Joint Price and QoS Market Share Game with Adversarial Service Providers and Migrating Customers

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Abstract. In order to attract more and more customers, the price war between service providers (SPs) is becoming increasingly fierce. This kind of conflict situations has been analyzed and hugely studied by a plenty of works in the related literature. However unfortunately, almost all prior works neglect an important decision parameter, it is the promised quality of service which can be a serious advantage to register to an operator rather than the others. In this paper, we formulate the interaction between service providers as a non-cooperative game. First, each SP chooses the Quality of Service (QoS) to guarantee (it depends on the amount of requested bandwidth) and the corresponding price. Second, the customers decide to which SP to subscribe defining the market share for each SP. Then, each customers may migrate/churn to another SP or alternatively switch to "no subscription state" depending on the observed price/QoS. Furthermore, we build a Markovian model to derive the behaviour of customers depending on the strategic actions of the SPs. Finally, we provide extensive numerical examples to show the importance of taking price and QoS as a joint decision parameters and provide thereby some insights/heuristic on how to set them.

Keywords: Price, QoS, Behaviours of customers, Competition, Migration customers, Nash equilibrium.

1 Introduction

Compared to the earlier communication systems that are data centric, the current customers often transmit voice and video along with data. Definitively, voice and video (e.g., VoD, VoIP, streaming, mobile TV, $\cdots$) communications are more stringent in their quality requirements than raw data. These services are more sensitive to delay and require reliable transmission of the packets. Because the characteristics of traffic have changed, we need some reconsideration of the implications of architecture, service classes, and design principles on the pricing models of service providers. From customers point of view, a service provider

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becomes more attractive as the quality it guarantee tends to 100% service quality (full delivery rate and reasonable delay). On one hand, when the network is over-provisioned, i.e., the service quality is 100%, more capacity does not lead to any performance improvement for the traffic. But on the other hand, and from service providers perspective this generate extra and useless fee which reduces their net revenue.

It is clear now that both price and promised quality encourage the migration/churn of customers from a service provider to another. Indeed, this phenomenon becomes relevant since the liberalization of telecommunications service and continuous proliferation of service providers. Churn is especially large in mobile networks, where yearly migration rates as high as 25% [1,9]. The migration of each customer, to the benefit of another service provider, implies both the loss of future revenues associated to that customer (this conflict situation can be modeled to a good extent by a zero-sum game). Service providers are therefore very keen on retaining their customers as well as on attracting new ones.

1.1 Related Works and Their Drawbacks

The interactions between the strategies of operators, seeking compromise between price and quality of service, are modeled by several works, most of these have been achieved by the use of non-cooperative game theory and assume a single characteristic through which an equilibrium is studied, such as price [1], [2], [8] and [11], QoS in terms of delay [5], or even loss probabilities [4]. However, to take in consideration the QoS on a network, and develop an real model, it is better to invest more than one parameter, an example is to incorporate jointly price and some measure of Quality of service, [3] assuming that the demand function for SPs is linear, authors establish conditions for existence and uniqueness of the equilibria for both measures of loss and delay of QoS. For more details on surveys techniques of competitive game in telecommunications, see [6,7].

This research examines the basic issue of designing a joint pricing and QoS model for service provider (SP) selection. By formulating a utility formula based on pricing schema drawn from related literature, see for example [1]. The novelty here is that we consider the quality of services (can be expressed in term of average throughput, expected delay, loss probability or any other performance metric) as an extra parameter that may potentially define the strategic decision of SPs. Nowadays, thanks to the democratization of telecommunications service, we observe that all SPs propose similar services at same price (e.g., in France, all SPs offer the triple-play service at 29,99Euros/month). Some SPs may consider this situation as unfair. Then and to attract more and more customers, they announce a better service quality compared to other competitors. This becomes a typical conflict situation that could be modeled as a non-cooperative game. This research provides new insights to understand how to fine-tune the pricing as well as the quality promised by service providers. This pricing-quality model is pretty flexible, however it bases the study on customers steady behaviour which is not easy to evaluate.

1.2 Organization of the Paper

The rest of this paper is organized as follows : in Section 2, we present the joint price-QoS problem and provide an analysis of the customers steady behaviour. Later, we study the non-cooperative game between the competitive service providers in Section 3. Finally, we provide an extensive numerical results to assist our analysis and conclude our paper in Section 4 and Section 5, respectively.

2 Problem Formulation : Customers Steady Behaviour

We consider a system with N customers and 2 service providers (SPs). Each customer seeks to find the operator which allows him to meet a QoS (this metric can be throughput, delay, loss probability, $\cdots$) sufficient to satisfy his needs, at suitable price. Let p_i and q_i be the pricing policy and the QoS guaranteed by service provider i[1], respectively. Based on this two parameters policy, each customer decides to register to one of the two operators or to stay at no subscription state, see Fig. 1.

