A Game Theoretic Approach for Multi-hop Power Line Communications*

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Abstract. In this paper, a model for multi-hop power line communication is studied in which a number of smart sensors, e.g., smart meters, seek to minimize the delay experienced during the transmission of their data to a common control center through multi-hop power line communications. This problem is modeled as a network formation game and an algorithm is proposed for modeling the dynamics of network formation. The proposed algorithm is based on a myopic best response process in which each smart sensor can autonomously choose the path that connects it to the control center through other smart sensors. Using the proposed algorithm, the smart sensors can choose their transmission path while optimizing a cost that is a function of the overall achieved transmission delay. This transmission delay captures a tradeoff between the improved channel conditions yielded by multi-hop transmission and the increase in the number of hops. It is shown that, using this network formation process, the smart sensors can self-organize into a tree structure which constitutes a Nash network. Simulation results show that the proposed algorithm presents significant gains in terms of reducing the average achieved delay per smart sensor of at least 28.7% and 60.2%, relative to the star network and a nearest neighbor algorithm, respectively.

1 Introduction

The use of power lines as a means for communications has been adopted by utility companies for many decades in order to transmit control and monitoring data in power systems. Recently, power line communication (PLC) has emerged as a key technology that enables the delivery of new applications such as broadband Internet, telephony, automation, remote metering, as well as in-home delivery of a variety of data and multimedia services [1, 2, 3, 4]. While the full potential of PLC is yet to be exploited in the market, it is expected that PLC will play a major role as an enabler for efficient communications in the emerging smart grid and "Internet of things" networks.

In particular, within large-scale networks such as the smart grid, PLC is one of the main candidate technologies that can be used to ensure data communication between the smart sensors that are typically used to collect data (e.g., household loads, monitoring

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data, maintenance, prices inquiry, etc.) and transmit it to a control center or other central nodes within the network [2]. In this respect, enabling such PLC-based applications faces a variety of challenges such as channel modeling, medium access, security, efficient data transmission, and advanced network planning. For instance, in [4], a survey of smart grid applications for PLC is provided. The authors discuss a variety of usage scenarios for PLC such as advanced metering infrastructure, vehicle-to-grid communications, and substations interconnection in the medium voltage part of the grid. The work in [5] discusses the use of relaying techniques, such as decode-and-forward, for improving the capacity and coverage of an in-home broadband PLC network. In [6], the authors propose a space-time coding technique for improving retransmissions through repeaters using PLC channels. Performance analysis of a variety of channel models for PLC is done in [7, 8, 9, 10] and the references therein. The use of PLC for demand response in the smart grid is analyzed in [11]. An extensive treatment of communications and other networking issues in the smart grid is found in [2].

In essence, PLC can operate on either broadband or narrowband frequencies [1]. Depending on the mode of operation, the potential PLC applications can vary. While broadband PLC is suitable for Internet services, in-home entertainment, or demand response applications, narrowband PLC is bound to be used for advanced metering, vehicle-to-grid networking, as well as other smart grid applications [1, 2, 4]. In particular, narrowband PLC is a suitable means for interconnecting the smart sensors that are used for control, load monitoring, price inquiry, and other metering purposes in the smart grid. In fact, narrowband PLC has been widely used for advanced metering infrastructure in Europe [4]. One of the key challenges of adopting narrowband PLC as a communication technology between the sensors of the smart grid stems from the limited capacity of PLC channels which decreases quickly with distance as discussed in [1, Chap. 5] and [7]. Overcoming these limitations in existing PLC networks is typically done by dimensioning the network, prior to deployment, so as to ensure a reasonable capacity for every point-to-point PLC communication [1, 7].

However, for large-scale heterogeneous networks, such as the smart grid, a predetermined dimensioning may not be possible. For example, the deployment of smart meters is restricted by the locations of the related homes or businesses, irrespective of the communication technology that will be adopted between these meters. Hence, the ability to control the potential PLC capacity within a large cyber-physical network such as the smart grid faces several constraints and practical restrictions. In such networks, the PLC capacity limitations can lead to large delays or limited coverage, which constitute key quality-of-service (QoS) requirements for most smart grid applications [12]. As a result, it is of central interest to design intelligent and advanced algorithms that enable narrowband PLC communication in networks such as the smart grid or the Internet of things, while maintaining a reasonable QoS (e.g., low delays) given the large-scale, heterogeneous, and decentralized nature of these networks.

