"To Sense" or "Not to Sense"
in Energy-Efficient Power Control Games

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Abstract. A network of cognitive transmitters is considered. Each transmitter has to decide his power control policy in order to maximize energy-efficiency of his transmission. For this, a transmitter has two actions to take. He has to decide whether to sense the power levels of the others or not (which corresponds to a finite sensing game), and to choose his transmit power level for each block (which corresponds to a compact power control game). The sensing game is shown to be a weighted potential game and its set of correlated equilibria is studied. Interestingly, it is shown that the general hybrid game where each transmitter can jointly choose the hybrid pair of actions (to sense or not to sense, transmit power level) leads to an outcome which is worse than the one obtained by playing the sensing game first, and then playing the power control game. This is an interesting Braess-type paradox to be aware of for energy-efficient power control in cognitive networks.

1 Introduction

In fixed communication networks, the paradigm of peer-to-peer communications has known a powerful surge of interest during the the two past decades with applications such as the Internet. Remarkably, this paradigm has also been found to be very useful for wireless networks. Wireless ad hoc and sensor networks are two illustrative examples of this. One important typical feature of these networks is that the terminals have to take some decisions in an autonomous (quasi-autonomous) manner. Typically, they have to choose their power control and resources allocation policy. The corresponding framework, which is the one of this paper, is the one of distributed power control or resources allocation. More specifically, the scenario of interest is the case of power control in cognitive

networks. Transmitters are assumed to be able to sense the power levels of neighboring transmitters and adapt their power level accordingly. The performance metric for a transmitter is the energy-efficiency of the transmission [5] that is, the number of bits successfully decoded by the receiver per Joule consumed at the transmitter.

The model of multiuser networks which is considered is a multiple access channel with time-selective non-frequency selective links. Therefore, the focus is not on the problem of resources allocation but only on the problem of controlling the transmit power over quasi-static channels. The approach of the paper is related to the one of [8][7] where some hierarchy is present in the network in the sense that some transmitters can observe the others or not; also the problem is modeled by a game where the players are the transmitters and the strategies are the power control policies. One the differences with [8][7] is that *every* transmitter can be cognitive and sense the others but observing/sensing the others has a cost. Additionally, a new type of power control games is introduced (called hybrid power control games) in which an action for a player has a discrete component namely, to sense or not to sense, and a compact component namely, the transmit power level. There are no general results for equilibrium analysis in the game-theoretic literature. This is a reason why some results are given in the 2-player case only, as a starting point for other studies. In particular, it is shown that it is more beneficial for every transmitter to choose his discrete action first and then his power level. The (finite) sensing game is therefore introduced here for the first time and an equilibrium analysis is conducted for it. Correlated equilibria are considered because they allow the network designer to play with fairness, which is not possible with pure or mixed Nash equilibria.

This paper is structured as follows. A review of the previous results regarding the one-shot energy efficient power control game is presented in Sec. 2. The sensing game is formally defined and some equilibrium results are stated in Sec. 3. A detailed analysis of the 2-players sensing is provided in Sec. 4 and the conclusion appears in Sec. 5.

2   Review of Known Results

2.1   Review of the One-Shot Energy-Efficient Power Control Game (Without Sensing)

We review a few key results from [6] concerning the static non-cooperative PC game. In order to define the static PC game some notations need to be introduced. We denote by R_i the transmission information rate (in bps) for user i and f an efficiency function representing the block success rate, which is assumed to be sigmoidal and identical for all the users; the sigmoidness assumption is a reasonable assumption, which is well justified in [11][4]. Recently, [3] has shown that this assumption is also justified from an information-theoretic standpoint. At a given instant, the SINR at receiver $i \in \mathcal{K}$ writes as:

$$\text{SINR}_i = \frac{p_i|g_i|^2}{\sum_{j\neq i} p_j|g_j|^2 + \sigma^2} \tag{1}$$

where p_i is the power level for transmitter i, g_i the channel gain of the link between transmitter i and the receiver, σ^2 the noise level at the receiver, and f is a sigmodial efficiency function corresponding to the block success rate. With these notations, the static PC game, called $\mathcal{G}$, is defined in its normal form as follows.

Definition 2.1 (Static PC game). The static PC game is a triplet $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_i\}_{i\in\mathcal{K}}, \{u_i\}_{i\in\mathcal{K}})$ where $\mathcal{K}$ is the set of players, $\mathcal{A}_1, ..., \mathcal{A}_K$ are the corresponding sets of actions, $\mathcal{A}_i = [0, P_i^{\max}]$, $P_i^{\max}$ is the maximum transmit power for player i, and $u_1, ..., u_k$ are the utilities of the different players which are defined by:

$$u_i(p_1, ..., p_K) = \frac{R_i f(\mathrm{SINR}_i)}{p_i} \ [\mathrm{bit}/\mathrm{J}]. \tag{2}$$

