Technology Choices and Pricing Policies in Wireless Networks

Yuanzhang Xiao[1], William R. Zame[2], and Mihaela van der Schaar[1]

[1] Department of Electrical Engineering, UCLA, Los Angeles CA 90095, USA
`{yxiao,mihaela}@ee.ucla.edu`
[2] Department of Economics, UCLA, Los Angeles CA 90095, USA
`zame@econ.ucla.edu`

Abstract. This paper studies the provision of a wireless network by a monopolistic provider who may be either benevolent (seeking to maximize social welfare) or selfish (seeking to maximize provider profit). The paper addresses the following questions: Under what circumstances is it feasible for a provider, either benevolent or selfish, to operate a network in such a way as to cover costs? How is the optimal behavior of a benevolent provider different from the optimal behavior of a selfish provider, and how does this difference affect social welfare? And, most importantly, how does the medium access control (MAC) technology influence the answers to these questions? To address these questions, we build a general model, and provide analysis and simulations for simplified but typical scenarios; the focus in these scenarios is on the contrast between the outcomes obtained under carrier-sensing multiple access (CSMA) and outcomes obtained under time-division multiple access (TDMA). Simulation results demonstrate that differences in MAC technology can have a significant effect on social welfare, on provider profit, and even on the (financial) feasibility of a wireless network.

Keywords: network economics, pricing, wireless networks.

1 Introduction

There has been much recent debate about the deployment of wireless networks that would allow Internet access in public areas. Central to this debate is the tradeoff between costs and benefits. Surprisingly, this debate seems to have ignored that the costs and benefits of such wireless networks depend crucially on the technology that is or could be employed. The purpose of this paper is to provide a framework for exploring the influence of technology on the costs and benefits of wireless networks and to demonstrate in a simple scenario that the feasibility and desirability of such a network may depend on the technology chosen. We show that the analysis depends crucially on the technology layer, the application layer, and the economic layer, and most crucially of all, on the interactions between these layers.

To see why the analysis depends crucially on the interactions between the various layers, consider a simple scenario that seems typical. There are two classes

R. Jain and R. Kannan (Eds.): GameNets 2011, LNICST 75, pp. 92–107, 2012.

of (potential) users: *data users*, who are sensitive to throughput but relatively insensitive to delay, and *video users*, who are sensitive to both throughput and delay. In managing the network, the service provider can offer a pricing policy and a scheduling policy, but the service provider's range of choices depends on the technology – in particular, on the medium access control (MAC) protocol – employed. If time-division multiple access (TDMA) is employed, the service provider will be able to guarantee quality of service (QoS) and monitor the usage of each user in order to charge per bit. Hence, the service provider can use a tiered pricing policy to screen the users into a number of types and offer performance guarantees to those users willing to pay for such guarantees. If carrier-sensing multiple access (CSMA) is employed, the service provider will be unable to guarantee QoS. Absent such performance guarantees, video users who require higher throughput or less delay may be unwilling to pay more than data users who will accept lower throughput and more delay. In this case, it is more reasonable for the service provider to adopt a flat fee for both types of users. As we will show, there are large regions within the range of plausible parameters in which employing TDMA rather than CSMA makes possible large improvements in social welfare. Indeed, there are regions in which employing TDMA would be consistent with operating a self-financing network while employing CSMA would not be.

1.1 Related Work

Two substantial bodies of work in the engineering literature ask about optimal behavior of the provider of a wireless network. The first considers a benevolent provider whose objective is to maximize social welfare [1]- [6]; the second considers a selfish provider whose objective is to maximize profit [7]- [12]. What we do here is to ask different (although related) questions that do not seem to have been studied before: Under what circumstances is it possible for a provider to operate a network in such a way as to cover costs? How is optimal behavior of a benevolent provider different from optimal behavior of a selfish provider and how does the difference affect social welfare? And, perhaps most importantly, how does the MAC protocol influence the answers to these questions?

Among the papers that focus on optimal pricing in networks, Palomar and Chiang [1] and Kelly *et al.* [2] [3] consider a network with one service provider serving multiple users and propose charging in proportion to the flow rates of the users in order to maximize social utility. Johari and Tsitsiklis [5] [6] focus on the efficiency loss under this pricing scheme and its variant with price differentiation. Gibbens and Kelly [4] propose a packet-based pricing policy for more effective flow control. Under the same scenario, Basar *et al.* [7] [8] [9] propose linear and nonlinear differentiated pricing schemes to control the network usage and maximize the provider's revenue. For cellular networks, Mandayam *et al.* [10] and Alpcan *et al.* [11] propose pricing for power control to reduce interference. It should be noted, however, that the prices in the above papers are not actually paid by the users; rather, they are signals used for the purpose of controlling the network congestion. In Paschalidis and Tsitsiklis [12], which studies a dynamic network with users arriving and leaving the network and derive the optimal pricing strategy and its static approximation, prices are actually paid by users, but – as in [2]- [11] – the technology layer is highly abstracted (as a constraint on the resource allocation). Other papers use different models and have a different focus. Friedman and Parkes [13] study the existence of implementable mechanisms for the users to truthfully announce their arrivals in WiFi networks. Musacchio and Walrand [14] model WiFi pricing as a dynamic game involving one access point and one user, and study the Nash equilibrium (NE) of this game. van der Schaar [15] and Sarkar [16] focus on competition among multiple service providers with simplified user subscription models.

Our work differs from this literature in that we model prices as actually paid by users and collected by the service providers, and we provide a much more detailed and less abstracted description of technology. We make use of both of these differences to study the interaction between technology and pricing and their impacts on performance.[1] In particular, we consider various technologies and pricing policies (closely modeled as those used in the real world by wireless carriers) to study the interactions between technology and pricing.