The main contribution of this paper is to propose a novel multi-hop protocol for narrowband PLC communication suitable for cyber-physical networks such as the smart grid. Hence, our objective is to develop an algorithm that enables multi-hop PLC communication between a number of smart sensors that need to send their data (e.g., meter readings, load reports, control and monitoring data, power quality, or pricing information) to a common access point (e.g., a control center or a repeater). For this purpose, we formulate a network formation game in which the smart sensors are the players and the strategy of each smart sensor is to select the preferred next hop for transmitting its data, using narrowband PLC. Then, we propose a network formation algorithm which enables the smart sensors to take distributed decisions regarding their PLC transmission path while minimizing their cost function which captures the overall experienced transmission delay. Using the proposed algorithm, the smart sensors can self-organize into a Nash network, i.e., a stable tree structure which connects them to the common control center or repeater. Simulation results show that the proposed algorithm yields a significant reduction of the average delay per smart sensor when compared to the star network or a nearest neighbor approach.

The rest of this paper is organized as follows: Section 2 presents the system model. In Section 3, we formulate the network formation game between the smart sensors while in Section 4, we present the proposed algorithm. Simulation results are presented and analyzed in Section 5. Finally, conclusions are drawn in Section 6.

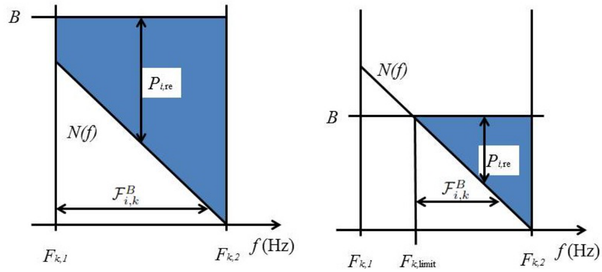
2 System Model

Consider an area of a smart grid network composed of M physically interconnected smart sensors, e.g., smart meters, and let $\mathcal{M}$ denote the set of all of these smart sensors. Each smart sensor in the set $\mathcal{M}$ needs to communicate different information such as meter readings, load reports, pricing inquiries, event detection data, or network repair information to a common access point (CAP). This access point can be either a control center installed by the grid operator or a repeater that connects this area to other parts of the smart grid. In order to communicate, these smart sensors operate using high data rate (HDR) narrowband power line communication. Hence, all frequencies used throughout this paper are chosen from within the typical narrowband PLC values which range between 3 kHz and 148.5 kHz in Europe and up to 500 kHz in the USA [1].

We consider that the CAP allows each smart sensor $i \in \mathcal{M}$ to communicate using a channel having a bandwidth W, assumed to be the same for all smart sensors, without loss of generality. To do so, the CAP divides its range of frequencies $F_{\text{CAP},1} \leq |f| \leq F_{\text{CAP},2}$ into $K_{\text{CAP}} \triangleq \frac{F_{\text{CAP},2}-F_{\text{CAP},1}}{W}$ channels with equal bandwidth W. Subsequently, the CAP assigns each one of these channels to a requesting smart sensor $i \in \mathcal{M}$. We let $\mathcal{K}_{\text{CAP}}$ denote the set of all K_{CAP} channels that the CAP offers. Each channel $k \in \mathcal{K}_{\text{CAP}}$ is defined by a range of frequencies $F_{k,1} \leq |f| \leq F_{k,2} = F_{k,1} + W$. We consider a general case in which the CAP assigns the channels randomly depending on when a certain smart sensor requests to communicate. Further, hereinafter, we assume that the CAP is able to provide one channel for every smart sensor in the network and, thus, $M \leq K_{\text{CAP}}$. Nonetheless, the analysis can be easily extended to the case in which $M > K_{\text{CAP}}$ by adding extra control centers or by adopting advanced techniques for multiple PLC access which are outside the scope of this paper.

In narrowband PLC communication, the colored background noise has a (double sided) power spectral density $N(f)$ which decreases for increasing frequencies and can be expressed by [1, Chap. 5] (in W/Hz):

$$N(f) = 1/2 \cdot 10^{\gamma - 4 \cdot 10^{-5} f}, \tag{1}$$



(a) Region 1, $F_{k,1} \leq |f| \leq F_{k,2}$ (b) Region 2, $F_{k,\text{limit}} \leq |f| \leq F_{k,2}$

Fig. 1. Two regions (dependent on the frequency range $\mathcal{F}^B_{i,k}$) used for finding the capacity of a PLC transmission, based on (2)-(4), for a smart sensor i using a certain channel k

where γ is normally distributed with mean -8.64 and standard deviation 0.5. As observed in [7], the value of γ is often estimated from measurements and has a worst case (high noise) value of -7.64 and a best case (low noise) value of -9.64.