In this game with complete information ($\mathcal{G}$ is known to every player) and rational players (every player does the best for himself and knows the others do so and so on), an important game solution concept is the NE (i.e., a point from which no player has interest in unilaterally deviating). When it exists, the non-saturated NE of this game can by obtained by setting $\frac{\partial u_i}{\partial p_i}$ to zero, which gives an equivalent condition on the SINR: the best SINR in terms of energy-efficiency for transmitter i has to be a solution of $xf'(x) - f(x) = 0$ (this solution is independent of the player index since a common efficiency function is assumed, see [4] for more details). This leads to:

$$\forall i \in \{1, ..., K\}, \ p_i^* = \frac{\sigma^2}{|g_i|^2} \frac{\beta^*}{1 - (K-1)\beta^*} \tag{3}$$

where β^* is the unique solution of the equation $xf'(x) - f(x) = 0$. By using the term "non-saturated NE" we mean that the maximum transmit power for each user, denoted by $P_i^{\max}$, is assumed to be sufficiently high not to be reached at the equilibrium i.e., each user maximizes his energy-efficiency for a value less than $P_i^{\max}$ (see [8] for more details). An important property of the NE given by (3) is that transmitters only need to know their individual channel gain $|g_i|$ to play their equilibrium strategy. One of the interesting results of this paper is that it is possible to obtain a more efficient equilibrium point by repeating the game $\mathcal{G}$ while keeping this key property.

2.2 Review of the Stackelberg Energy-Efficient Power Control Game (With Sensing)

Here we review a few key results from [7]. The framework addressed in [7] is that the existence of two classes of transmitters are considered: those who can sense and observe the others and those who cannot observe. This establishes a certain hierarchy between the transmitters in terms of observation. A suited model to study this is the Stackelberg game model [13]: some players choose their transmit power level (these are the leaders of the power control game) and the others

observe the played action and react accordingly (these are the followers of the game). Note that the leaders know they are observed and take this into account for deciding. This leads to a game outcome (namely a Stackelberg equilibrium) which Pareto-dominates the one-shot game Nash equilibrium (given by (3)) when there is no cost for sensing [8]. However, when the fraction of time to sense is taken to be $\alpha > 0$, the data rate is weighted by $(1 - \alpha)$ and it is not always beneficial for a transmitter to sense [7]. The equilibrium action and utility for player i when he is a game leader (L) are respectively given by

$$p_i^L = \frac{\sigma^2}{|g_i|^2} \frac{\gamma^*(1 + \beta^*)}{1 - (K - 1)\gamma^*\beta^* - (K - 2)\beta^*} \tag{4}$$

where γ^* is the unique solution of $x \left[1 - \frac{(K-1)\beta^*}{1-(K-2)\beta^*} x \right] f'(x) - f(x) = 0$ and

$$u_i^L = \frac{|g_i|^2}{\sigma^2} \frac{1 - (K - 1)\gamma^*\beta^* - (K - 2)\beta^*}{\gamma^*(1 + \beta^*)} f(\gamma^*). \tag{5}$$

On the other hand, if player i is a follower (F) we have that:

$$p_i^F = \frac{\sigma^2}{|g_i|^2} \frac{\beta^*(1 + \gamma^*)}{1 - (K - 1)\gamma^*\beta^* - (K - 2)\beta^*} \tag{6}$$

and

$$u_i^F = (1 - \alpha) \frac{|g_i|^2}{\sigma^2} \frac{1 - (K - 1)\gamma^*\beta^* - (K - 2)\beta^*}{\beta^*(1 + \gamma^*)} f(\beta^*). \tag{7}$$

3   A New Game: The $K-$Player Sensing Game

3.1   Sensing Game Description

In the two hierarchical power control described above, the transmitter is, by construction, either a cognitive transmitter or a non-cognitive one and the action of a player consists in choosing a power level. Here, we consider that all transmitters can sense, the power level to be the one at the Stackelberg equilibrium, and the action for a player consists in choosing to sense (S) or not to sense (NS). This game is well defined only if at least one player is a follower (i.e., he senses) and one other is the leader (i.e., he does not sense). We assume in the following that the total number of transmitters is $K + 2$, where K transmitters are considered as usual players and the two last are a follower and a leader. Define the $K-$player sensing game as a triplet:

$$G = (K, (\mathcal{S})_{i \in K}, (U_i)_{i \in K}) \tag{8}$$

where the actions set are the same for each player $i \in K$, sense or not sense: $\mathcal{S} = (S, NS)$. The utility function of each player $i \in K$ depends on his own channel state g_i and transmission rate R_i but also on the total number of players F playing the sensing action and the number of players that non sense denoted

L. Denote $U_i^S(F, L)$ the utility of player i when playing action sensing S whereas $F - 1$ other players are also sensing and L other players are non-sensing. The total number of player is $F + L = K$.

$$U_i^S(F, L) = \frac{g_i R_i}{\sigma^2} \frac{f(\beta^*)}{N\beta^\star(N + \gamma_{L+1}^\star)}$$
$$\left(N^2 - N\beta^\star - [(N+\beta^\star)L + (F+1)\beta^\star]\gamma_{L+1}^\star\right)$$

$$U_i^{NS}(F, L) = \frac{g_i R_i}{\sigma^2} \frac{f(\gamma_L^*)}{N\gamma_{L+1}^\star(N + \beta^\star)}$$
$$\left(N^2 - N\beta^\star - [(N+\beta^\star)L + (F+1)\beta^\star]\gamma_{L+1}^\star\right)$$

with γ_L^* solution of $x(1 - \epsilon_L x)f'(x) = f(x)$ with:

$$\epsilon_L = \frac{(K + 2 - L)\beta^\star}{N^2 - N(K + 1 - L)\beta^\star}. \tag{9}$$

3.2   The Sensing Game Is a Weighted Potential Game

The purpose of this section is to show that the sensing game may be an exact potential game. However, this holds under restrictive assumptions on the channel gains. It is then shown, as a second step, that the game is a weighted potential game. For making this paper sufficiently self-containing we review important definitions to know on potential games.

