare with respect to the CP entry cost c_e for two different values of $t_e = \{5, 15\}$. The figures show that social welfare decreases with c_e in both cases. The welfare superiority of neutral vs. non-neutral (one- vs. two-sided) is not changed significantly by changing c_e – neutral is better for $t_e = 5$ and non-neutral is better for $t_e = 15$. The last panel



(a) S.W. w.r.t c_e, $t_e = 5$



(b) S.W. w.r.t c_e, $t_e = 15$



(c) S.W. w.r.t. k

Fig. 4. Panels (a) and (b): The social welfare and consumer surplus as a function of c_e for $t_e = 5$ and $t_e = 15$ respectively. Panel (c): The social welfare as k varies while $c_e = 0.1$ and $t_e = 8$.

of Fig. 4 illustrates how social welfare changes with respect to k. In the non-neutral case, for small k, no providers enter the market in equilibrium. When k passes a threshold, 1 ISP enters. As k rises further, a 2nd ISP enters and social welfare drops sharply, but then increases. Social welfare with respect to k in the neutral case seems to fluctuate less.

4 Conclusions

Our results suggest that welfare superior regime depends on the following key parameters. As in [6], we observe that a larger number of ISPs tend to reduce social welfare in the two-sided case. An explanation is that if a large number of ISPs try to extract revenue from each CP, the IPSs collective charges on the CPs exceed the socially optimal ones. This effect strengthens with a higher number of ISPs. Thus for low ISP entry costs t_e, we observe a significantly lower social welfare in the two-sided case.

A higher advertising rate a appears to favor the two-sided (non-neutral) case. An explanation for this is that if a large portion of the network's revenue is realized by advertising, allowing the ISPs to capture some of this revenue improves their incentive to invest, which leads to a higher social welfare. The effects of increased a and the effect of an increased number of ISPs in the two-sided case can interact in an interesting way. The lower panels of Fig. 3 show that the welfare of a two-sided market roughly increases as a increases, but it also exhibits step decreases every time the improved profitability of the ISP market permits an extra ISP entrant. Changes in the content provider entry cost c_e effect social welfare in both the one- and two-sided regimes, but it seems to have much less effect on the relative superiority of the two regimes than the parameters t_e or a.

Our results suggest that regulatory authorities should be cautious of restricting the pricing in the internet, and special attention ought to be paid to: the relative ability of CPs and ISPs to earn revenue (a in our model), the concentration of the ISP market (N), and the entry cost of ISPs.

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5 Appendix

5.1 Analysis of the Two-Sided Case

Consider the game $G(M,N)$ (fixed N and M). We use Π_{Cm} (given by (3)), combined with R_{mn} defined by (2) and B_n is given by (1) to find:

$$\Pi_{Cm}(M,N) = \mu\eta c_m^v \sum_n (a-q_n) t_n^w e^{-p_n/\theta} - \beta c_m.$$

For a given (p_n, q_n, t_n), the c_m that maximizes the above satisfies

$$c_m = c = \left(\frac{v\mu\eta}{\beta} \sum_n (a-q_n) t_n^w e^{-p_n/\theta} \right)^{\frac{1}{(1-v)}}. \tag{16}$$

Substituting the above, $\Pi_{Tn}(M,N)$ assuming optimal content investment is

$$\Pi_{Tn}(M,N) = M\mu\eta(q_n+p_n) t_n^w e^{-p_n/\theta} \times \left[\frac{v\mu\eta}{\beta} \sum_{k=1}^{N} (a-q_k) e^{-p_k/\theta} t_k^w \right]^{\frac{v}{(1-v)}} - \alpha t_n. \tag{17}$$