Each smart sensor $i \in \mathcal{N}$ has a transmit power of P_i and needs to send L_i packets of R bits each to the CAP. In this respect, given the noise in (1) the capacity $C^k_{i,\text{CAP}}$of any point-to-point PLC communication link between a smart sensor $i \in \mathcal{N}$ and the CAP, using a channel $k \in \mathcal{K}_{\text{CAP}}$ is given by the so-called *water-filling* solution [7],[1, Chap. 5]:

$$C^k_{i,\text{CAP}} = \int_{f \in \mathcal{F}^B_{i,k}} 1/2 \log_2 \left[\frac{B}{N(f)}\right] df, \tag{2}$$

where $\mathcal{F}^B_{i,k}$ is the range of frequencies for channel k (i.e., within $F_{k,1} \leq |f| \leq F_{k,2} = F_{k,1} + W$) for which we have:

$$N(f) \leq B, \tag{3}$$

where B is the solution to

$$P_{i,\text{re}} = P_i \cdot 10^{-\kappa d_{i,\text{CAP}}} = \int_{f \in \mathcal{F}^B_{i,k}} [B - N(f)] \,. \tag{4}$$

Here, κ is the attenuation factor which ranges between 40 dB/km and 100 dB/km and $d_{i,\text{CAP}}$ is the distance between smart sensor i and the CAP. Note that, (2), (3), and (4) are derived based on the assumption that the transfer function $H(f)$ is chosen such that, within the frequency range of channel k, the channel gain is equal to one[1], i.e., $|H(f)|^2 = 1$. However, (2)-(4) can be easily modified to accommodate any other transfer function $H(f)$ as shown in [7].

As discussed in [1, Chap. 5], the solution to (2)-(4) can be found by splitting the problem into the two regions of Figure 1 in order to find the frequency range $\mathcal{F}^B_{i,k}$. In the first region shown in Figure 1(a), the level B is larger than the noise level and, thus, this region corresponds to the case in which the received signal power is relatively large,

[1] This consideration on the transfer function is inspired from the channel filter of the well-known CENELEC A-band [7].

i.e., there exists enough power to fill the entire frequency region. In contrast, the second region, shown in Figure 1(b), has a level B smaller than the noise level within a portion of the frequency range. This region corresponds to the case in which the received signal power is relatively small (due to attenuation) and, hence, it is used to fill the portion of the spectrum with a small noise level, i.e., below B.

The expression of the capacity in (2) differs between the two regions. In order to compute the expression within each region, it is of interest to find the critical distance $\tilde{d}^k_{i,\text{CAP}}$ such that for $d_{i,\text{CAP}} \leq \tilde{d}^k_{i,\text{CAP}}$ the operation is in Region 1 and for $d_{i,\text{CAP}} > \tilde{d}^k_{i,\text{CAP}}$ the operation is in Region 2. In other words, the critical distance represents the distance after which we move from the analysis of Region 1, i.e., Figure 1(a), to that of Region 2, i.e., Figure 1(b). For the transmission between a smart sensor $i \in \mathcal{M}$ to the CAP over a channel $k \in \mathcal{K}_{\text{CAP}}$, this critical distance $\tilde{d}^k_{i,\text{CAP}}$ can be given by [7]:

$$\tilde{d}^k_{i,\text{CAP}} = -\kappa^{-1} \cdot \log_{10}\left[\frac{(W-2)N(F_{k,1}) - 2N(F_{k,2})}{4 \cdot 10^{-5} \cdot \ln 10 \cdot P_i}\right], \tag{5}$$

where $N(f)$ is given by (1).

In Region 1 where $d_{i,\text{CAP}} \leq \tilde{d}^k_{i,\text{CAP}}$, as clearly seen from Figure 1(a), $\mathcal{F}^B_{i,k}$ is simply the entire frequency band available at channel k, i.e., $F_{k,1} \leq |f| \leq F_{k,2}$. Therefore, given the critical distance in (5), determining the capacity in Region 1 is straightforward and can be computed from (2) and (4) as follows [7]:

$$C^k_{i,\text{CAP}} = (1 + \log_2 B + 3.3\gamma)W + 6.65 \cdot 10^{-5} \cdot \left(F^2_{k,2} - F^2_{k,1}\right), \tag{6}$$

where B is given by:

$$B = \frac{[N(F_{k,1}) - N(F_{k,2})]}{9.2 \cdot 10^{-5} \cdot W} + \frac{10^{-\kappa d_{i,\text{CAP}}} \cdot P_i}{2W}. \tag{7}$$