The remainder of this paper is organized as follows. In Section 2 we introduce the system model for the three-layer network. In Section 3 we formulate the design problem for the benevolent and selfish providers and the decision process of the users as a two stage game (with the provider acting in the first stage and the users acting in the second stage). In Section 4, we focus our analysis on a typical scenario to gain insights into this problem, and provide simulation results in this typical scenario. Finally, in Section 5 we conclude the paper.

2 System Model

We consider a wireless network with a single access point (AP), created by a service provider to enable Internet connections to potential users. Keeping in mind that a single access point will typically serve a relatively small number of potential users who may come and go at any moment in time, we build a dynamic continuous-time framework in which a finite number of potential users arrive and depart randomly.

Before we begin with the description of the service provider, we first introduce the basic concept of the user type. The *users* are categorized into K types according to their utility functions and arrival and departure processes. There are N_k identical users of type k.

2.1 The Service Provider

The service provider must choose a MAC protocol and a pricing policy.

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[1] The interplay of technology and pricing policy is discussed by Lehr *et al.* [17], but their paper provides no quantitative analysis. To our best knowledge, no previous work has ever mathematically modeled and explicitly studied this problem.

The Medium Access Control Protocol. The MAC protocol determines which users will have access to which resources in which way. In principle, the service provider might be able to choose among many MAC protocols. CSMA and TDMA are the canonical MAC protocols. CSMA is representative of the protocols without a central controller, where the packets contend to get access to the medium. TDMA is representative of the protocols with a central controller, where the packets access the medium in non-overlapping periods of time. The key difference between CSMA and TDMA is the ability to offer QoS guarantee, which will probably result in different selections of pricing policies. The lack of QoS guarantee in CSMA may prevent the provider from charging by bit. Imagine a video user who pays for some video frames but loses subsequent frames due to network congestion. Since those paid video frames may be useless because of the loss of subsequent frames, the video users may be unwilling to pay for those bits without QoS guarantee. *Therefore, the provider using TDMA is able to charge both a subscription fee and a per-bit fee, while the provider using CSMA is more likely to charge a subscription fee only.* We write θ for a particular protocol.

Pricing Plans, Pricing Policies, and Pricing States. A *pricing plan* is a schedule of charges to users. We assume that charges consist of a subscription fee (paid once per billing period) p_s and a per-bit surcharge q for usage in excess of some specified threshold number of bits β. Thus a pricing plan is a triple

$$\mathbf{p} = (p_s, q, \beta).$$

To allow for the possibility that some users choose not to belong to the network at all, let $\phi = (0, 0, 0)$ be a dummy plan that imposes no costs. A user choosing ϕ does not subscribe to the network.

A *pricing policy* is a vector of pricing plans; for simplicity, we assume here that each pricing policy is a vector of exactly $L+1$ pricing plans: $\mathbf{P} = (\mathbf{p}^0, \mathbf{p}^1, \ldots, \mathbf{p}^L)$; by convention we assume that $\mathbf{p}^0 = \phi$.

Given a pricing policy $\mathbf{P} = (\mathbf{p}^0, \mathbf{p}^1, \ldots, \mathbf{p}^L)$, each user type k chooses a pricing plan from $\mathbf{P}$ by randomizing over all the choices according to a probability distribution. We define the *pricing state* to be the vector $\mathbf{v} = (v^0, v^1, \ldots, v^L)$, where v^ℓ is the number of users who are currently online and choose the pricing plan $\mathbf{p}^\ell$. We write $\mathcal{V}$ for the set of pricing states.

2.2 Users

The users are characterized by their utility functions, arrival processes, and service times. Given user characteristics and the technology and the pricing policy adopted by the service provider, each user determines a probability distribution on the choices of pricing plans that maximizes its expected utility (which will depend on the choices of all the other users). At the beginning of time, each user chooses a pricing plan randomly according to the prescribed probability distribution, and every time a user arrives at the network, the user reports the chosen plan to the service provider. The service provider will make the scheduling according to the current pricing state and the choice of a particular user.

Choices of Pricing Plans. Users choose pricing plans to maximize their expected utility, given the menu of pricing plans, the MAC protocol of the provider and the choices of other users. We allow for the possibility that users *randomize*, so users of type k choose a probability distribution over pricing plans. We write $\pi_{k,\ell}$ for the probability that a user of type k chooses plan ℓ.

Allowing for randomization guarantees that equilibrium exists. We may interpret randomization literally: users who are indifferent over various plans break their indifference in a random way. Alternatively, we may interpret randomization simply as uncertainty in the minds of the provider and other users. If the number of users is large, we can also interpret the probability distribution over pricing plans as the distributions of plans among the population [20].

The randomization is realized at the beginning of time. Upon arrival, each user tells the service provider the pricing plan it chooses, and the provider uses this information for scheduling. Write $\pi_k = [\pi_{k,0}, \ldots, \pi_{k,L}]$ for the (random) action of users of type k, and $\pi = (\pi_1, \ldots, \pi_K)$ for the vector of actions of all users. Represent the result of the randomization by a set of vectors $\mathbf{n} = (\mathbf{n}_1, \ldots, \mathbf{n}_K) = ([n_{1,0}, \ldots, n_{1,L}], \ldots, [n_{K,0}, \ldots, n_{K,L}])$ with $n_{k,\ell}$ being the number of type-k users choosing plan ℓ.

System State. The *system state*, or the *true state*, is defined as the number of users of each type choosing each pricing plan. Specifically, the system state $\mathbf{X}(t)$ at time t is a $K \times (L+1)$ matrix, with $x_{k,\ell}$ as the element at the kth row and $(\ell+1)$th column, representing the number of type-k users who choose plan ℓ.