If the derivative of (17) w.r.t. q_n is zero, then

$$A^{\frac{v}{(1-v)}} - (q_n+p_n)\frac{v}{1-v} A^{\frac{v}{(1-v)}-1} t_n^w e^{-p_n/\theta} = 0,$$

where $A := \sum_k (a-q_k) e^{-p_k/\theta} t_k^w$. Solving the above for A we have,

$$A = (q_n+p_n)\frac{v}{1-v} t_n^w e^{-p_n/\theta}. \tag{18}$$

The two aforementioned expressions for A imply

$$(q_n+p_n)\frac{v}{1-v} t_n^w e^{-p_n/\theta} = \sum_k (a-q_k) e^{-p_k/\theta} t_k^w, \tag{19}$$

and thus $(q_1+p)t_1^w = (q_2+p)t_2^w$. Thus $t_1 \le t_2$ if $q_1 \ge q_2$.

Setting the derivative of (17) w.r.t. p_n is zero yields

$$e^{-p_n/\theta} A^{\frac{v}{(1-v)}} - \frac{1}{\theta}(q_n+p_n) e^{-p_n/\theta} A^{\frac{v}{(1-v)}} - (q_n+p_n) e^{-p_n/\theta} \frac{v}{1-v} A^{\frac{v}{(1-v)}-1} \frac{1}{\theta}(a-q_n) t_n^w e^{-p_n/\theta} = 0.$$

Substituting relation (18) for A, multiplying the resulting expression by θ and dividing it by $e^{-p_n/\theta}A^{\frac{v}{(1-v)}}$, we find $p_n = \theta - a$, which is (7). Thus in equilibrium, all ISPs choose identical user prices.

Taking the derivative of (17) w.r.t t_n and setting it to zero, we find that

$$F \cdot \frac{1}{1-v} t_n^{\frac{wv}{(1-v)}} w t_n^{w-1} = \alpha, \tag{20}$$

$$\text{where } F = M\mu\eta \left[(q_n + p_n)e^{-p_n/\theta}\right]^{\frac{1}{(1-v)}} \left(\frac{v\mu\eta}{\beta}\frac{v}{1-v}\right)^{\frac{v}{(1-v)}}.$$

We are now ready to prove, Proposition 1, that the equilibrium of $G(M,N)$ is symmetric in the two-sided case.

Proof (Proposition 1). Suppose the proposition were not true and $q_1 > q_2$. Recall that from the ISPs' FOCs wrt q_n we have $(q_1 + p)t_1^w = (q_2 + p)t_2^w$. Therefore

$$t_1 < t_2 \text{ when } q_1 > q_2. \tag{21}$$

From the ISPs' FOCs wrt t_n (we raise (20) to the power $(1-v)$ and rearrange):

$$(q_1 + p)t_1^{wv} t_1^{(w-1)(1-v)} = (q_2 + p)t_2^{wv} t_2^{(w-1)(1-v)} = H,$$

$$\text{where } H = \left[\frac{\alpha}{Mw\mu\eta}\right]^{(1-v)} \left(\frac{\beta}{v\mu\eta}\frac{1-v}{v}\right)^{v} e^{p^{\ddagger}/\theta}. \tag{22}$$

Thus $(q_1+p)t_1^{w+v-1} = (q_2+p)t_2^{w+v-1}$, from which we must have $t_1 \geq t_2$ when $q_1 \geq q_2$. But from (21) we have $t_1 \leq t_2$ when $q_1 \geq q_2$, which is a contradiction, unless $t_1 = t_2$ and $q_1 = q_2$. Thus Proposition (1) is proven.

Having shown the equilibrium is unique, we turn to deriving expressions for the equilibrium. From (19) and the fact that $p_n = \theta - a$ we obtain

$$(q_n + \theta - a)\frac{v}{1-v} t_n^w - (a - q_n)t_n^w = \sum_{k \neq n} (a - q_k)t_k^w.$$

Thus, $t_n^w[\theta v + q_n - a] = (1-v)\sum_{k \neq n}(a - q_k)t_k^w$. Writing this for $n = 1, ...N$ and summing we find $\sum_n t_n^w[\theta v + (q_n - a)(N(1-v) + v)] = 0$. Combining this with $q_n = q$ and $t_n = t$ we have $q^{\ddagger}$ as given in (7).