In Region 2 where $d_{i,\text{CAP}} > \tilde{d}^k_{i,\text{CAP}}$, $\mathcal{F}^B_{i,k}$ is a region such that $F_{k,\text{limit}} \leq |f| \leq F_{k,2}$, where $F_{k,\text{limit}} > F_{k,1}$ is the point after which we have (3) verified (see Figure 1(b)). In this case, determining the capacity using (2)-(4) requires finding $F_{k,\text{limit}}$ first. While the computation can be analytically complex as discussed in [7], first, one can find an expression that links the distance and $F_{k,\text{limit}}$ as follows [7]:

$$\begin{aligned} P_i \cdot 10^{-\kappa d_{i,\text{CAP}}} - 2.2 \cdot 10^4 N(F_{k,2}) &= -2F_{k,\text{limit}} N(F_{k,\text{limit}}) \\ &\quad + 2N(F_{k,\text{limit}})(F_{k,2} - 1.1 \cdot 10^4) \end{aligned} \tag{8}$$

An analytical solution for (8) in which $F_{k,\text{limit}}$ is a real number expressed as a function of $d_{i,\text{CAP}}$ can be found as a function of the "Lambert W" function, as follows:

$$F_{k,\text{limit}} = (F_{k,2} - 1.1 \cdot 10^4) - \frac{W_L(g(d_{i,\text{CAP}}) 10^{(F_{k,2} - 1.1 \cdot 10^4) 4 \cdot 10^{-5}})}{4 \cdot 10^{-5} \ln 10} \tag{9}$$

where $W_L(\cdot)$ is the "Lambert W" function and $g(d_{i,\text{CAP}})$ is given by

$$g(d_{i,\text{CAP}}) = 9.2 \cdot 10^{-5}(P_i \cdot 10^{-\gamma - \kappa d_{i,\text{CAP}}} - 1.1 \cdot 10^{4 - 4 \cdot 10^{-5} \cdot F_{k,2}}), \tag{10}$$

Then, by using (9) and (2), we can find a closed-form expression for the capacity in Region 2 as follows:

$$C_{i,\text{CAP}}^{k} = (F_{k,2} - F_{k,\text{limit}}) \log_2 (10^{-4 \cdot 10^{-5}} F_{k,\text{limit}}) \\ +6.65 \cdot 10^{-5} (F_{k,2}^2 - F_{k,\text{limit}}^2) \tag{11}$$

Given any smart sensor $i \in \mathcal{M}$ that needs to send L_i packets of R bits each to the CAP while using a certain channel $k \in \mathcal{K}_{\text{CAP}}$, we define a cost function that captures the transmission delay as follows:

$$r_i(G) = \tau_{i,\text{CAP}} = \frac{R \cdot L_i}{C_{i,\text{CAP}}^{k}}, \tag{12}$$

where G is a star network graph centered at the CAP and connecting it to the smart sensors with direct transmission links (i.e., edges).

For narrowband PLC, it is well known that the capacities, as given in (6) and (11), are large for small distances, however, they decay very fast with distance. Subsequently, the delay in (12) can increase significantly, notably for a large-scale smart grid network in which the smart sensors need to communicate with a relatively far CAP. For many of the emerging applications within smart grid networks, delay and capacity are key QoS requirements [12], and, thus, it is of interest to design an approach that enables the smart sensors to utilize narrowband PLC for sending their data, while maintaining reasonable delays. For instance, by exploiting the fact that the capacities in (6) and (11) can be large for small to medium distances, one can develop a multi-hop scheme that enables the smart sensors to relay each others' data, while optimizing the delay in (12).

3 A Game Theoretic Approach for Multi-hop PLC Transmission

In order to improve their delays while communicating with the CAP, the smart sensors in $\mathcal{M}$ can interact with one another in order to perform multi-hop transmission. By doing so, the smart sensors can exploit the fact that the capacity of a narrowband PLC channel as captured by (2) is large for small distances but decays fast as the communication distances become larger. To perform multi-hop PLC communication, the smart sensors will essentially try to interact with their neighbors and decide on which hop to use given their traffic and potential PLC capacity.