Increasing market share is the most important objectives of each operator. Then, from operator point of view, the question is to define the best pricing strategy and the best amount of bandwidth to request from the network owner. The service providers are supposed to know the effect of their policy on the customers registration policy. Whereas from customers point of view, the question is to set the best probabilities vector to register to an operator. Conceptually, this situation is a typical Stackelberg game [13] where the operators are the leaders and the customers are the followers. This way, the providers play first, but using backward induction, they anticipate the resulting strategy of end users who actually make the last move. Sometimes customers may leave an operator (migration) in order to register under the services of another. We turn first to analyze the behavior of one customer, see figure 1. Here state "Service provider 1" means that the customer is with provider 1, state "Service provider 2" that he is with provider 2 and state "No subscription" that he does not use any service. We assume that customer behavior is represented by a continuous time Markov chain. The transition from state i to state j depends not only on parameters of current SP i that are p_i and q_i, but also on the price and QoS offered by its competitors, that is, π_i depends upon the entire price vector $\mathbf{p} = (p_1, p_2)$ and the entire QoS vector $\mathbf{q} = (q_1, q_2)$. To avoid negative profits, we consider here that both price and QoS indicator are bounded and positives, i.e., $0 < p_i^{min} \leq p_i \leq p_i^{max}$ and $0 < q_i^{min} \leq q_i \leq q_i^{max}$.

Let us denote by $\lambda_{i,j}$ the transition probability from state i to state j, the transition probabilities are depicted in figure 1. The resulting infinitesimal generator of Markov chain corresponding to the figure 1 is the following

[1] This QoS is strongly depending on the total bandwidth yield by service provider i and the number of customers registered to this latter.

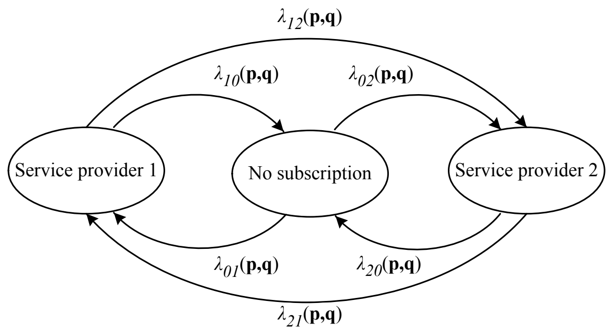

Fig. 1. Customer's behaviour as a continuous time Markov chain

$$\mathbf{Q} = \begin{pmatrix} -\left[\lambda_{01}(\mathbf{p},\mathbf{q}) + \lambda_{02}(\mathbf{p},\mathbf{q})\right] & \lambda_{01}(\mathbf{p},\mathbf{q}) & \lambda_{02}(\mathbf{p},\mathbf{q}) \\ \lambda_{10}(\mathbf{p},\mathbf{q}) & -\left[\lambda_{10}(\mathbf{p},\mathbf{q}) + \lambda_{12}(\mathbf{p},\mathbf{q})\right] & \lambda_{12}(\mathbf{p},\mathbf{q}) \\ \lambda_{20}(\mathbf{p},\mathbf{q}) & \lambda_{21}(\mathbf{p},\mathbf{q}) & -\left[\lambda_{20}(\mathbf{p},\mathbf{q}) + \lambda_{21}(\mathbf{p},\mathbf{q})\right] \end{pmatrix}$$

Let $\pi(\mathbf{p},\mathbf{q}) = \{\pi_i(\mathbf{p},\mathbf{q}),\ i \in \{0,1,2\}\}$ denotes the steady state of the Markovian system, where the probability that a given customer is with service provider i is π_i. From non-cooperative game theory perspective, π is the equilibrium mixed strategy of any given customer. Equivalently, π_i can also be seen as the market share of SP i, it follows that the average number of customers that are with SP i is $N\pi_i$. The customers problem is then a solution of system :

$$\begin{cases} \pi Q = 0, \\ \sum_{i=0}^{2} \pi_i = 1, \\ \pi_i \geq 0, \quad i = 0,1,2. \end{cases} \tag{1}$$

The solution of this system is quite easy to obtain for two SPs, it is given by :

$$\pi_0 = \left[\lambda_{10}(\mathbf{p},\mathbf{q})\lambda_{20}(\mathbf{p},\mathbf{q}) + \lambda_{10}(\mathbf{p},\mathbf{q})\lambda_{21}(\mathbf{p},\mathbf{q}) + \lambda_{12}(\mathbf{p},\mathbf{q})\lambda_{20}(\mathbf{p},\mathbf{q})\right] \div C,$$
$$\pi_1 = \left[\lambda_{01}(\mathbf{p},\mathbf{q})\lambda_{21}(\mathbf{p},\mathbf{q}) + \lambda_{21}(\mathbf{p},\mathbf{q})\lambda_{02}(\mathbf{p},\mathbf{q}) + \lambda_{20}(\mathbf{p},\mathbf{q})\lambda_{01}(\mathbf{p},\mathbf{q})\right] \div C,$$
$$\pi_2 = \left[\lambda_{02}(\mathbf{p},\mathbf{q})\lambda_{12}(\mathbf{p},\mathbf{q}) + \lambda_{10}(\mathbf{p},\mathbf{q})\lambda_{02}(\mathbf{p},\mathbf{q}) + \lambda_{01}(\mathbf{p},\mathbf{q})\lambda_{12}(\mathbf{p},\mathbf{q})\right] \div C,$$