To find the optimal ISP investment $t = t^{\ddagger}$, we use $q_n = q$ and $t_n = t$ to express $\Pi_{Tn}(M,N)$ as $\Pi_{Tn}(M,N) = E \cdot t^w \cdot [Nt^w]^{\frac{v}{(1-v)}} - \alpha t_n$, where E is defined by

$$E^{1-v} = M^{1-v}(\mu\eta)(p+q)^{1-v}e^{-p/\theta}\left(\frac{v(a-q)}{\beta}\right)^{v}. \tag{23}$$

Writing the partial derivative of $\Pi_{Tn}(M,N)$ wrt t_n and equating it to zero, we find:

$$E \cdot \left[w t_n^{w-1}[t_1^w + \cdots + t_N^w]^{\frac{v}{(1-v)}} + t_n^w \frac{v}{1-v} w t_n^{w-1}[t_1^w + \cdots + t_N^w]^{\frac{v}{(1-v)}-1}\right] - \alpha = 0,$$

the symmetric solution $t_n = t$ is:

$$E \cdot \left[wt^{w-1}[Nt^w]^{\frac{v}{(1-v)}} + t^w \frac{v}{1-v} wt^{w-1}[Nt^w]^{\frac{v}{(1-v)}-1} \right] - \alpha = 0,$$

which simplifies to

$$E \cdot \left(N + \frac{v}{(1-v)} \right) wt^w t^{w-1} (Nt^w)^{\frac{v}{(1-v)}-1} = \alpha. \tag{24}$$

Lastly, we substitute our expressions for $p^\ddagger$ and $q^\ddagger$ into (24) to obtain

$$\begin{aligned} E^{1-v} &= (\mu\eta) \cdot \left[\frac{\theta N(1-v)}{N(1-v)+v} \right]^{1-v} e^{\frac{-(\theta-a)}{\theta}} \cdot \left(\frac{v}{\beta} \frac{\theta v}{N(1-v)+v} \right)^v \\ &= (\mu\eta)\, e^{\frac{-(\theta-a)}{\theta}} \cdot y^v \cdot \left[\frac{\theta N(1-v)}{N(1-v)+v} \right]^{1-v}. \end{aligned}$$

Then, we combine with (24) to find (8) with $x^\ddagger$and $y^\ddagger$ as defined in (10). Thus, we demonstrated that only a symmetric equilibrium with $p_n = p$, $q_n = q$, $t_n = t$ and $c_m = c$ exists, and it is unique (by construction). Finally, to calculate $c^\ddagger$, we substitute our expressions for equilibrium q and t into (16) to find (9).

5.2 Equilibrium Uniqueness in G

Before proceeding, we define some notation. Let π and δ be defined as:

$$\pi := \frac{v}{N(1-v)+v} \text{ and } \delta := \frac{a}{\theta}. \tag{25}$$

We assume that with today's Internet parameters, end user prices $p^\dagger$ are positive[2], which gives $p^\dagger = \theta(1-\pi) > 0$ and $p^\ddagger = \theta - a > 0$, and $q^\ddagger = a - \theta\frac{v}{N(1-v)+v} = a - \theta\pi$, from which $\pi < 1$, $\delta < 1$ and $\pi < \delta$. Also we infer $\pi < v$ and decreases with N In addition, from the expressions of the equilibrium parameters, one can show that the average (per each provider) returns in the equilibrium of the game $G(M, N)$ are:

$$\frac{\Pi_C(M,N)}{c} = \frac{\beta(1-v)}{v} \text{ and } \frac{\Pi_T(M,N)}{t} = \frac{\alpha}{w}[1-w-\pi]. \tag{26}$$

We are ready to prove Proposition 3.