Arrival Process and Service Time. We use a continuous-time model for the arrival and departure processes[2] (reflecting the fact that users might arrive/depart at any moment); as in [21], we assume that the arrival process of type-k users choosing plan ℓ is Poisson with arrival rate

$$\lambda_{k,\ell}(t) = \lambda_k \cdot (n_{k,\ell} - x_{k,\ell}(t)),$$

where λ_k is the individual arrival rate of a type-k user. We also assume that the service time of one type-k user is exponentially distributed with mean $1/\mu_k$.

Billing Period. We fix a *billing period* of length ΔT, which is typically one month. Subscription fees are charged at the beginning of each billing period; other fees are charged at the end of each billing period. This is consistent with the usual billing methods: people pay a subscription fee prospectively and other charges retrospectively. For convenience, we assume that neither the provider nor the users discount utility and cost over the billing period.

Expected Utility. The service provider and the users evaluate the social welfare and their satisfaction, respectively, by the *expected utility*, defined as the

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[2] Here, the arrival process characterizes the arrival of users, but not the arrival of users' packets. Similarly, the service time is the duration of users staying in the system.

expectation of the total utility over a billing period when the stochastic process of the system state $\mathbf{X}(t)$ reaches the steady state. Each user's total utility consists of two components: utility of use and disutility of cost. To keep the model simple, we assume that total utility is simply the sum of utility of use and disutility of cost and is linear in cost with marginal utility of cost equal to 1 [22]:

$$\text{total utility} \;=\; \text{utility of use} \;-\; \text{cost}\ . \tag{1}$$

We denote the expected utility of use of a type-k user by $U_k(\theta, \pi)$, if the MAC protocol is θ and the joint probability distribution over pricing plans is π. We can calculate the expected utility of use $U_k(\theta, \pi)$ as follows

$$U_k(\theta, \pi) = \sum_{\ell=1}^{L} \pi_{k,\ell} \cdot \sum_{\mathbf{n}:n_{k,\ell}\geq 1} \Pr(\mathbf{n}) \cdot V_k^{\ell}(\theta, \mathbf{n}), \tag{2}$$

where $\Pr(\mathbf{n})$ is the probability that the randomization results in $\mathbf{n}$, and $V_k^{\ell}(\theta, \mathbf{n})$ is the steady-state utility of use of a type-k user, if the MAC protocol is θ and the result of the randomization is $\mathbf{n}$.

We denote the expected cost of a type-k user by $C_k(\theta, \mathbf{P}, \pi)$, if the MAC protocol is θ, the pricing policy is $\mathbf{P}$, and the joint probability distribution over pricing plans is π. The details for the calculation of $\Pr(\mathbf{n})$, $V_k^{\ell}(\theta, \mathbf{n})$, and $C_k(\theta, \mathbf{P}, \pi)$ can be found in [23, Sec. II-B].

Users' Decision Process. Each user determines the randomizing probability that maximizes its own expected utility. The optimal action for a type-k user satisfies

$$\pi_k = \arg\max_{\pi'_k} \left\{ \widetilde{U}_k(\theta, (\pi; \pi'_k)) - \widetilde{C}_k(\theta, \mathbf{P}, (\pi; \pi'_k)) \right\}, \tag{3}$$

where $(\pi; \pi'_k)$ is the joint action profile π with one type-k user changing its action from π_k to π'_k, and $\widetilde{U}_k(\theta, (\pi; \pi'_k))$ and $\widetilde{C}_k(\theta, \mathbf{P}, (\pi; \pi'_k))$ are the utility of use and cost of that deviating user, respectively, calculated in [23, Sec. II-B].

Since each user maximizes their own expected utility, the outcome of the users' decision process is naturally the Nash equilibrium of the *plan selection game* defined as

$$\mathcal{G}_{\mathbf{P}} = \left\{ \mathcal{K} = \{1, \ldots, K\}, \{\pi_k\}_{k=1}^{K}, \{U_k - C_k\}_{k=1}^{K} \right\}.$$

Here we put $\mathbf{P}$ in the subscript of $\mathcal{G}$ to emphasize that the plan selection game depends on the pricing policy of the provider. We denote $\pi^{\mathrm{NE}}(\mathbf{P})$ as the Nash equilibrium of $\mathcal{G}_{\mathbf{P}}$.

Proposition 1. *There exists a symmetric Nash equilibrium in the plan selection game $\mathcal{G}_{\mathbf{P}}$.*

Proof. The plan selection game $\mathcal{G}_{\mathbf{P}}$ is a finite game; Nash [22], [24] shows that each such game has an Nash equilibrium in which players of the same type choose the same strategy.

3 Problem Formulation

In this section, we formulate the design problem of the service provider as a Stackelberg game. The service provider tries to find a MAC protocol θ and a pricing policy $\mathbf{P}$, so that at the equilibrium of the plan selection game $\mathcal{G}_{\mathbf{P}}$, the social welfare (for the benevolent provider) or the total revenue (for the selfish provider) is maximized, subject to the constraint that costs be covered.

Before doing this, however, we must note that our notion of solution assumes that the service provider knows the arrival rates, service times, and utility functions of all types of users (but does not know the type of a particular user), and foresees the behavior of the users. The users in turn must also know the behavior of other users. Implicitly, therefore, we view the outcome as involving some learning process that is not modeled here. We intend to address this issue in later work, while focusing on characterizing the system performance at the equilibria in this paper.