where

$$\begin{aligned} C = \ & \lambda_{01}(\mathbf{p},\mathbf{q})\lambda_{12}(\mathbf{p},\mathbf{q}) + \lambda_{21}(\mathbf{p},\mathbf{q})\lambda_{02}(\mathbf{p},\mathbf{q}) + \lambda_{02}(\mathbf{p},\mathbf{q})\lambda_{10}(\mathbf{p},\mathbf{q}) \\ & + \lambda_{02}(\mathbf{p},\mathbf{q})\lambda_{12}(\mathbf{p},\mathbf{q}) + \lambda_{20}(\mathbf{p},\mathbf{q})\lambda_{01}(\mathbf{p},\mathbf{q}) + \lambda_{20}(\mathbf{p},\mathbf{q})\lambda_{10}(\mathbf{p},\mathbf{q}) \\ & + \lambda_{20}(\mathbf{p},\mathbf{q})\lambda_{12}(\mathbf{p},\mathbf{q}) + \lambda_{21}(\mathbf{p},\mathbf{q})\lambda_{01}(\mathbf{p},\mathbf{q}) + \lambda_{21}(\mathbf{p},\mathbf{q})\lambda_{10}(\mathbf{p},\mathbf{q}). \end{aligned}$$

Remark 1. It is important to note that the market share π_i is exactly the experienced demand by SP i.

3 Service Providers Strategic Decision

After we have defined the customers behavior, taking into consideration both the unit price and the guaranteed QoS, we turn now to define the best policy (price and QoS) for each SP. We first define the utility function and then analyze the equilibrium concept of interest. Each SP seeks to attract the maximum possible number of customers among a population of N customers. On one hand increasing the price (and/or decreasing the quality to guarantee) will increase the revenue per customer. On the other hand, this policy may potentially reduce the number of customers (equivalently the market share). Henceforth, there is a tradeoff to be analyzed. Having a market share π_i, the total revenue of SP i is then $N\pi_i p_i$. We assume that we have a single network owner, this latter charges each SP i a cost ϑ_i per unit of requested bandwidth. In order to insure the customers loyalty, the amount of bandwidth μ_i required by SP i should depend on π_i, N and on the QoS q_i it wishes to offer to its customers. Therefore, the net profit of SP i is simply the difference between the total revenue and the fee paid to the network owner :

$$U_i(\mathbf{p}, \mathbf{q}) = N\pi_i p_i - F_i(q_i, \pi_i), \quad \forall i \in \{1, 2\}. \tag{2}$$

The fee paid by SP i can be written as [3] : $F_i = \vartheta_i \mu_i\left(N, \pi_i, q_i\right)$, where $\mu_i\left(N, \pi_i, q_i\right)$ is the amount of bandwidth required by SP i to guarantee the announced QoS q_i, which has the following form :

$$\mu_i\left(N, \pi_i, q_i\right) = N\pi_i g_i\left(q_i\right) + h_i\left(q_i\right) \tag{3}$$

where $g_i\left(q_i\right)$ and $h_i\left(q_i\right)$ are positive functions, which mean that the profile function of SP i becomes :

$$U_i(\mathbf{p}, \mathbf{q}) = N\pi_i\left(p_i - \vartheta_i g_i\left(q_i\right)\right) - \vartheta_i h_i\left(q_i\right), \forall i \in \{1, 2\}. \tag{4}$$

Expected Delay as QoS Indicator: In the rest of this paper, we assume that the measure defining the QoS corresponds to some function of the expected delay. We consider the Kleinrock delay function which is a common delay function used in networking games, see [3] and [5]. This way, the maximization of QoS requires minimization of the delay. For this reason, and instead of minimizing delays, we consider the maximization of the reciprocal of its square root :

$$q_i := \frac{1}{\sqrt{Delay_i}} = \sqrt{\mu_i - N\pi_i}. \tag{5}$$

Therefore, $\mu_i\left(N, \pi_i, q_i\right) = N\pi_i g_i\left(q_i\right) + h_i\left(q_i\right)$. We will focus on the rest of this paper to the simple case where $g_i\left(q_i\right) = 1$ and$h_i\left(q_i\right) = q_i^2$. Thus, equation (4) becomes :

$$U_i(\mathbf{p}, \mathbf{q}) = N\pi_i\left(p_i - \vartheta_i\right) - \vartheta_i q_i^2, \quad i \in \{1, 2\}. \tag{6}$$

Each service provider strives to find its best strategy, i.e., its price and QoS to guarantee maximizing its revenue, which can be modified by the strategy of the competitor. The solution concept to adopt here is naturally that of a Nash equilibrium[2].

Definition 1 (Nash equilibrium). Since the customers steady state depends only on the fixed prices and the offered QoS by the two adversarial SP, then $(\boldsymbol{p}^*, \boldsymbol{q}^*) = (p_1^*, p_2^*, q_1^*, q_2^*)$ is a service providers Nash Equilibrium if it satisfies

1) $(p_i^*, q_i^*) \in \underset{p_i, q_i}{\operatorname{argmax}}\, U_i\left(p_i, p_{-i}^*, q_i, q_{-i}^*\right), \quad i = 1, 2$ and
2) $(\mathbf{p}^*, \mathbf{q}^*)$ is a feasible strategy profile.