Proof (Proposition 3). Consider subgame $G(M, N)$. Since $\pi < \delta$ and $y^\ddagger = y^\dagger \frac{\pi}{\delta}$ we have $y^\ddagger < y^\dagger$. Using these equations, we obtain:

$$t^\ddagger = \frac{1}{N} \left[\pi^v \left(\frac{w}{\alpha} \right)^{1-v} \cdot (\frac{v}{\beta})^v \cdot \theta e^{\frac{-(\theta-a)}{\theta}} \cdot \left[1 - e^{-kM} \right] \right]^{\frac{1}{(1-w-v)}},$$

[2] Despite widely available free internet access, average end-user access price is clearly strictly positive.

$$c^{\ddagger} = \frac{1}{M}\left[\pi^{1-w}\cdot\left(\frac{w}{\alpha}\right)^{w}\cdot(\frac{v}{\beta})^{1-w}\cdot\theta e^{\frac{-(\theta-a)}{\theta}}\cdot\left[1-e^{-kM}\right]\right]^{\frac{1}{(1-v-w)}}, \qquad (27)$$

$$t^{\dagger} = \frac{1}{N}\left[\left(\frac{\theta w}{\alpha}\right)^{1-v}\cdot(\frac{av}{\beta})^{v}\cdot e^{\pi-1}\left[1-e^{-kM}\right]\right]^{\frac{1}{(1-w-v)}},$$

$$c^{\dagger} = \frac{1}{M}\left[\left(\frac{\theta w}{\alpha}\right)^{w}\cdot(\frac{av}{\beta})^{1-w}e^{\pi-1}\left[1-e^{-kM}\right]\right]^{\frac{1}{(1-v-w)}}, \qquad (28)$$

and

$$t^{\ddagger} = \left(\frac{\beta}{\alpha}\frac{w}{v}\right)\cdot\pi^{-1}\frac{M}{N}c^{\ddagger};\quad t^{\dagger} = \frac{\theta}{a}\left(\frac{\beta}{\alpha}\frac{w}{v}\right)\cdot\frac{M}{N}c^{\dagger}.$$

Expression (28) and (27) have derivatives with respect to M that either transitions from being positive to being negative for one M, or is always negative. This property combined with (26) gives us that for any fixed N there exists unique $M(N)$ s.t.

$$\Pi_C(M(N),N) \geq c_e \text{ and } \Pi_C(M(N)+1,N) < c_e,$$

for $M(N) > 0$ or $\Pi_C(1,N) < c_e$ in the $M(N) = 0$ case. Moreover if $\tilde{M} > M(N)$, $\Pi_C(\tilde{M},N)$ decreases with $\tilde{M}$.

Next we claim $\frac{dM(N)}{dN} \leq 0$ by the following reasoning. Assume the reverse and let $N_1 < N_2$, and $M(N_1) < M(N_2)$. Then, since (28), (27) and (26) show that content provider profits decrease w.r.t. N for fixed M we have

$$\Pi_C(M(N_2),N_1) \geq \Pi_C(M(N_2),N_2) \geq c_e.$$

This contradicts the fact that $\Pi_C(\tilde{M},N)$ decreases with $\tilde{M}$ for any $\tilde{M} > M(N_1)$, and thus $\frac{dM(N)}{dN} \leq 0$ is proven. Lastly, we notice that in both regimes:

$$\frac{d\Pi_T(M(N),N))}{dN} = \frac{\alpha}{w}\frac{d\left\{[1-w-\pi(N)]t(M(N),N))\right\}}{dN} < 0,$$

which can be shown by differentiation. (We elaborate on this below.) The uniqueness and existence of the equilibrium follows immediately. Thus, we have proven Proposition 3.