Definition 3.1 (Monderer and Shapley 1996 [9]). *The normal form game G is a potential game) if there is a potential function $V : S \longrightarrow \mathbb{R}$ such that*

$$U_i(s_i, s_{-i}) - U_i(t_i, s_{-i}) = V(s_i, s_{-i}) - V(t_i, s_{-i}), \tag{10}$$
$$\forall i \in K, \ s_i, t_i \in \mathcal{S}_\rangle \tag{11}$$

Theorem 3.2. *The sensing game $G = (K, (\mathcal{S})_{i \in K}, (U_i)_{i \in K})$ is an exact potential game if and only if one of the two following conditions is satisfied.*

1) $\forall i, j \in K \quad R_i g_i = R_j g_j$

2) $\forall i, j \in K, \ s_i, t_i \in S_i, \ \forall s_j, t_j \in S_j, \ \forall s_k \in S_{K \setminus \{i,j\}}$
$U^T(t_i, s_j, s_k) - U^S(s_i, s_j, s_k)$
$+U^S(s_i, t_j, s_k) - U^T(t_i, t_j, s_k) = 0$

The Proof is given in the Appendix 4.

The potential functions of our game depends on which condition is satisfied in the above theorem. Suppose that the first condition is satisfied $\forall i, j \in K \quad R_i g_i = R_j g_j$. Then the Rosenthal's potential function writes :

$$\Phi(F, L) = \sum_{i=1}^{F} U^S(i, K - i) + \sum_{j=1}^{L} U^{NS}(K - j, j)$$

Theorem 3.3 (Potential Game [9]). *Every finite potential game is isomorphic to a congestion game.*

Definition 3.4 (Monderer and Shapley 1996 [9]). *The normal form game G is a weighted potential game if there is a vector $(w_i)_{i \in K}$ and a potential function $V : S \longrightarrow \mathbb{R}$ such that:*

$$U_i(s_i, s_{-i}) - U_i(t_i, s_{-i}) = w_i(V(s_i, s_{-i}) - V(t_i, s_{-i})),$$
$$\forall i \in K,\ s_i, t_i \in \mathcal{S}_i$$

Theorem 3.5. *The sensing game $G = (K, (\mathcal{S}_i)_{i \in K}, (U_i)_{i \in K})$ is a weighted potential game with the weight vector:*

$$\forall i \in K \quad w_i = \frac{R_i g_i}{\sigma^2} \tag{12}$$

The Proof is given in the Appendix 5.

3.3   Equilibrium Analysis

First of all, note that since the game is finite (i.e., both the number of players and the sets of actions are finite), the existence of at least one mixed Nash equilibrium is guaranteed [10]. Now, since we know that the game is weighted potential we know that there is at least one pure Nash equilibrium [9]. Indeed, the following theorem holds.

Theorem 3.6. *The equilibria of the above potential game is the set of maximizers of the Rosenthal potential function [12].*

$$\{S = (S_1, \ldots, S_K) | S \in NE\} = \arg\max_{(F,L)} \Phi(F, L)$$

$$= \arg\max_{(F,L)} \left[\sum_{i=1}^{F} U(S, i, K - i) + \sum_{j=1}^{L} U(NS, K - j, j) \right]$$

The proof follows directly the one of Rosenthal's theorem [12].

We may restrict our attention to pure and mixed Nash equilibria. However, as it will be clearly seen in the 2-player case study (Sec. 4.2), this may pose a problem of fairness. This is the main reason why we study the set of correlated equilibria of the sensing game. We introduce the concept of correlated equilibrium [1] in order to enlarge the set of equilibrium utilities. Every utility vector inside the convex hull of the equilibrium utilities is a correlated equilibrium. The convexification property of the correlated equilibrium allow the system to better chose an optimal sensing. The concept of correlated equilibrium is a generalization of the Nash equilibrium. It consist in the stage game G extended with a signalling structure Γ. A correlated equilibrium (CE) of a stage game correspond to a Nash equilibrium (NE) of the same game extended with an adequate signalling structure Γ. A canonical correlated equilibrium is a probability distribution $Q \in \Delta(A)$, $A = A_1 \times \ldots \times A_K$ over the action product of the players that satisfy some incentives conditions.