In order to model these interactions between the smart sensors, we use the analytical framework of network formation games [13, 14, 15, 16]. Network formation games involve situations in which a number of players need to interact in order to decide on the formation of a network graph among them. In a network formation game, the outcome is essentially a graph structure that interconnects the various players while capturing their individual objectives. In this respect, to overcome the capacity limitations inherent to narrowband PLC-based networks, we propose a network formation game in which the smart sensors are the players and the objective is to form a multi-hop tree structure that enables each smart sensor to reduce its delay.

Hence, the result of the proposed smart sensors network formation game is a *directed* graph $G(\mathcal{M}, \mathcal{E})$ with $\mathcal{M}$ being the set of vertices of the graph (i.e., the smart sensors) and $\mathcal{E}$ being the set of all edges (links) between pairs of smart sensors. Note that, for the scope of this paper, we limit our attention to *tree structures* in which each smart sensor selects only one parent node for transmission. Each directed link between two smart sensors $i \in \mathcal{M}$ and $j \in \mathcal{M}$, denoted $(i, j) \in \mathcal{E}$, corresponds to a traffic flow over the narrowband PLC channel from smart sensor i to smart sensor j. Prior to delving into the details of the proposed network formation game, we will first define the notion of a path:

Definition 1. A *path* between two smart sensors i and j in a graph structure G is a sequence of smart sensors $i_1, \ldots, i_L$ such that $i_1 = i$, $i_L = j$ and each directed link $(i_l, i_{l+1}) \in G$ for each $l \in \{1, \ldots, L-1\}$.

In the proposed network formation game, each smart sensor will have a single path to the CAP due to the fact that we consider multi-hop tree structures between the smart sensors. As a result, we have a network formation game between the smart sensors in which the strategy of each smart sensor is to select its preferred path to destination. Formally, we can delineate the possible actions or strategies that a smart sensor can take in the proposed PLC network formation game as follows. The strategy space of any smart sensor $i \in \mathcal{M}$ is the set of possible smart sensors (or the CAP) that i can connect to. Consequently, the strategy of the smart sensor i is to select the link that it wants to form out of its available strategy space. Essentially, a smart sensor $i \in \mathcal{M}$ can connect either directly to the CAP or through any other smart sensor $j \in \mathcal{M},\ j \neq i$ as long as j is not, itself, connected to i. In other words, a smart sensor i cannot connect to another smart sensor j which is already connected to i, i.e., if $(j, i) \in G$, then $(i, j) \notin G$.

Hence, for a given network graph G, let $\mathcal{A}_i = \{j \in \mathcal{M} \setminus \{i\} | (j, i) \in G\}$ be the set of smart sensors from which smart sensor i accepted a link (j, i) and $\mathcal{S}_i = \{(i, j) | j \in \mathcal{V} \setminus (\{i\} \bigcup \mathcal{A}_i)\}$ be the set of links corresponding to the nodes (smart sensors or the CAP) that i can connect to, with $\mathcal{V}$ defined as the set of all smart sensors *and* the CAP. In consequence, the strategy of a smart sensor i can be formally defined as the link $s_i \in \mathcal{S}_i$ that it wants to form. As we consider tree structures, the strategy of a smart sensor can be seen as a *replace* operation using which a smart sensor $i \in \mathcal{M}$ replaces its current link with a new link from $\mathcal{S}_i$.

For narrowband PLC transmission, whenever a smart sensor needs to select a strategy, i.e., connect to another smart sensor (or the CAP), it needs to obtain an appropriate channel for transmission. In this respect, we define, for each smart sensor $i \in \mathcal{M}$, a set $\mathcal{K}_i$ that represents the set of *all* channels that i is able to offer to other smart sensor wishing to use i for multi-hop communication. In essence, for any given channel $k \in \mathcal{K}_i$ defined with a range of frequencies $F_{k,1} \leq |f| \leq F_{k,1} + W$, as the frequency $F_{k,1}$ increases, the capacity in (2) increases, thus reducing the delay (given that the bandwidth W is assumed to be the same for all channels). Thus, whenever a smart sensor i connects to a smart sensor j, we assume that j would assign the channel $k_i \in \mathcal{K}_j$ having the largest frequency $F_{k_i,1}$ to i, i.e., $k_i \in \arg\max_{l \in \mathcal{K}_j} F_{l,1}$. This best available channel is, thus, the channel that yields the lowest delay (among other *available* channels at j) for smart sensor i as per (13).