To verify that $\frac{d\Pi_T(M(N),N))}{dN}$ is non-positive, we use $\frac{d\pi}{dN} = -\frac{(1-v)}{v}\pi^2$. In the two-sided case:

$$\frac{d\Pi_T(M(N),N))}{dN} = \left[1-e^{-kM}\right]^{\frac{1}{(1-w-v)}}\frac{\partial}{\partial N}\left\{\frac{1}{N}\left[1-w-\pi\right]\pi^{\frac{v}{1-w-v}}\right\}$$
$$+\left\{\frac{1}{N}\left[1-w-\pi\right]\pi^{\frac{v}{1-w-v}}\right\}\times\frac{1}{(1-w-v)}\left[1-e^{-kM}\right]^{\frac{1}{(1-w-v)}-1}k\underbrace{\frac{dM}{dN}}_{[-]},$$

where the $[-]$ notation is a reminder that $\frac{dM}{dN}$ is nonpositive. The second curly brackets term is positive. Expanding the first curly brackets term we get

$$\frac{\partial}{\partial N}\left\{\frac{1}{N}\left[1-w-\pi\right]\pi^{\frac{v}{1-w-v}}\right\} = -\frac{1}{N^2}\left[1-w-\pi\right]\pi^{\frac{v}{1-w-v}} +$$
$$\left\{\frac{1}{N}\left[1-w-\pi\right]\right\}\frac{v}{1-w-v}\pi^{\frac{v}{1-w-v}-1}\frac{d\pi}{dN} - \frac{1}{N}\pi^{\frac{v}{1-w-v}}\frac{d\pi}{dN}$$

Collecting terms, this becomes

$$-\frac{1}{N^2}\left[1-w-\pi\right]\pi^{\frac{v}{1-w-v}} + \left\{\frac{(1-v)}{v}\frac{1}{N}\pi^{\frac{1-w}{1-w-v}}\right\}\left\{-\frac{v\left[1-w-\pi\right]}{1-w-v} + \pi\right\} < 0,$$

because from $\pi < v < 1$ the last curly bracket is negative.

The derivations to show $\frac{d\Pi_T(M(N),N))}{dN}$ is negative for the one-sided case are similar so we omit them.

Proof (Proposition 4). Suppose there exists a pure strategy equilibrium of $\tilde{G}$, and let $(\tilde{M}^*, \tilde{N}^*)$ denote the respective equilibrium numbers of CPs and ISPs. Let (M^*, N^*) denote the respective number of CPs and ISPs in the unique equilibrium of the game G. Then it must be that $\tilde{M}^* = M(\tilde{N}^*)$, where $M(\cdot)$ is the function described in the proof of Prop. 3. The equilibrium actions and payoffs in the subgame $\tilde{G}(\tilde{M}^*, \tilde{N}^*)$ are the same as in subgame $G(\tilde{M}^*, \tilde{N}^*)$ since the games are identical after the entry stage and the subgame admits a unique equilibrium. From the proof of Prop. 3, per ISP profit $\Pi_T(M(N), N)$ decreases with N, and for any $\tilde{N}^* > N^*$, we have $\Pi_T(M(\tilde{N}^*), \tilde{N}^*) < t_e$, and thus, $\tilde{N}^* > N^*$ cannot occur as an equilibrium of the game $\tilde{G}$. If $\tilde{N}^* < N^* - 1$ another ISP will enter because the entering ISP sees profit of

$$\Pi_T(M(\tilde{N}^*), \tilde{N}^* + 1) \geq \Pi_T(M(\tilde{N}^* + 1), \tilde{N}^* + 1) > t_e$$

since $M(\cdot)$ and $\Pi_T(\cdot, N)$ are monotone. Finally, suppose $\tilde{N}^* = N^* - 1$. Then it must be that $\Pi_T(M(\tilde{N}^*), \tilde{N}^*+1) = \Pi_T(M(N^*-1), N^*) < t_e$ or else another ISP would have entered in the game $\tilde{G}$. But, $t_e \leq \Pi_T(M(N^*), N^*) \leq \Pi_T(M(N^* - 1), N^*)$ which is a contradiction.