Definition 3.7. *A probability distribution $Q \in \Delta(A)$ is a canonical correlated equilibrium if for each player i, for each action $a_i \in A_i$ that satisfies $Q(a_i) > 0$ we have:*

$$\sum_{a_{-i} \in A_{-i}} Q(a_{-i} \mid a_i) u_i(a_i, a_{-i})$$

$$\geq \sum_{a-i \in A_{-i}} Q(a_{-i} \mid a_i) u_i(b_i, a_{-i}),$$

$$\forall b_i \in A_i$$

The result of Aumann 1987 [2] states that for any correlated equilibrium, it correspond a canonical correlated equilibrium.

Theorem 3.8 (Aumann 1987, prop. 2.3 [2]). *The utility vector u is a correlated equilibrium utility if and only if there exists a distribution $Q \in \Delta(A)$ satisfying the linear inequality contraint 13 with $u = E_Q U$.*

The convexification property of the correlated equilibrium allow the system to better chose an optimal sensing. Denote E the set of pure or mixed equilibrium utility vectors and Conv E the convex hull of the set E.

Theorem 3.9. *Every utility vector $u \in$ Conv E is a correlated equilibrium utility of the sensing game.*

Any convex combination of Nash equilibria is a correlated equilibrium. As example, let $(\underline{U}_j)_{j \in J}$ a family of equilibrium utilities and $(\lambda_j)_{j \in J}$ a family of positive parameters with $\sum_{j \in J} \lambda_j = 1$ such that:

$$\underline{U} = \sum_{j \in J} \lambda_j \underline{U}_j \tag{13}$$

Then $\underline{U}$ is a correlated equilibrium utility vector.

4   Detailed Analysis for the 2-Player Case

4.1   The 2-Player Hybrid Power Control Game

In the previous section, we consider the sensing game as if the players do not chose their own power control policy. Indeed, when a player chooses to sense, he cannot choose its own power control because, it would depend on whether the other transmitters sense or not. We investigate the case where the players are choosing their sensing and power control policy in a joint manner. It enlarges the set of actions of the sensing game and it turns that, as a Braess-type paradox, that the set of equilibria is dramatically reduced. The sensing game with power

control has a stricly dominated strategy: the sensing strategy. It implies that the equilibria of such a game boils down to the Nash equilibrium without sensing.

We consider that the action set for player i consists in choosing to sense or not and the transmit power level. The action set of player i writes :

$$A_i = \{S_i, NS_i\} \times [0, \bar{P}_i] \tag{14}$$

Before to characterize the set of equilibria of such a game, remark that the two pure equilibria of the previous matrix game are no longer equilibria. Indeed, assume that player 2 will not sense its environment and transmit using the leading power p_2^L. Then player 1 best response would be to play the following transmit power p_1^F as for the classical Stackelberg equilibrium. Nevertheless in the above formulation, the player 1 has a sensing cost α that correspond to the fraction of time to sense its environment. In this context, player 1 is incited to play the following transition power without sensing. The strategy (S_1, p_1^F) and (NS_2, p_1^L) is not an equilibrium of the game with Discrete and Compact Action Set.

Theorem 4.1. *The unique Nash equilibrium of the Power Control and Sensing Game is the Nash equilibrium without sensing.*

Proof. This result comes from the cost of sensing activity. Indeed, the strategy (S_1, p_1) is always dominated by the strategy (NS_1, p_1). It turns out that the sensing is a dominated actions for both players 1 and 2. Thus every equilibria is of the form $(NS_1, p_1), (NS_2, p_2)$ with the reduced action spaces $p_1 \in [0, \bar{P}_1]$ and $p_2 \in [0, \bar{P}_2]$. The previous analysis applies in that case, showing that the unique Nash equilibrium of the Power Control and Sensing Game is the Nash of the game without sensing (p_1^*, p_2^*). □

As a conclusion, we see that letting the choice to the transmitters to choose jointly their discrete and continuous actions lead to a performance which is less than the one obtained by choosing his discrete action first, and then choosing his continuous action. This is the reason why we assume, from now on, the existence of a mechanism imposing this order in the decision taking.

4.2   The 2-Player Sensing Game

We consider the following two players-two strategies matrix game where players 1 and 2 choose to sense the channel (action S) or not (action NS) before transmitting his data. We denote by x_i the mixed strategy of user i, that is the probability that user i takes action S (sense the channel). Sensing activity provide the possibility to play as a follower, knowing in advance the action of the leaders. Let α denote the sensing cost, we compare the strategic behavior of sensing by considering the equilibrium utilities at the Nash and at the Stackelberg equilibria as payoff functions.

	NS_2	S_2
NS_1	$\frac{R_1 g_1 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*}$, $\frac{R_2 g_2 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*}$	$\frac{R_1 g_1 f(\gamma^*)(1-\gamma^*\beta^*)}{\sigma^2 \gamma^*(1+\beta^*)}$, $(1-\alpha)\frac{R_2 g_2 f(\beta^*)(1-\gamma^*\beta^*)}{\sigma^2 \beta^*(1+\gamma^*)}$
S_1	$(1-\alpha)\frac{R_1 g_1 f(\beta^*)(1-\gamma^*\beta^*)}{\sigma^2 \beta^*(1+\gamma^*)}$, $\frac{R_2 g_2 f(\gamma^*)(1-\gamma^*\beta^*)}{\sigma^2 \gamma^*(1+\beta^*)}$	$(1-\alpha)\frac{R_1 g_1 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*}$, $(1-\alpha)\frac{R_2 g_2 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*}$

Fig. 1. The Utility Matrix of the Two-Player Sensing Game

The equilibria of this game are strongly related to the sensing parameter α.