Under the above assumptions, we can formulate the design problem of the service provider as follows. For a benevolent service provider aiming at maximizing the social welfare, its design problem (PB) can be written as

$$\begin{aligned} \max_{\theta,\mathbf{P}} \quad & \sum_{k=1}^{K} \left(U_k(\theta, \pi^{\mathrm{NE}}(\mathbf{P})) - C_k(\theta, \mathbf{P}, \pi^{\mathrm{NE}}(\mathbf{P})) \right) \cdot N_k \\ s.t. \quad & \mathrm{IR} : \textstyle\sum_{k=1}^{K} C_k(\theta, \mathbf{P}, \pi^{\mathrm{NE}}(\mathbf{P})) \cdot N_k \geq C_0, \end{aligned}$$

where C_0 is the fixed cost for the service provider during a billing period due to the maintenance of the network. The objective function is the social welfare defined as the sum utility of all the users. The constraint is the individual rationality (IR) constraint (or participation constraint) for the service provider. The solution $\mathbf{P}^*$ to the above problem provides the users with a set of pricing plans to choose from. After each user chooses the pricing plan that maximizes its own expected utility, the system reaches the maximum social welfare.

Similarly, for a selfish service provider aiming at maximizing its own revenue, its design problem (PS) can be written as

$$\begin{aligned} \max_{\theta,\mathbf{P}} \; & \textstyle\sum_{k=1}^{K} C_k(\theta, \mathbf{P}, \pi^{\mathrm{NE}}(\mathbf{P})) \cdot N_k \\ s.t. \; & \mathrm{IR} : \textstyle\sum_{k=1}^{K} C_k(\theta, \mathbf{P}, \pi^{\mathrm{NE}}(\mathbf{P})) \cdot N_k \geq C_0. \end{aligned}$$

Here, the only difference between the problem (PB) and (PS) is the objective function.

Because our focus is the influence of technology on the economic layer and system performance, we will first find the optimal pricing policy of the problems (PB) and (PS) with fixed MAC protocol, and then compare the optimal pricing policies and the resulting system performance under different MAC protocols.

4 Two Simple Scenarios

In this section, we study two simple scenarios. In each scenario, there are two types of users: type-1 users are video users with stringent throughput and delay requirements, while type-2 users are data users, who require low throughput and can tolerate large delay. In the first scenario, the service provider uses CSMA and only charges the same subscription fee for all the active users. In the second scenario, the service provider uses TDMA and charges for a per-bit surcharge in addition to the subscription fee.

4.1 CSMA with Subscription Fee Only

The provider using CSMA offers the dummy pricing plan $\mathbf{p}^0 = \phi$ and a single non-dummy pricing plan $\mathbf{p}^1 = (p_s, 0, 0)$. The design problem of the provider can be analyzed using backward induction. In the plan selection game, there can be three types of Nash equilibria depending on the value of $\pi_{k,1}$: $\pi_{k,1} = 0$, $\pi_{k,1} = 1$, or $0 \leq \pi_{k,1} \leq 1$. We can calculate the optimal pricing policy that induces the desired equilibrium, and the corresponding social welfare and provider revenue. The benevolent (selfish) provider compares all the possible equilibria and adopts the subscription fee that induces the NE with the highest social welfare (revenue). In both cases, the constraint is that revenue must cover cost – else the network will not operate at all.

Theorem 1. *Suppose that the service provider uses CSMA and offers the following pricing policy*

$$\mathbf{P} = \left(\mathbf{p}^0 = \phi, \mathbf{p}^1 = (p_s, 0, 0)\right).$$

For the pure Nash equilibria, we show the optimal pricing policies of both providers and the resulting social welfare and provider revenue, as well as the existence conditions for the NE, as follows:

- *Type-1 NE:* $\pi_{1,1} = 1$, $\pi_{2,1} = 1$. *See Table 1.*
- *Type-2 NE:* $\pi_{k,1} = 1$, $\pi_{3-k,1} = 0$. *See Table 2 for the case with* $\pi_{1,1} = 1$ *and* $\pi_{2,1} = 0$. *The case with* $\pi_{1,1} = 0$ *and* $\pi_{2,1} = 1$ *is symmetric.*
- *Type-3 NE:* $\pi_{1,1} = 0$, $\pi_{2,1} = 0$. *This NE is a trivial one that can be achieved by setting the subscription fee high enough.*

Proof. See [23, Appendix A].

Remark 1: In the above theorem, we only characterize the system performance at the pure Nash equilibria, because pure Nash equilibria seem to be a more reasonable outcome in terms of information availability. As we can see from Table 1-2, information on the users' probability distribution over pricing plans π is *not* required for service providers and especially for the users. However, for the mixed Nash equilibrium, providers and users need to know the actions of all

Table 1. CSMA, Type-1 NE: $\pi_{1,1} = 1$, $\pi_{2,1} = 1$, $\mathbf{n} = ([0, N_1], [0, N_2])$, $k^\star = \arg\min_k V_k^1(\theta, \mathbf{n})$

Provider Type	Benevolent	Selfish
Pricing Policy	$p_s = \frac{C_0}{N_1+N_2}$	$p_s = V_{k^\star}^1(\theta, \mathbf{n})$
Social Welfare	$\sum_{i=1}^2 V_i^1(\theta, \mathbf{n}) \cdot N_i - C_0$	$(V_{3-k^\star}^1(\theta, \mathbf{n}) - V_{k^\star}^1(\theta, \mathbf{n})) \cdot N_{3-k^\star}$
Provider Revenue	C_0	$V_{k^\star}^1(\theta, \mathbf{n}) \cdot (N_1 + N_2)$
Existence Conditions	$V_{k^\star}^1(\theta, \mathbf{n}) \cdot (N_1 + N_2) \geq C_0$	

Table 2. CSMA, Type-2 NE: $\pi_{1,1} = 1$, $\pi_{2,1} = 0$, $\mathbf{n} = ([0, N_1], [N_2, 0])$, $\mathbf{n}' = ([0, N_1], [N_2 - 1, 1])$