In order to solve this non-cooperative game we use a backward induction technique. We start with the customers registration and derive their steady behaviour as a function of the price set and the QoS offered by the service providers. Without extra assumptions, existence of a Nash equilibrium cannot be ensured, nor its uniqueness when existence is shown. In the case where utilities and rates functions are simple enough in terms of prices and QoS, we may find the form of the Nash equilibria analytically. Otherwise, the computations can be performed numerically using the following algorithm 1. We define the best response of each provider as a function of the strategy of its opponent by

$$BR_1(p_2, q_2) = \underset{p_1, q_1}{\operatorname{argmax}}\, U_1(\mathbf{p}, \mathbf{q}), \tag{7}$$

$$BR_2(p_1, q_1) = \underset{p_2, q_2}{\operatorname{argmax}}\, U_2(\mathbf{p}, \mathbf{q}). \tag{8}$$

Algorithm 1. Numerically finding the Nash equilibria of the joint Price-QoS game

1: Input : Transition rates of the Markov chain as function of $\mathbf{p}$ and $\mathbf{q}$,
2: For all feasible couple of values (p_2, q_2) of SP 2, find the set $BR_1(p_2, q_2)$,
3: For all feasible couple of values (p_1, q_1) of SP 1, find the set $BR_2(p_1, q_1)$,
4: Each 4-tuple $(p_1^*, p_2^*, q_1^*, q_2^*)$ such that $(p_1^*, q_1^*) \in BR_1(p_2^*, q_2^*)$ and $(p_2^*, q_2^*) \in BR_2(p_1^*, q_1^*)$, is a Nash equilibrium. Note that this game needs not have a Nash equilibrium.

3.1 Discussion : Special But Realistic Cases

Price Game with Fixed QoS: When the market price is high or similar for all service providers, it changes the customers perceptions about the quality of the product, see [12] . A different pricing strategy provides an "imaginary effect" on perceptions of quality and leads to a willingness to buy. However with presence

[2] A Nash equilibrium is a strategy profile such that no service provider can unilaterally increase its revenue.

of adversarial competitors, in particular when providing same quality of service, this turns to be false. This motivates us to study the pricing game with fixed QoS.

QoS Game with Fixed Price: In earlier days, quality of service meant delivering packets from a source to a destination without any transmission errors. As the Internet has become commercialized, however, QoS has started to become an important strategic tool for market competition [10]. Most network pricing researchers assume that when the price is high the arrival rate or demand for that service class is low. However, this assumption may not be always true. Some customers value high quality services more than lower-class services. Therefore, some people buy more expensive services because they believe that they can get better quality of service by paying more. Nowadays, all service providers propose services at same cost. To convince customers to join an SP i, this latter exposes its ability to provide a good QoS and invents many new services to attract more and more customers. This way, considering a game ruled only by the QoS is reasonable and realistic enough. This scheme is also used to analyze the interplay of quality of service on customers loyalty and service providers revenue.

4 Numerical Investigation

4.1 Migration Rate Model

The dependence of migration/transition rates on provider prices and qualities are too complicated. To solve the problem analytically is then a hard issue. Even with simple expressions of the transition rates, the problem remains too complicated to solve due to expression of steady market share expression. Obviously a service provider attracts more customers as it decreases the price and/or increase the promised QoS. Thus any function increasing with respect to p_i and decreasing with respect to q_i can be used to model the migration rate into SP i. Here, we consider a the following specific but realistic enough transition rate functions :

$$\begin{aligned}
\lambda_{01}(\mathbf{p},\mathbf{q}) &= \lambda_{02}(\mathbf{p},\mathbf{q}) = \alpha,\\
\lambda_{10}(\mathbf{p},\mathbf{q}) &= \alpha_1 \tfrac{p_1}{q_1},\\
\lambda_{20}(\mathbf{p},\mathbf{q}) &= \alpha_2 \tfrac{p_2}{q_2},\\
\lambda_{12}(\mathbf{p},\mathbf{q}) &= \beta_1 \tfrac{p_1 q_2}{p_2 q_1},\\
\lambda_{21}(\mathbf{p},\mathbf{q}) &= \beta_2 \tfrac{p_2 q_1}{p_1 q_2}.
\end{aligned}$$

$\alpha_1, \alpha_2, \beta_1, \beta_2$ are strictly positive constants. The expressions of λ_{10} and λ_{20} shows that probability that customer leaves the SP $i \in \{1, 2\}$ to state 0 (no subscription) if their prices become excessive, or alternatively their quality of service tends to become low. Incorporating these latter parameters helps us to introduce some asymmetry between service providers and then can be used to model the reputation of each service provider. Substituting the transition rates by their expressions, we can compute the steady state vector $\pi = (\pi_i)_{i\in\{0,1,2\}}$. The net revenues of each operator is then obtained using equation (6). Until contraindication, the parameter values considered for numerical examples are the following: $N = 1000$ (total number of customers), $p_i^{min} = 2, p_i^{max} = 100, q_i^{min} = 0, q_i^{max} = 120, \vartheta_1 = \vartheta_2 = 1$, $\lambda_{01} = \lambda_{02} = 3$, $\alpha = 3$ and $\beta_1 = \beta_2 = 20$. For sake of illustration, we perform an extensive numerical study with a focus on the two special but realistic schemes : Price game with fixed QoS and QoS game with fixed price. For the two-parameters game, and based on numerical analysis we noticed that an equilibrium point needs not exist. Moreover, the game may potentially has several equilibria.