Theorem 4.2. *The matrix game has three equilibria if and only if*

$$\alpha < \frac{\beta^* - \gamma^*}{1 - \beta^*\gamma^*} \tag{15}$$

Let us characterize the three equilibria. From Appendix 1, is it easy to see that:

$$\alpha < \frac{\beta^* - \gamma^*}{1 - \beta^*\gamma^*} \iff$$
$$(1-\alpha)\frac{R_1 g_1 f(\beta^*)(1 - \gamma^*\beta^*)}{\sigma^2 \beta^*(1 + \gamma^*)} > \frac{R_1 g_1 f(\beta^*)(1 - \beta^*)}{\sigma^2 \beta^*}$$

We conclude that the joint actions (NS_1, NS_2) and (S_1, S_2) are not Nash Equilibria:

$$U_1(NS_1, NS_2) < U_1(S_1, NS_2) \tag{16}$$
$$U_2(NS_1, NS_2) < U_2(NS_1, S_2) \tag{17}$$
$$U_1(S_1, S_2) < U_1(NS_1, S_2) \tag{18}$$
$$U_2(S_1, S_2) < U_2(S_1, NS_2) \tag{19}$$

The sensing parameter determines which one of the two options is optimal between leading and following.

Corollary 4.3. *Following is better than leading if and only if*

$$\alpha < \frac{f(\beta^*) - f(\gamma^*) + \frac{f(\beta^*)}{\beta^*} - \frac{f(\gamma^*)}{\gamma^*}}{f(\beta^*)\frac{1+\beta^*}{\beta^*}} \tag{20}$$

The proof is given in Appendix 3.

The above matrix game has two pure equilibria (NS_1, S_2) and (S_1, NS_2). There is also a completely mixed equilibrium we compute using the indifference principle. Let $(x, 1 - x)$ a mixed strategy of player 1 and $(y, 1 - y)$ a mixed strategy of player 2. We aim at characterize the optimal joint mixed strategy

$$x^* = y^* = \frac{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*}}{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*} + \frac{f(\beta^*)}{\beta^*}(1-\beta^*) - (1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}$$

(x^*, y^*) satisfying the indifference principle (see Appendix 2 for more details). The above joint mixed strategy $(x^*, 1-x^*)$ and $(y^*, 1-y^*)$ is an equilibrium strategy. The corresponding utilities are computed in Appendix 2. and writes with Δ defined in(4.2).

$$U_1(x^*, y^*) = \frac{R_1 g_1}{\sigma^2}\Delta$$

$$U_2(x^*, y^*) = \frac{R_2 g_2}{\sigma^2}\Delta$$

$$\Delta = \frac{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*}(1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*} + \frac{f(\beta^*)}{\beta^*}(1-\beta^*) - (1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}$$

The equilibrium utilities are represented on the following figure. The two pure Nash equilibrium utilities are represented by a circle whereas the mixed Nash utility is represented by a square.
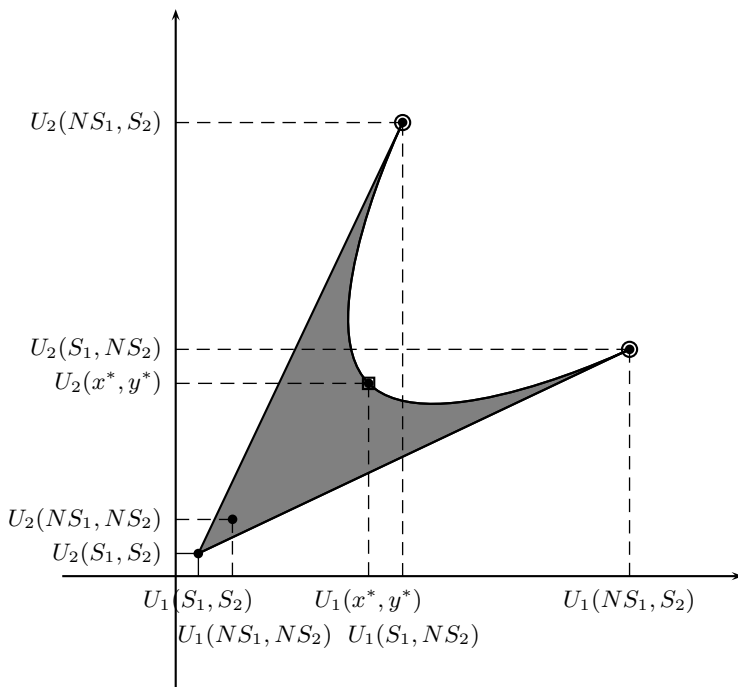
Fig. 2. The Equilibrium and Feasible Utilities

We also provide a characterization of the equilibria for the cases where α is greater or equal than $\frac{\beta^* - \gamma^*}{1 - \beta^* \gamma^*}$.