Provider Type	Benevolent	Selfish
Pricing Policy	$p_s = \max\left\{\frac{C_0}{N_1}, V_2^1(\theta, ([0, N_1], [N_2 - 1, 1]))\right\}$	$p_s = V_1^1(\theta, \mathbf{n})$
Social Welfare	$V_1^1(\theta, \mathbf{n}) \cdot N_1 - \max\left\{C_0, V_2^1(\theta, \mathbf{n}') \cdot N_1\right\}$	0
Provider Revenue	$\max\left\{C_0, V_2^1(\theta, \mathbf{n}') \cdot N_1\right\}$	$V_1^1(\theta, \mathbf{n}) \cdot N_1$
Existence Conditions	$V_1^1(\theta, \mathbf{n}) \cdot N_1 \geq C_0$, $V_1^1(\theta, \mathbf{n}) > V_2^1(\theta, \mathbf{n}')$	

the users. Take the equilibrium $\pi_{1,1} = 1$, $\pi_{2,1} \in (0, 1)$ for example. In this case, both benevolent and selfish providers should set the subscription fee as

$$p_s = \sum_{n_{2,1}=1}^{N_2} \binom{N_2 - 1}{n_{2,1} - 1} \pi_{2,1}^{n_{2,1}-1} (1 - \pi_{2,1})^{N_2 - n_{2,1}} V_2^1(\theta, \{\mathbf{n}_1, [N_2 - n_{2,1}, n_{2,1}]\}),$$

where $\pi_{2,1}$ is required to compute p_s. The same argument applies to Theorem 2, which only characterizes the pure NE.

Remark 2: As seems obvious, the benevolent provider charges as little as possible, subject to revenue being at least as great as cost; the selfish provider charges as much as possible, subject to the cost to each user being no greater than utility. As the simulations in Sec. IV-C make clear, there are ranges of the user number and demand parameters for which the outcome when the provider is benevolent and the outcome when the provider is selfish do not lead to the usage by the same types.

4.2 TDMA with Subscription Fee and Per-bit Surcharge

Similar to the case with CSMA, we can get the following theorem about the pure equilibria when the service provider uses TDMA and can charge a subscription fee plus a per-bit surcharge.

Theorem 2. *Suppose that the service provider uses TDMA and offers the following pricing policy*

$$\mathbf{P} = \left(\mathbf{p}^0 = \phi, \mathbf{p}^1 = (p_s^1, 0, 0), \mathbf{p}^2 = (p_s^2, q, \beta)\right).$$

For the pure Nash equilibria, we show the optimal social welfare and provider revenue, as well as the existence conditions for the NE, as follows:[3]

- *Type-1 NE:* $\pi_{1,2} = 1$, $\pi_{2,2} = 1$. *See Table 3.*
- *Type-2 NE:* $\pi_{k,2} = 1$, $\pi_{3-k,1} = 1$. *See Table 4 for the case with* $\pi_{1,2} = 1$ *and* $\pi_{2,1} = 1$. *The case with* $\pi_{1,1} = 1$ *and* $\pi_{2,2} = 1$ *is symmetric.*
- *Type-3 NE:* $\pi_{k,2} = 1$, $\pi_{3-k,0} = 1$. *See Table 5 for the case with* $\pi_{1,2} = 1$ *and* $\pi_{2,0} = 1$. *The case with* $\pi_{1,0} = 1$ *and* $\pi_{2,2} = 1$ *is symmetric.*
- *Type-4 NE:* $\pi_{1,0} = 1$, $\pi_{2,0} = 1$. *This NE is a trivial one that can be achieved by setting the subscription fees high enough.*

Table 3. TDMA, Type-1 NE: $\pi_{1,2} = 1$, $\pi_{2,2} = 1$, $\mathbf{n} = ([0, 0, N_1], [0, 0, N_2])$, $k^\star = \arg\min_k V_k^2(\theta, \mathbf{n})$, $j^\star = \arg\max_j \hat{B}_j^2(\theta, \mathbf{n})$

Provider Type	Benevolent	Selfish
Social Welfare	$\sum_{i=1}^{2} V_i^2(\theta, \mathbf{n}) \cdot N_i - C_0$	$(V_{3-j^\star}^2(\theta, \mathbf{n}) - V_{k^\star}^2(\theta, \mathbf{n})) \cdot N_{3-j^\star}$
Provider Revenue	C_0	$V_{j^\star}^2(\theta, \mathbf{n}) \cdot N_{j^\star} + V_{k^\star}^2(\theta, \mathbf{n}) \cdot N_{3-j^\star}$
Existence Conditions	$\exists\ i : \hat{B}_i^2(\theta, \mathbf{n}) \geq \hat{B}_{3-i}^2(\theta, \mathbf{n})$ and $V_i^2(\theta, \mathbf{n}) \cdot N_i + V_{k^\star}^2(\theta, \mathbf{n}) \cdot N_{3-i} \geq C_0$	

Table 4. TDMA, Type-2 NE: $\pi_{1,2} = 1$, $\pi_{1,1} = 1$, $\mathbf{n} = ([0, 0, N_1], [0, N_2, 0])$, $\mathbf{n}' = ([0, 1, N_1 - 1], [0, N_2, 1])$, $\gamma = \max\left\{0, V_1^1(\theta, \mathbf{n}') - V_2^1(\theta, \mathbf{n})\right\}$

Provider Type	Benevolent	Selfish
Social Welfare	$V_1^2(\theta, \mathbf{n}) \cdot N_1 +$ $V_2^1(\theta, \mathbf{n}) \cdot N_2 - C_0$	$\gamma \cdot N_1$
Provider Revenue	C_0	$\left(V_1^2(\theta, \mathbf{n}) - \gamma\right) \cdot N_1 +$ $V_2^1(\theta, \mathbf{n}) \cdot N_2$
Existence Conditions	$\left(V_1^2(\theta, \mathbf{n}) - \gamma\right) \cdot N_1 + V_2^1(\theta, \mathbf{n}) \cdot N_2 \geq C_0$	

Proof. See [23, Appendix B].