Remark 2. We recall that the quality of service can be expressed as average throughput, expected delay or any other performance metric. Here, increasing the quality indicator q_i is equivalent to promise shorter delay.

4.2 Price Game with Fixed QoS

We consider here the classic scheme where the QoS is fixed and the price is the only action of the game. Later, we study the Nash equilibrium of this game and show how the net revenue and the market share evolves. Figure 2 plots the utility U_1 of provider 1 when its price p_1 varies, for different values of the adversary price p_2. We remark that the net revenue of provider 1 is first increasing, then decreasing in p_1. Also that the utility of provider 1 decreases when its price p_1 tends to infinity, which implies existence of a finite price p_1^o maximizing U_1. This price value constitutes the best response of provider 1 against the price set by provider 2. We remark also that the net revenue of SP_1 increase when adversary price p_2 increase, which is likely intuitive. Indeed, if the opponent chooses a higher pricing strategy, then it may loose a fraction of its customers. Moreover, service provider SP 1 may attract those latter and make incite them to churn. Consequently, its net revenue is increased. Surprisingly, we observed an interesting feature characterizing our proposed price game : Through hundreds of numerical example runs, we 1) checked existence of a unique best response, and 2) checked that as the price defined by SP 2 increases, the best response of SP 1 tends to increase also ! This motivated us to work about proving supermodularity of this game.

Figure 3 and 4 present respectively the influence of QoS q_1 and q_2 of both providers on the net revenue U_1 of SP_1. In terms of service provider's own quality of service, the net revenue behaves in two ways. When SP 1 sets a low price, it tends to be more beneficial to request relatively low amount of bandwidth providing low quality of service. This is a quite surprising behaviour, but can have a realistic good extent. Indeed, when an SP offers some given service at a low cost, its market share goes to 1. Absorbing the total number of customers, the tagged SP tends to no investing anymore and the offered service quality becomes poor. Another important result is that as the offered QOS increases as the best response increase respecting by the way "If you want more, you have to pay more". We depict in figure 4 the impact of QoS offered by the opponent. It shows that when SP 2 investment increases, i.e., its offered QoS, the best move for SP 1 is to decrease the cost of its services.

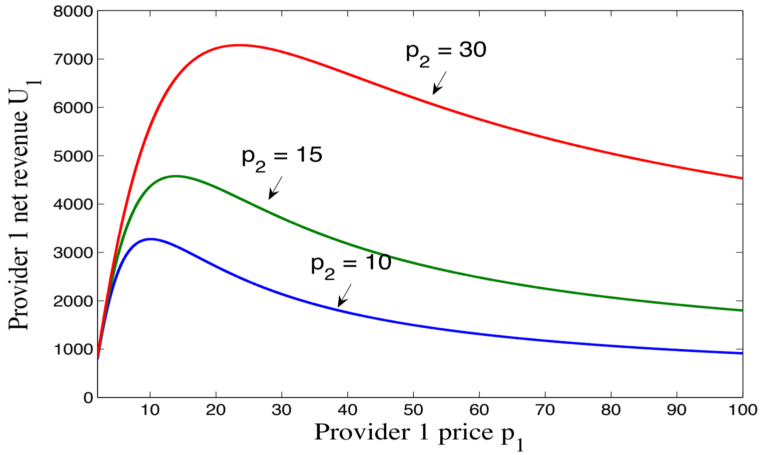

Fig. 2. Net revenue of provider 1 versus p_1 for different values of p_2, with $q_1 = 10$ and $q_2 = 10$

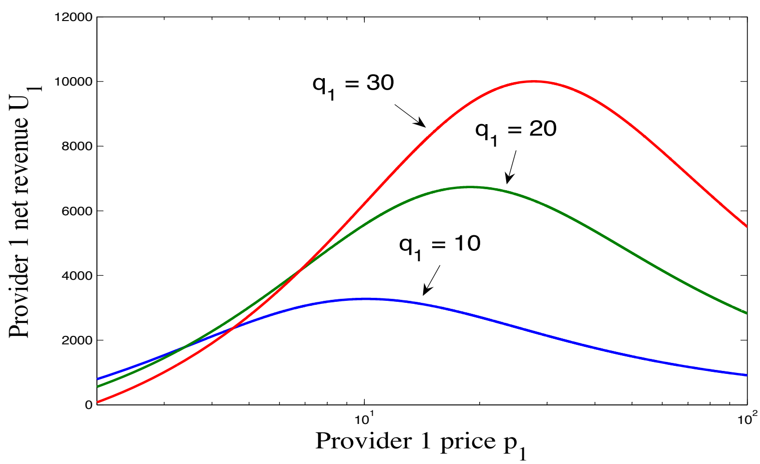

Fig. 3. Net revenue of provider 1 versus p_1 for different values of q_1, with $q_2 = 10$ and $p_2 = 10$