Corollary 4.4. *The matrix game has a unique equilibrium if and only if*

$$\alpha > \frac{\beta^* - \gamma^*}{1 - \beta^* \gamma^*} \tag{21}$$

It has a infinity of equilibria if and only if

$$\alpha = \frac{\beta^* - \gamma^*}{1 - \beta^* \gamma^*} \tag{22}$$

First note that if the sensing cost is too high, the gain in terms of utility at Stackelberg instead of Nash equilibrium would be dominated by the loss of utility due to the sensing activity. In that case, the Nash equilibrium would be more efficient. Second remark that in case of equality, the action profiles (NS_1, NS_2), (NS_1, S_2), (S_1, NS_2) and every convex combination of the corresponding payoffs are all equilibrium payoffs.

Now that we have fully characterized the pure and mixed equilibria of the game, let us turn our attention to correlated equilibria.

Theorem (3.8) allows us to characterize the correlated equilibrium utility using the system of linear inequalities (13). We investigate the situation where the stage game has three Nash equilibria and following is better than leading. We suppose that the parameter α satisfies.

$$\alpha < \min\left(\frac{\beta^* - \gamma^*}{1 - \beta^* \gamma^*}, \frac{f(\beta^*) - f(\gamma^*) + \frac{f(\beta^*)}{\beta^*} - \frac{f(\gamma^*)}{\gamma^*}}{f(\beta^*)\frac{1 + \beta^*}{\beta^*}}\right) \tag{23}$$

Note that the analysis is similar in the case where Leading is better than Following. However, if the parameter $\alpha > \frac{\beta^* - \gamma^*}{1 - \beta^* \gamma^*}$ we have seen that the stage game has only one Nash equilibrium corresponding to play the Nash equilibrium power in the one-shot game. In such a case, no signalling device can increase the set of equilibria. The unique correlated equilibrium is the Nash equilibrium. We characterize an infinity of correlated equilibria.

Theorem 4.5. *Any convex combination of Nash equilibria is a correlated equilibrium. In particular if there exists a utility vector u and a parameter $\lambda \in [0, 1]$ such that:*

$$u_1 = \lambda U_1(S_1, NS_2) + (1 - \lambda)U_1(NS_1, S_2) \tag{24}$$

$$u_2 = \lambda U_2(S_1, NS_2) + (1 - \lambda)U_2(NS_1, S_2) \tag{25}$$

Then u is a correlated equilibrium.

The above result state that any distribution Q defined as follows with $\lambda \in [0, 1]$ is a correlated equilibrium. The canonical signalling device which should be added to the game consist in a lottery with parameter λ over the actions (S_1, NS_2)

	NS_2	S_2
NS_1	0	$1 - \lambda$
S_1	λ	0

and (NS_1, S_2) and of signalling structure such that each player receives her component. For example, if (S_1, NS_2) is chosen the player 1 receives the signal "play S_1" whereas player 2 receives the signal "play NS_2".

The correlated equilibrium utilities are represented by the bold line. The signalling device increase the achievable utility region by adding the light gray area.
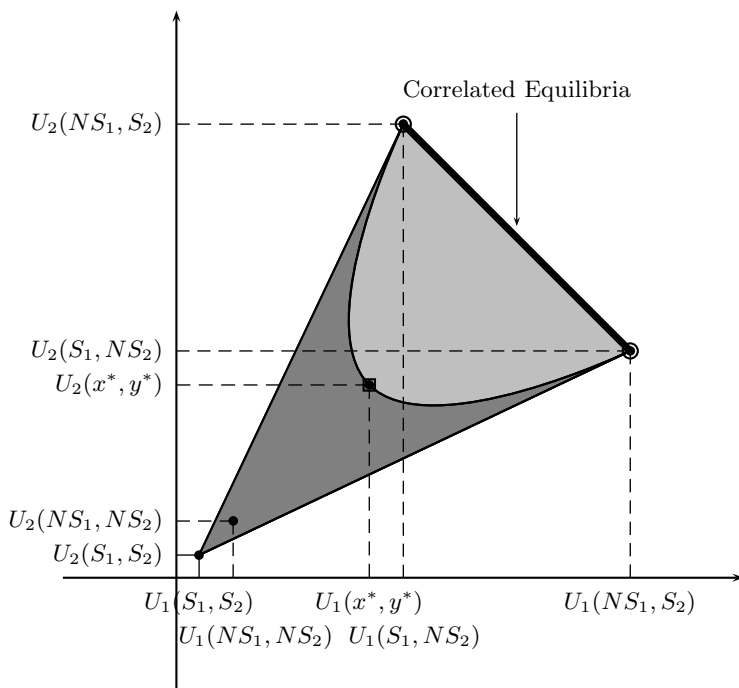

Fig. 3. The Correlated Equilibria

5 Conclusion

In this paper we have introduced a new power control game where the action of a player is hybrid, one component is discrete while the other is continuous. Whereas the general study of these games remains to be done, it turns out that in our case we can prove the existence of a Braess paradox which allows us to restrict our attention to two separate games played consecutively: a finite game where the players decide to sense or not and a compact game where the transmitter chooses his power level. We have studied in details the sensing game. In particular, it is proved it is weighted potential. Also, by characterizing the correlated equilibria of

this game we show what is achievable in terms of fairness. Much work remains to be done to generalize all these results to games with arbitrary number of players and conduct simulations in relevant wireless scenarios.