Remark 3: From the above theorem, we can predict the equilibrium point induced by both providers under TDMA. First, if the utility of one type of users alone in the system is higher than the sum utility of two types of users coexisting in the system, both providers will admit only the high-utility users (most likely the video users), resulting in the type-3 scenario. However, the type-3 scenario may not be common under TDMA, because the providers can charge video users for a high surcharge to control their data usage, such that they will

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[3] In Table 3-5, $\hat{B}_k^\ell(\theta, \mathbf{n})$ is the expected amount of excessive data usage consumed by a type-k user choosing plan ℓ over a billing period at the steady state; see [23, Eqn. (5)] for the detailed definition and calculation.

Table 5. TDMA, Type-3 NE: $\pi_{1,2} = 1$, $\pi_{2,0} = 1$, $\mathbf{n} = ([0, 0, N_1], [N_2, 0, 0])$, $\mathbf{n}' = ([0, 0, N_1], [N_2 - 1, 0, 1])$

Provider Type	Benevolent	Selfish
Social Welfare	if $\hat{B}_1^2(\theta, \mathbf{n}) < \hat{B}_2^2(\theta, \mathbf{n}')$: $V_1^2(\theta, \mathbf{n}) \cdot N_1 - C_0$; else : $V_1^2(\theta, \mathbf{n}) \cdot N_1 - \max\left\{C_0, V_2^2(\theta, \mathbf{n}') \cdot N_1\right\}$.	0
Provider Revenue	0	$V_1^2(\theta, \mathbf{n}) \cdot N_1$
Existence Conditions	$V_1^2(\theta, \mathbf{n}) \cdot N_1 \geq C_0$, $\left\{\hat{B}_1^2(\theta, \mathbf{n}) < \hat{B}_2^2(\theta, \mathbf{n}') \text{ or } \left(\hat{B}_1^2(\theta, \mathbf{n}) \geq \hat{B}_2^2(\theta, \mathbf{n}') \text{ and} V_1^2(\theta, \mathbf{n}) \geq V_2^2(\theta, \mathbf{n}')\right)\right\}$	

not consume a large amount of data to congest the network. Both type-1 and type-2 scenarios characterize the cases when the providers admit both types of users. For type-1 scenario, both providers set a very high p_s^1 so that no users choose $\mathbf{p}^1$. Then the benevolent provider charges a small p_s^2 and q just to cover the cost, while the selfish one set appropriate p_s^2 and q so that both types of users receive zero total utility. The selfish provider can do that as long as the high-usage users have higher utility of use than the low-usage users. For type-2 scenario, both providers set appropriate plans so that low-usage users choose $\mathbf{p}^2$ and high-usage users choose $\mathbf{p}^1$.

Remark 4: By comparison between the scenarios under CSMA and TDMA, we can see that the feasible region under TDMA becomes larger because the service provider can measure the data usage and charge for the excessive bits used by the users. Intuitively, if the SP can only charge the same subscription fee for all the users, the high-usage users, such as the video users, will have the incentives to use unlimited amount of data, which will congest the network and result in a negative utility for the low-usage users that are online. By imposing the surcharge, the benevolent provider can charge less for the data users and more for the video users so that both types of users have positive utility. The selfish provider can use the surcharge to maximize its own revenue. In particular, if the high-usage users have higher utility of use than the low-usage users do, the selfish provider can gain so much revenue that both types of users get zero utility.

4.3 Numerical Simulation

Now we use numerical simulations to observe more details about the impact of the technology on the system performance. The key parameters in the simulation are described as follows:

- The service provider uses CSMA protocol with constant backoff window of 16ms or TDMA protocol.
- The pricing policy is $\mathbf{P} = (\phi, \mathbf{p}^1 = (p_s, 0, 0))$ for CSMA and $\mathbf{P} = (\phi, \mathbf{p}^1 = (p_s^1, 0, 0), \mathbf{p}^2 = (p_s^2, q, \beta))$ for TDMA.

- The total throughput of the AP is $B = 54\ Mbps$.
- The utility of type-1 users, the video users, is the Peak Signal-to-Noise Ratio (PSNR) of the video sequences. Here we use the *Foreman* video (CIF 15Hz), whose operational utility-rate-delay function is calculated by experiment. The details can be found in [25].
- The utility of type-2 users, the data users, is [8] [9]

$$u_2 = 10 \cdot \log(1 + \tau_2). \tag{4}$$

- The billing period is $\Delta T = 360$ hours/month, namely 12 hours/day times 30 days/month.
- The cost of the service provider is $C_0 = 1000$.

In the simulation, we change the numbers and arrival rates of the users and solve the problem of the benevolent and selfish providers under different tuples of user numbers and arrival rates. The simulation results and the corresponding analysis is as follows.