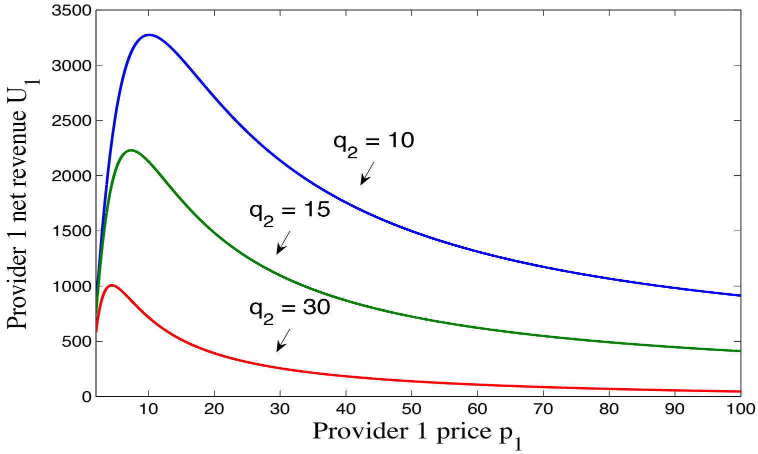

Fig. 4. Net revenue of provider 1 versus p_1 for different values of q_2, with $q_1 = 10$ and $p_2 = 10$

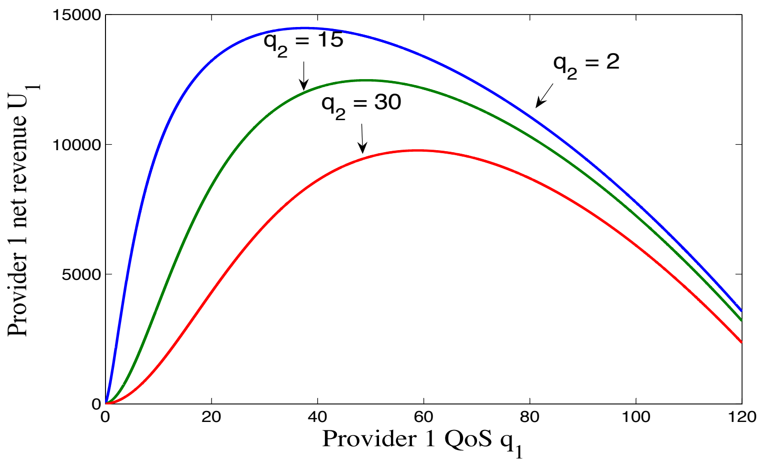

Fig. 5. Net revenue of provider 1 versus q_1 for several values of q_2, with $p_1 = 20$ and $p_2 = 20$

Next we depict the best response of the two adversarial service providers. We recall that this figure is obtained using algorithm 1 and the intersection between the two graphs represents the Nash equilibrium point of the game. Through several examples, we always obtained a unique Nash equilibrium. Another visible feature is that the fixed price should evolve in the same direction of the investment. In other words, the end price increases as the offered QoS increases.

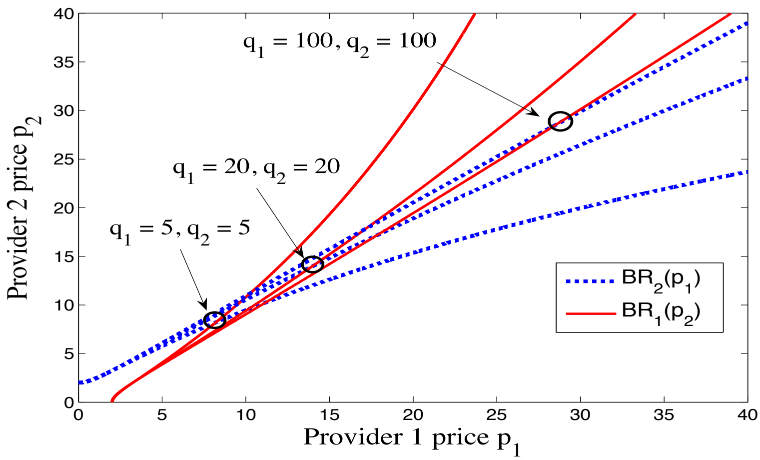

Fig. 6. Best response for both providers for different values of $q_1 = q_2$

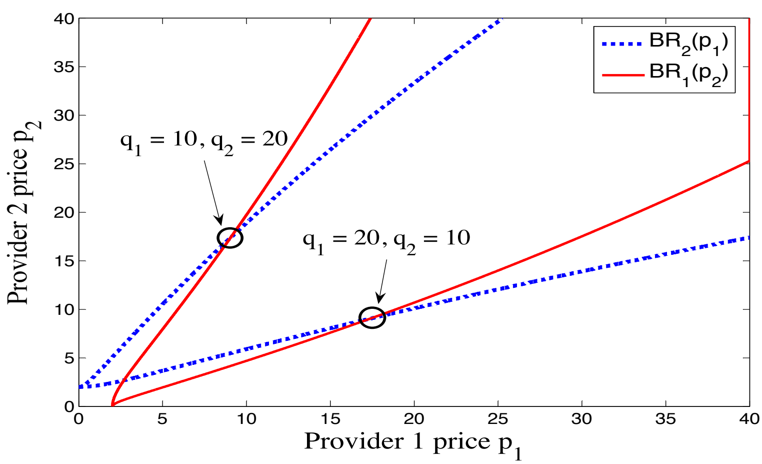

Fig. 7. Best responses for both providers for different values of $q_1 \neq q_2$

4.3 QoS Game with Fixed Price

Next we consider the QoS game while fixing the price for both SPs. We first notice that the net revenue is decreasing with the QoS. This is quite intuitive since it means for service provider to invest more and more while charging customers a constant price, see figure 5. It is clear from the same figure that, as the opponent SP 2 increases its quality of service, it is beneficial for SP 1 to increase its QoS. This latter result is well known in economics. Figures 8 and 9 shows the need to define a higher price as the own QoS improves and the need investment