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Appendix 1

$$\alpha < \frac{\beta^* - \gamma^*}{1 - \beta^* \gamma^*}$$
$$\iff \frac{1 - \gamma^* \beta^* - \beta^* - \gamma^*}{(1 - \gamma^* \beta^*)} < 1 - \alpha$$
$$\iff (1 - \beta^*)(1 + \gamma^*) < (1 - \alpha)[(1 - \beta^*)(1 + \gamma^*) + \gamma^* + \beta^*]$$
$$\iff \frac{f(\beta^*)}{\beta^*}(1 - \beta^*) < (1 - \alpha)\frac{f(\beta^*)}{\beta^*}\frac{1 - \beta^* \gamma^*}{1 + \gamma^*}$$
$$\iff \frac{R_1 g_1 f(\beta^*)(1 - \beta^*)}{\sigma^2 \beta^*} < (1 - \alpha)\frac{R_1 g_1 f(\beta^*)(1 - \gamma^* \beta^*)}{\sigma^2 \beta^*(1 + \gamma^*)}$$

Appendix 2

$$\frac{R_1 g_1 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*} \cdot y^* + \frac{R_1 g_1 f(\gamma^*)(1-\gamma^*\beta^*)}{\sigma^2 \gamma^*(1+\beta^*)} \cdot (1-y^*)$$

$$= (1-\alpha)\frac{R_1 g_1 f(\beta^*)(1-\gamma^*\beta^*)}{\sigma^2 \beta^*(1+\gamma^*)} \cdot y^* + (1-\alpha)\frac{R_1 g_1 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*} \cdot (1-y^*)$$

$$\iff \quad y^* \cdot \Big[\frac{R_1 g_1 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*} - (1-\alpha)\frac{R_1 g_1 f(\beta^*)(1-\gamma^*\beta^*)}{\sigma^2 \beta^*(1+\gamma^*)}$$

$$+ (1-\alpha)\frac{R_1 g_1 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*} - \frac{R_1 g_1 f(\gamma^*)(1-\gamma^*\beta^*)}{\sigma^2 \gamma^*(1+\beta^*)}\Big]$$

$$= (1-\alpha)\frac{R_1 g_1 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*} - \frac{R_1 g_1 f(\gamma^*)(1-\gamma^*\beta^*)}{\sigma^2 \gamma^*(1+\beta^*)}$$

$$\iff \quad y^* = \frac{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*}}{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*} + \frac{f(\beta^*)}{\beta^*}(1-\beta^*) - (1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}$$

Replacing the above y^* into the indifference equation, we obtain the utility of player 1 at the mixed equilibrium.

$$U_1(x^*, y^*) = \frac{\frac{R_1 g_1 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*}\frac{R_1 g_1 f(\gamma^*)(1-\gamma^*\beta^*)}{\sigma^2 \gamma^*(1+\beta^*)} - \frac{R_1 g_1}{\sigma^2}\frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*}\frac{R_1 g_1}{\sigma^2}(1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}{\frac{R_1 g_1}{\sigma^2}(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{R_1 g_1}{\sigma^2}\frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*} + \frac{R_1 g_1}{\sigma^2}\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{R_1 g_1}{\sigma^2}(1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}$$

$$+ \frac{\frac{R_1 g_1}{\sigma^2}(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*)\frac{R_1 g_1}{\sigma^2}\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{R_1 g_1 f(\beta^*)(1-\beta^*)}{\sigma^2 \beta^*}\frac{R_1 g_1 f(\gamma^*)(1-\gamma^*\beta^*)}{\sigma^2 \gamma^*(1+\beta^*)}}{\frac{R_1 g_1}{\sigma^2}(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{R_1 g_1}{\sigma^2}\frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*} + \frac{R_1 g_1}{\sigma^2}\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{R_1 g_1}{\sigma^2}(1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}$$

$$= \frac{R_1 g_1}{\sigma^2}\frac{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*}(1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*} + \frac{f(\beta^*)}{\beta^*}(1-\beta^*) - (1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}$$

The same argument applies:

$$U_2(x^*, y^*) = \frac{R_2 g_2}{\sigma^2}\frac{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*}(1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}{(1-\alpha)\frac{f(\beta^*)}{\beta^*}(1-\beta^*) - \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\beta^*} + \frac{f(\beta^*)}{\beta^*}(1-\beta^*) - (1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}}$$

Appendix 3

$$\alpha < \frac{f(\beta^*) - f(\gamma^*) + \frac{f(\beta^*)}{\beta^*} - \frac{f(\gamma^*)}{\gamma^*}}{f(\beta^*)\frac{1+\beta^*}{\beta^*}}$$

$$\iff 1 - \alpha > \frac{f(\beta^*)\frac{1+\beta^*}{\beta^*} - f(\gamma^*)\frac{1+\gamma^*}{\gamma^*}}{f(\beta^*)\frac{1+\beta^*}{\beta^*}}$$

$$\iff (1-\alpha)\frac{f(\beta^*)}{\beta^*}\frac{1-\gamma^*\beta^*}{1+\beta^*} > \frac{f(\gamma^*)}{\gamma^*}\frac{1-\gamma^*\beta^*}{1+\gamma^*}$$