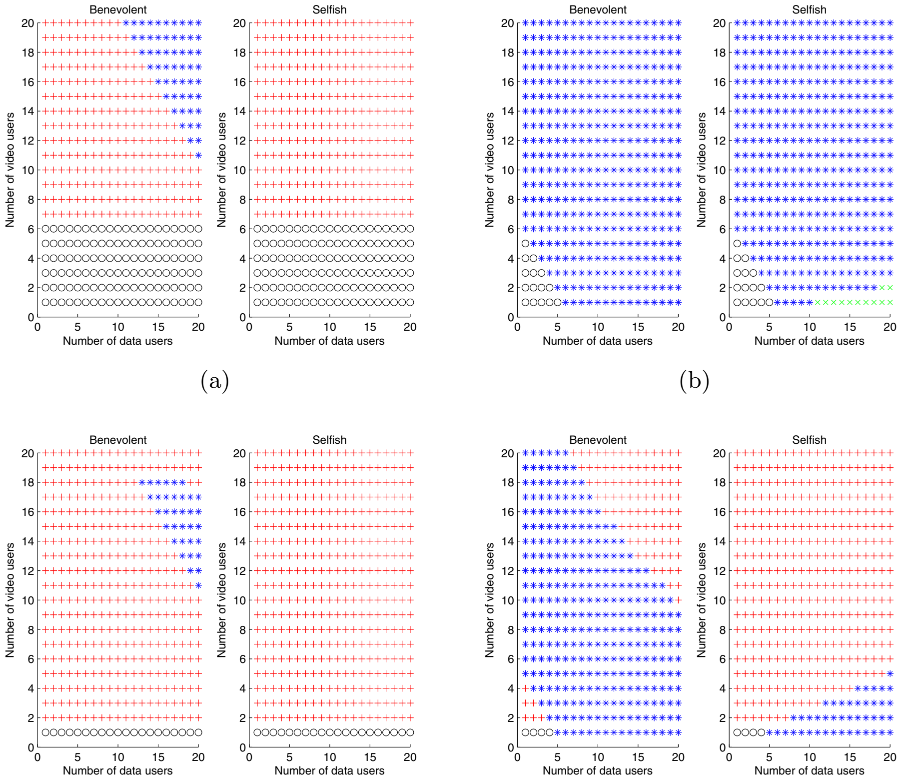
Numbers of Users. Here we show the phase diagram of the types of users in the system at the equilibrium under different user numbers. The phase diagram here illustrates which type or types of users are admitted to the system, given different numbers of video and data users in the system with other parameters fixed.

Fig. 1 show the phase diagrams with low-demand video users and low-demand data users, low-demand video users and high-demand data users, high-demand video users and low-demand data users, and high-demand video users and high-demand data users, respectively. We can see from the figures that in general, the benevolent provider admits more types of users than the selfish one does, whenever it is possible. The phase diagram under TDMA with both users having low demand is also shown as a representative scenario under TDMA protocol. We omit the TDMA scenarios with other user demands here due to space limitation. More detailed analysis on each scenario is presented as below.

Fig. 1(a) shows that, when the video users and data users both have low demands, the benevolent provider tends to admit both types of users to maximize the social welfare if the numbers of both types of users are large. On the contrary, the selfish one tends to admit video users to give the entire bandwidth to the highly profitable video users and denies access for the data users with low utility.

Fig. 1(b) shows that, when the demand of video users remains low and the data users have higher demand, both providers begin to admit some data users, in addition to video users, to achieve larger social welfare or gain more revenue, since the data users occupy the channels more often and thus have higher total utility now. When the data users significantly outnumber the video users, the selfish provider will admit only the data users.

Fig. 1(c) and Fig. 1(d) show that, when the video users have high demand, the benevolent provider drops all the data users when their demand is low, and tries to admit some data users when their demand is high. This means that the benevolent provider chooses the high-utility video users, when both users have



(a)

(b)

(c)

(d)

Fig. 1. Phase diagrams of the types of users in the system at the equilibrium under CSMA. 'blue *': both video and data users, 'red +': only video users, 'green ×': only data users, 'black ○': none. (a): low-demand video users and low-demand data users; (b): low-demand video users and high-demand data users; (c): high-demand video users and low-demand data users; (d): high-demand video users and high-demand data users. Here low demand means $\lambda_1/\mu_1 = 0.1$ and high demand means $\lambda_1/\mu_1 = 1$.

high demands and it has to choose one from the two types to reduce congestion. For the selfish provider, it always tends to drop the data users to allocate the entire bandwidth to the video users to maximize the revenue.

We also show the phase diagram under TDMA protocol with low-demand video and data users in [23, Fig. 3], which we omit here due to space limit. We observe that both providers admit both types of users under most configurations of user numbers: the benevolent provider admits both users to maximize social welfare, while the selfish one admits both users to maximize revenue. Compared to CSMA, TDMA enables both providers to admit both users by setting different plans for different types of users, when the difference between the utility of different users is large. This trend of admitting more users remains the same with other user demands.

Arrival Rates of Users. In Fig. 2, we show the phase diagram on what types of users are in the system at the equilibrium under different arrival rates of users of both types. We fix the number of users of each type at 20. From the figure, we can see that the benevolent provider admits both types of users under a large range of arrival rates. In particular, when the data users have large arrival rates and the video users have medium arrival rates, the relatively low subscription fee set by the benevolent provider draws a large number of users, resulting in low throughput and thus low utility of use of video users. Hence, only data users choose to join the network in the charge of the benevolent provider. On the contrary, the selfish provider sets a high subscription fee to squeeze out the data users, leaving only video users in the system, in order to gain more revenue.

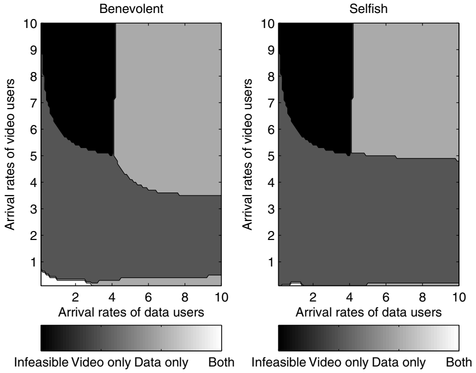


Fig. 2. Phase diagrams of the types of users in the system at the equilibrium with different arrival rates under CSMA. The number of users of each type is 20.