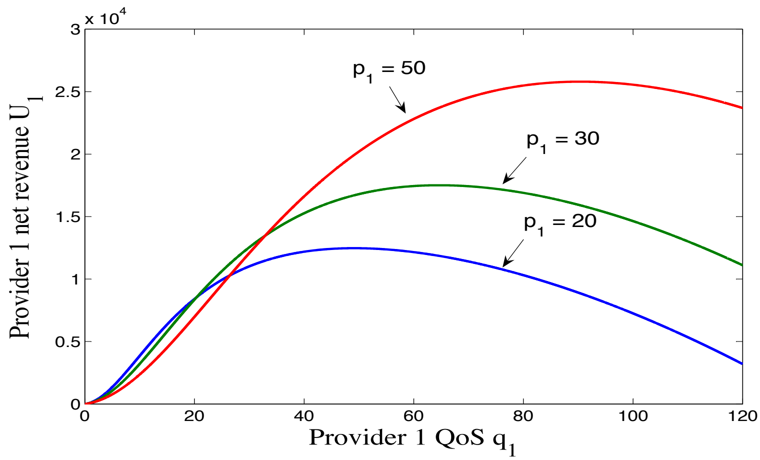

Fig. 8. Net revenue of provider 1 versus q_1 for several values of p_1, with $q_2 = 15$ and $p_2 = 20$

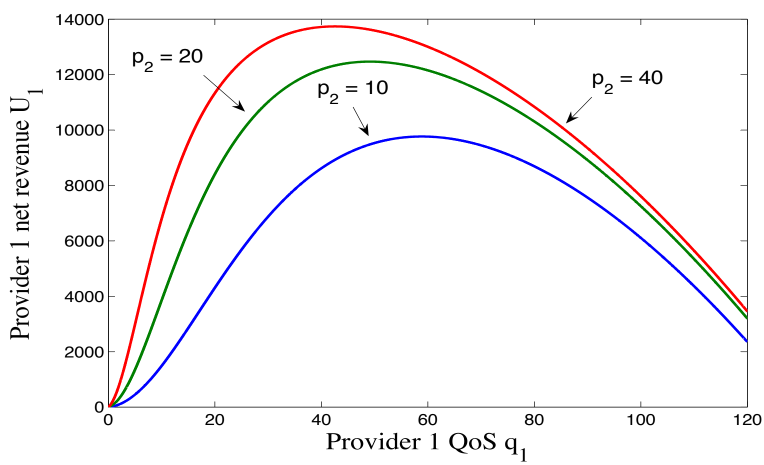

Fig. 9. Net revenue of provider 1 versus q_1 for several values of p_2, with $q_2 = 15$ and $p_1 = 20$

(improve promised QoS) as the competitor reduces its price. Later, we plot the best response in terms of QoS while the two service providers set the same price (figure 10) and where they set adopt different pricing strategies (figure 11).