Appendix 4

The proof comes from the theorem of Monderer and Shapley 1996 (see Sandholm "Decomposition of Potential" 2010)

Theorem 5.1. *The game G is a potential game if and only if for every players $i, j \in K$, every pair of actions $s_i, t_i \in S_i$ and $s_j, t_j \in S_j$ and every joint action $s_k \in S_{K \setminus \{i,j\}}$, we have that*

$$U_i(t_i, s_j, s_k) - U_i(s_i, s_j, s_k) + U_i(s_i, t_j, s_k) - U_i(t_i, t_j, s_k) +$$
$$U_j(t_i, t_j, s_k) - U_j(t_i, s_j, s_k) + U_j(s_i, s_j, s_k) - U_j(s_i, t_j, s_k) = 0$$

Let us prove that the two conditions provided by our theorem are equivalent to the one of Monderer and Shapley's theorem. We introduce the following notation defined for each player $i \in K$ and each action $T \in \mathcal{S}$.

$$w_i = R_i g_i \tag{26}$$

$$U^T(t_i, t_j, s_k) = \frac{U_i^T(t_i, t_j, s_k)}{w_i} \tag{27}$$

For every players $i, j \in K$, every pair of actions $s_i, t_i \in S_i$ and $s_j, t_j \in S_j$ and every joint action $s_k \in S_{K \setminus \{i,j\}}$, we have the following equivalences:

$$
\begin{aligned}
& U_i(t_i, s_j, s_k) - U_i(s_i, s_j, s_k) \\
& +U_i(s_i, t_j, s_k) - U_i(t_i, t_j, s_k) \\
& + U_j(t_i, t_j, s_k) - U_j(t_i, s_j, s_k) \\
& +U_j(s_i, s_j, s_k) - U_j(s_i, t_j, s_k) = 0 \\
\iff \quad & w_i(U^T(t_i, s_j, s_k) - U^S(s_i, s_j, s_k) \\
& +U^S(s_i, t_j, s_k) - U^T(t_i, t_j, s_k)) \\
& + w_j(U^T(t_i, t_j, s_k) - U^S(t_i, s_j, s_k) \\
& +U^S(s_i, s_j, s_k) - U^T(s_i, t_j, s_k)) = 0 \\
\iff \quad & (w_i - w_j)(U^T(t_i, s_j, s_k) - U^S(s_i, s_j, s_k) \\
& +U^S(s_i, t_j, s_k) - U^T(t_i, t_j, s_k)) = 0 \\
\iff \quad & \begin{cases} w_i = w_j \\ U^T(t_i, s_j, s_k) - U^S(s_i, s_j, s_k) \\ +U^S(s_i, t_j, s_k) - U^T(t_i, t_j, s_k) = 0 \end{cases}
\end{aligned}
$$

Thus the sensing game is a potential game if and only if one of the two following condition is satisfied:

$$\forall i, j \in K \quad R_i g_i = R_j g_j \tag{28}$$
$$\forall i, j \in K, \ s_i, t_i \in S_i, \ \forall s_j, t_j \in S_j, \ \forall s_k \in S_{K \setminus \{i,j\}} \tag{29}$$
$$U^T(t_i, s_j, s_k) - U^S(s_i, s_j, s_k) \tag{30}$$
$$+U^S(s_i, t_j, s_k) - U^T(t_i, t_j, s_k) = 0 \tag{31}$$

Appendix 5

The proof of this theorem follows the same line of the previous theorem. It suffices to show that the auxiliary game defined as follows is a potential game.

$$\widetilde{G} = (K, (\mathcal{S})_{i\in K}, (\tilde{U}_i)_{i\in K}) \tag{32}$$

Where the utility are defined by the following equations with $w_i = \frac{R_i g_i}{\sigma^2}$.

$$\tilde{U}_i(s_i, s_{-i}) = \frac{U_i(s_i, s_{-i})}{w_i} \tag{33}$$

From the above demonstration, it is easy to show that, for every players $i, j \in K$, every pair of actions $s_i, t_i \in S_i$ and $s_j, t_j \in S_j$ and every joint action $s_k \in S_{K\setminus\{i,j\}}$:

$$\tilde{U}_i(t_i, s_j, s_k) - \tilde{U}_i(s_i, s_j, s_k) \tag{34}$$
$$+ \tilde{U}_i(s_i, t_j, s_k) - \tilde{U}_i(t_i, t_j, s_k) \tag{35}$$
$$+ \tilde{U}_j(t_i, t_j, s_k) - \tilde{U}_j(t_i, s_j, s_k) \tag{36}$$
$$+ \tilde{U}_j(s_i, s_j, s_k) - \tilde{U}_j(s_i, t_j, s_k) = 0 \tag{37}$$

We conclude that the sensing game is a weighted potential game.