Further, we assume that the number of channels that a smart sensor i is able to offer is limited, due to the resource constrained nature of these smart sensors. Therefore, we have $|\mathcal{K}_i| < K_{\text{CAP}}$, $\forall i \in \mathcal{M}$, where $K_{\text{CAP}} = \mathcal{K}_{\text{CAP}}$ is the number of channels that the CAP can offer and $|\cdot|$ is the cardinality of a set operator. Thus, we can highlight the following property for our proposed smart sensors network formation game:

Property 1. For the proposed smart sensors network formation game, the number of nodes that a smart sensor $i \in \mathcal{M}$ serves within a graph G (i.e., the number of smart sensors in $\mathcal{A}_i$) is limited by the available channels in $\mathcal{K}_i$, and, thus, we have $|\mathcal{A}_i| \leq |\mathcal{K}_i|$.

As a result of Property 1, whenever a smart sensor $i \in \mathcal{M}$ has already accepted its maximum number of connections, i.e., $|\mathcal{A}_i| = |\mathcal{K}_i|$, it can no longer accept additional connections to serve. In this regard, denoting by $G + s_i$ as the graph G modified when a smart sensor i deletes its current link in G and adds a new link $s_i = (i, j)$, we define the concept of a *feasible* strategy as follows:

Definition 2. A strategy $s_i \in \mathcal{S}_i$, i.e., a link $s_i = (i, j)$, is a *feasible strategy* for a smart sensor $i \in \mathcal{V}$ if and only if smart sensor j can still offer a channel for i, i.e., $|\mathcal{A}_j| + 1 \leq |\mathcal{K}_j|$. For any smart sensor $i \in \mathcal{M}$, the set of all feasible strategies is denoted by $\hat{\mathcal{S}}_i \subseteq \mathcal{S}_i$.

Hence, a feasible strategy for any smart sensor i is simply a link $s_i = (i, j)$ in which the receiving smart sensor j has at least one channel available that can be assigned to smart sensor i. Whenever a smart sensor i plays a feasible strategy $s_i \in \hat{\mathcal{S}}_i$ while all the remaining smart sensors maintain a vector of strategies $\boldsymbol{s}_{-i}$, we let $G_{s_i, \boldsymbol{s}_{-i}}$ denote the resulting network graph.

In the proposed network formation game, the objective of each smart sensor $i \in \mathcal{M}$ is to select the path that minimizes its overall transmission delay when sending its data to the CAP, either directly or through multi-hop. Hence, given any tree structure $G_{s_i, \boldsymbol{s}_{-i}}$ resulting from the strategy selections of all the smart sensors in $\mathcal{M}$, the cost function of any smart sensor $i \in \mathcal{M}$ which selected a feasible strategy $s_i = (i, j) \in \hat{\mathcal{S}}_i$ having a corresponding path $q_i = \{i_1, \ldots, i_L\}$, with $i_1 = i$, $i_2 = j$ and i_L being the CAP, is captured by the total delay experienced by i which is given by:

$$c_i(G_{s_i, \boldsymbol{s}_{-i}}) = \tau_{i, s_i} = \sum_{(i_l, i_{l+1}) \in q_i} \tau_{i_l, i_{l+1}}, \tag{13}$$

where $\tau_{i_l, i_{l+1}}$ is the delay experienced during the transmission from smart sensor i_l to smart sensor i_{l+1} which can be given by:

$$\tau_{i_l, i_{l+1}} = \frac{R \cdot L_i}{C^k_{i_l, i_{l+1}}}, \tag{14}$$

where L_i is the number of packets of R bits that i needs to transmit and $C^k_{i_l, i_{l+1}}$ is the capacity for the narrowband PLC transmission between i_l and i_{l+1} over channel $k \in \mathcal{K}_{i_{l+1}}$. The capacity $C^k_{i_l, i_{l+1}}$ is computed using the method developed in Section 2 for the direct transmission to the CAP, i.e., using (2) - (4). Note that, whenever $G_{s_i, \boldsymbol{s}_{-i}}$ is a star network, i.e., all smart sensors are connected directly to the CAP, (13) reduces to (12).

Hence, in the proposed network formation game, the objective of each smart sensor is to interact with its neighbors in order to identify a strategy that can minimize its cost function in (13). These interactions are essentially non-cooperative as each smart sensor is selfish, i.e., interested in optimizing its individual cost as per (13). In this game, finding a suitable path to the CAP is a challenging task for each smart sensor, given the capacity limitations of narrowband PLC as well as the limited number of connections that a smart sensor can actually serve as highlighted in Property 1. Having formulated a network formation game among the smart sensors, our next step is to develop an algorithm that can model the interactions among the smart sensors that seek to form the network tree structure for multi-hop narrowband PLC transmission.