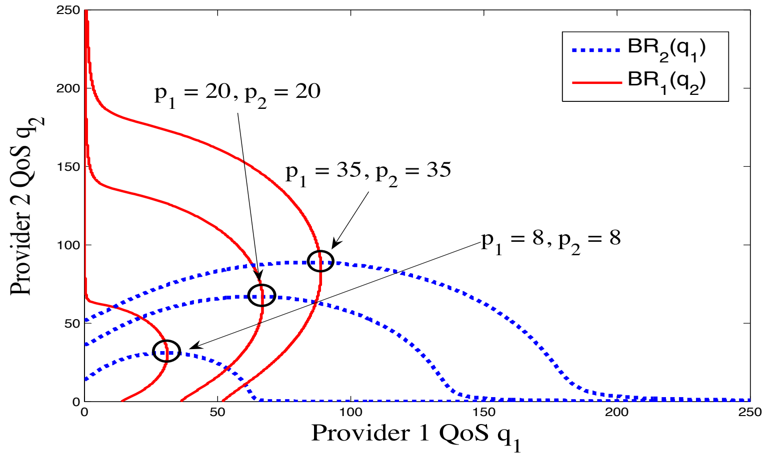

Fig. 10. Best responses QoSs of both provider for several values

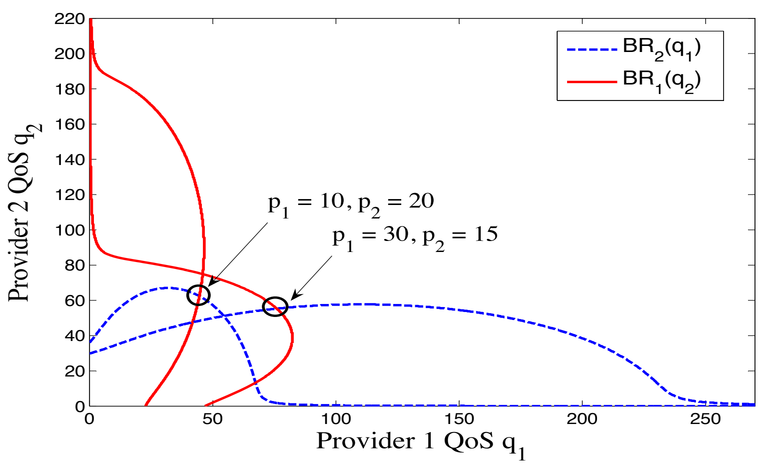

Fig. 11. Best responses Prices of both provider for several values

4.4 Convergence to Nash Equilibrium

If service providers allow free cost service with possibly null quality, i.e., price and quality may be 0, we notice that $(p_1, p_2) = (0, 0)$ is also a satisfying situation for both SPs, thus it represents also an equilibrium point. Since it brings a negative

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Algorithm 2. Convergence to Nash equilibrium (when exists)

1: $z_i^{t+1} = BR_i(z_{-i}^t), \qquad i = 1, 2.$
where $BR_i(\cdot)$ is the best response function of SP i, and z_i is a generic variable representing p_i or q_i (while maintaining the other parameter constant).

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revenue to the providers, and moreover it is not a stable Nash equilibrium : if any of the two providers slightly deviates from that situation by setting a strictly positive price, then an iterative best response-based algorithm leads to the other (stable) Nash equilibrium. By imposing non-null price and non-null

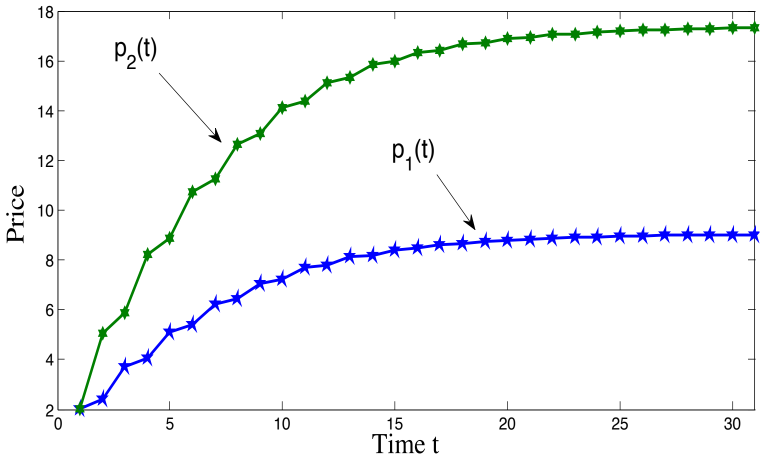

Fig. 12. Pricing game : Convergence to NE under $(q_1, q_2) = (10, 20)$

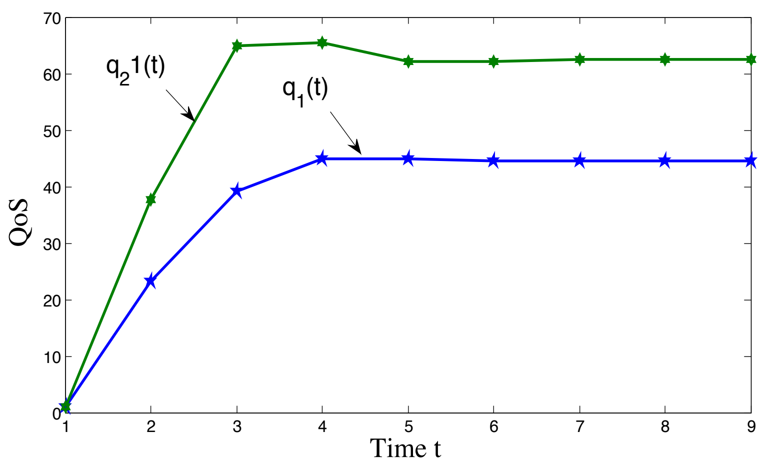

Fig. 13. QoS game : Convergence to NE under $(p_1, p_2) = (10, 20)$

quality, the Nash equilibrium would be a priori unique and the single-parameter game would be supermodular. We will consequently focus on that equilibrium in the following, when it exists. Algorithm 2 insures convergence of both SPs to the Nash equilibria points highlighted in previous figures, see figure 12 and figure 13. It is clear that the speed of convergence is relatively high (around 25 iterations to converge to price-baed equilibrium and around 8 iterations to converge to QoS-baed equilibrium).

5 Concluding Remarks

We have presented in this paper a non-cooperative game for pricing problem considering quality of service as an extra decision parameter. The service providers take into account the user migration/churn behavior to determine the best price and best quality of service to guarantee, so as to maximize their long term revenue. Through a numerical analysis, we remarked potential existence of many Nash equilibria or none for the joint price and QoS game. We remarked that the game my have a unique Nash equilibrium if minimum price p_{min} and minimum q_{min} are not zero. Furthermore, A special feature is obtained as the single game parameter seems to be supermodular. This way we checked that a simple successive best responses-based algorithm allows to learn the Nash equilibrium.

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