5 Conclusion

In this paper, we studied the provision of a public wireless network by a single (monopolistic) provider who may be either benevolent (seeking to maximize social welfare) or selfish (seeking to maximize provider profit). The paper presented a model for the public wireless network with three interdependent layers, namely the technology layer, the application layer, and the economic layer. Using the proposed model, we analyzed the influence of technology on the economic layer, and more importantly, the interaction of technology and economic layers that determines the feasibility and desirability of the network. We derived the feasibility conditions and the social welfare at the optimal operating points of the benevolent and selfish service providers for the public wireless network under different technologies. By simulation, we characterized different behaviors of a benevolent provider and a selfish provider at their optimal operating points, and the difference social welfare and revenue resulting from the different behaviors. Simulation results also demonstrated that differences in MAC technology can have a significant effect on the system performance. By using TDMA, which enables the providers to monitor the data usage of each user and charge per-bit rate, both the benevolent provider and the selfish provider can exploit the flexibility of differentiated pricing plans in order to maximize social welfare and revenue, respectively.

References

1. Palomar, D.P., Chiang, M.: A tutorial on decomposition methods for network utility maximization. IEEE J. Sel. Areas Commun. 24(8), 1439–1451 (2006)
2. Kelly, F.P.: Charging and rate control for elastic traffic. Eur. Trans. TeleCommn. 8, 33–37 (1997)
3. Kelly, F.P., Maulloo, A.K., Tan, D.K.H.: Rate control for communication networks: Shadow prices, proportional fairness and stability. J. Oper. Res. Soc. 49, 237–252 (1998)
4. Gibbens, R.J., Kelly, F.P.: Resource pricing and the evolution of congestion control. Automatica 35(12), 1969–1985 (1999)
5. Johari, R., Tsitsiklis, J.N.: Efficiency loss in a network resource allocation game. Math. Operations Research 29(3), 407–435 (2004)
6. Johari, R., Tsitsiklis, J.N.: Efficiency of scalar-parameterized mechanisms. Operations Research 57(4), 823–839 (2009)
7. Basar, T., Srikant, R.: Revenue-maximizing pricing and capacity expansion in a many-users regime. In: Proceedings IEEE INFOCOM 2002, pp. 1556–1563 (2002)
8. Shen, H., Basar, T.: Differentiated Internet pricing using a hierarchical network game model. In: Proc. 2004 American Control Conference, pp. 2322–2327 (2004)
9. Shen, H., Basar, T.: Optimal nonlinear pricing for a monopolistic network service provider with complete and incomplete information. IEEE J. Select. Areas Commun. 25, 1216–1223 (2007)
10. Saraydar, C.U., Mandayam, N.B., Goodman, D.J.: Efficient power control via pricing in wireless data networks. IEEE Trans. on Communications 50, 291–303 (2002)
11. Alpcan, T., Basar, T.: A hybrid noncooperative game model for wireless communications. In: Advances in Dynamic Games: Applications to Economics, Finance, Optimization, and Stochastic Control. Annals of Dynamic Games, vol. 9. Birkhauser (2006)
12. Paschalidis, I.C., Tsitsiklis, J.N.: Congestion-dependent pricing of network services. IEEE/ACM Trans. Networking 8(2), 171–184 (2000)
13. Friedman, E., Parkes, D.: Pricing WiFi at Starbucks - Issues in online mechanism design. Working Paper (2002), `http://www.eecs.harvard.edu/~parkes/pubs/online.pdf`
14. Musacchio, J., Walrand, J.: WiFi access point pricing as a dyanmic game. IEEE/ACM Trans. Networking 14(2), 289–301 (2006)
15. Ren, S., Park, J., van der Schaar, M.: User subscription dynamics and revenue maximization in communication markets. To appear in Infocom 2011 (2011)
16. Kasbekar, G., Sarkar, S.: Spectrum pricing games with bandwidth uncertainty and spatial reuse in cognitive radio networks. In: Proceedings of ACM MOBIHOC 2010, September 20-24 (2010)
17. Sirbua, M., Lehr, W., Gillett, S.: Evolving wireless access technologies for municipal broadband. Government Information Quarterly 23, 480–502 (2006)
18. IEEE 802.11b: Wireless LAN Medium Access Control (MAC) and Physical layer (PHY) Specifications, IEEE Standard (1999)

19. Draft Supplement to Part 11: WIreless Medium Access Control (MAC) and Physical Layer (PHY) Specifications: Medium Access Control (MAC) Enhancements for Quality of Service (QoS), IEEE 802.11e/D10.0 (November 2004)
20. Tembine, H., Altman, E., El-Azouzi, R., Hayel, Y.: Evolutionary games in wireless networks. IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics 40(3), 634–646 (2009)
21. Ross, K.W., Tsang, D.: The stochastic knapsack problem. IEEE Trans. on Commun. 37(7), 740–747 (1989)
22. Mas-Colell, A., Whinston, M., Green, J.: Microeconomic Theory. Oxford Univ. Press, Oxford (1995)
23. Xiao, Y., Zame, W.R., van der Schaar, M.: Technology choices and pricing policies in public and private wireless networks, `http://arxiv.org/abs/1011.3580`
24. Nash, J.F.: Non-cooperative games. The Annals of Mathematics 54(2), 286–295 (1951)
25. van der Schaar, M., Andreopoulos, Y., Hu, Z.: Optimized scalable video streaming over IEEE 802.11 a/e HCCA wireless networks under delay constraints. IEEE Trans. Mobile Comput. 5(6), 755–768 (2006)