4 Distributed Network Formation Algorithm

Before discussing the details of the algorithm, we highlight that, for any developed algorithm, the resulting network structure will always be a *connected* graph as follows:

Property 2. Any network graph resulting from a network formation algorithm applied to the smart sensors game formulated in Section 3 is a *connected* tree structure rooted at the CAP, as long as $M \leq K_{\text{CAP}}$.

Proof. Consider any network graph G in which there exists a certain smart sensor i that is disconnected from the CAP, i.e., no path of transmission (direct or multi-hop) exists between i and the CAP. In this case, the disconnected smart sensor i will experience an infinite delay as its data is not being transmitted, and, thus, its cost in (13) is maximized. As a result, no smart sensor has an incentive to disconnect from the CAP since such a disconnection will drastically increase its delay. Hence, as long as each smart sensor can always connect to the CAP, i.e., $M \leq K_{\text{CAP}}$, then any network graph G formed for the proposed game is a connected tree structure rooted at the CAP.

A direct result of this property is that any smart sensor that is unable to connect to other smart sensors for performing multi-hop PLC will eventually use a direct transmission channel to the CAP, as long as such a channel exists, i.e., $M \leq K_{\text{CAP}}$ (which is an assumption maintained throughout this paper). In this regard, we consider that the initial starting point for our network formation game is the star network in which all smart sensors are connected directly to the CAP, prior to interacting for further network formation decisions.

For any smart sensor $i \in \mathcal{M}$, given the set of feasible strategies $\hat{\mathcal{S}}_i$, we define the *best response* strategy as follows [15].

Definition 3. A strategy $s_i^* \in \hat{\mathcal{S}}_i$ is a *best response* for any smart sensor $i \in \mathcal{M}$ if $c_i(G_{s_i^*,\boldsymbol{s}_{-i}}) \leq c_i(G_{s_i,\boldsymbol{s}_{-i}}),\ \forall s_i \in \hat{\mathcal{S}}_i$. Thus, the best response for smart sensor i is to select the *feasible* link that minimizes its cost given that the other smart sensors maintain their vector of feasible strategies $\boldsymbol{s}_{-i}$.

Using the best responses of the smart sensors, we can develop a distributed network formation algorithm. To do so, we consider that the smart sensors are myopic, in the sense that each smart sensor seeks to reduce its delay by observing only the *current* *state* of the network without taking into account any potential future evolutions of the network graph. Developing algorithms for myopic network formation is a challenging task that has been receiving a significant attention in game theoretical research (e.g., see [13, 15, 16] and references therein). The challenging aspect of this problem stems from the fact that one deals with discrete strategy sets (i.e., forming links) and with the formation of network graphs in which adding or removing a single link can affect the overall network performance. The existing game theoretical literature on network formation games studies various myopic algorithms for different game models with directed and undirected graphs [13, 15, 16]. For our proposed smart sensors network formation algorithm, we construct an algorithm that is based on some of the models in [13] and [15], but modified to accommodate the specifics of the narrowband PLC multi-hop game. Hence, we define an algorithm where each round is mainly composed of three stages: a network discovery stage, a myopic network formation stage and a multi-hop PLC transmission phase.

Initially, the smart sensors start by using direct transmission within a star network. During the first stage of the proposed algorithm, the smart sensors attempt to discover some of their neighboring nodes, either by doing some monitoring of the communication in the star network or by using information downloaded from the network operator itself. Once each smart sensor obtains some information on the current nodes within the initial network, it can start with the second stage of the algorithm in which the main goal is to form the multi-hop tree structure. During the myopic network formation stage, the smart sensors perform pairwise negotiations (e.g., using some kind of dedicated PLC control channel), sequentially, in order to assess potential network formation decisions. In this stage, we consider that the smart sensors can make their decisions in a sequential, yet arbitrary order. In practice, this order can be dictated by which smart sensor requests first to form its link. Thus, in the myopic network formation stage, each smart sensor i selects a certain feasible strategy from its space $\hat{\mathcal{S}}_i$ so as to minimize its cost in (13). Each *iteration* in the network formation stage of the algorithm consists of a single sequence of plays during which all M smart sensors make their strategy choices to myopically react to the choices of the other smart sensors. The myopic network formation phase can consist of one or more iterations. In every iteration t, during its turn, each smart sensor i chooses to play its best response $s_i^* \in \hat{\mathcal{S}}_i$ in order to minimize its cost at each round given the current network graph resulting from the strategies of the other smart sensors. The best response of each smart sensor is a replace operation as the smart sensor disconnects its current link to the CAP while replacing it with another link that minimizes its cost (if such a link is available). Hence, the proposed network formation stage is based on the iterative feasible best responses of the smart sensors. When it converges, the network formation stage is guaranteed to reach a network in which no smart sensor can reduce its delay by changing its current link, i.e., a Nash network, defined as follows for the studied game [15]:

Definition 4. A network graph $G(\mathcal{M}, \mathcal{E})$ in which no smart sensor i can reduce its cost by unilaterally changing its feasible strategy $s_i \in \hat{\mathcal{S}}_i$ is a *Nash network*.

A Nash network is simply the concept of a Nash equilibrium applied to a network formation game. In the proposed game, a Nash network would, thus, be a network

Table 1. Proposed network formation algorithm

Initial State

The starting network is a star network in which each smart sensor is connected directly to the CAP.

The proposed algorithm consists of three stages

Stage 1 - Network Discovery:

Each smart sensor monitors the transmissions in the star network. Given the monitoring results and, possibly, assistance from the operator, each sensor gathers information on the other nodes.

Stage 2 - Myopic Network Formation:

repeat

In an arbitrary but sequential order, the smart sensors perform network formation.

a) In every iteration t of Stage 2, each smart sensor i plays its feasible best response $s_i^* \in \hat{\mathcal{S}}_i$, while minimizing its cost.

b) The best response s_i^* of each smart sensor is a *replace* operation using which a smart sensor i splits from its current parent smart sensor and replaces it with a new smart sensor that improves its cost.

until convergence to a final Nash tree G_{final}.

Stage 3 - Multi-hop PLC Transmission:

During this phase, data transmission from the smart sensors occurs using the assigned channels and hops within the formed network tree structure G_{final}.

where no smart sensor can improve its utility, by unilaterally changing its current link, given the current strategies of all other smart sensors.

Once a Nash network G_{final} forms, the last stage of the algorithm begins. This stage represents the actual data transmission phase, whereby the smart sensor can transmit their data using multi-hop PLC communication over the existing tree architecture G_{final}. A summary of the proposed algorithm is given in Table 1.

The proposed algorithm can be implemented in a distributed way within any network requiring PLC communication such as emerging smart grid networks. In essence, the smart sensors (e.g., meters) can perform the algorithm of Table 1 with little reliance on the CAP or other centralized control centers. For instance, the only role that may be required from the CAP is to provide the smart meters with some assistance in the network discovery phase, i.e., Stage 1 of the algorithm in Table 1. Once the sensors are aware of their environment, within every iteration t, during its turn, each smart sensor can engage in pairwise negotiations with the discovered nodes in order to find its feasible best response from the set of feasible strategies. During this process, the smart sensors need to only communicate in pairs and assess their potential cost function as per (13). Subsequently, each smart sensor can select its myopic best response, leading to a new iteration, until reaching the final Nash network. The worst case complexity for selecting the feasible best response (finding a preferred partner) for any smart sensor i is $O(M)$ where M is the total number of smart sensors. In practice, the complexity is much smaller as the smart sensors do not negotiate with the smart sensors that are connected to them, nor with the smart sensors that are too far away.

5 Simulation Results and Analysis

We consider a smart grid network deployed within a square area of 4 km $\times$ 4 km in which the control center (or a repeater) is placed at the middle. The smart sensors are deployed randomly inside this area and utilize narrowband PLC for transmission. Using typical parameters from [1] and [7], we choose a bandwidth of $W = 12.5$ kHz for every channel, we set the transmit power of any smart sensor $i \in \mathcal{M}$ to $P_i = 25$ W, we set the attenuation level to $\kappa = 0.007$ and we set $\gamma = -8.64$. The values of γ and κ are chosen within the typical best and worst cases [7]. For frequencies, we consider the narrowband PLC frequency range from 10 kHz to 235 kHz. Using this range, the CAP is able to offer 18 channels and, hence, it can accommodate up to $M = 18$ smart sensors. Each smart sensor can offer up to 5 channels for the nodes wishing to connect to it. The number of channels are picked randomly from integers between 1 and 5. All channels offered by the smart sensors are within the range of 10 kHz to 235 kHz. The packet size is set to $R = 2048$ bits and the number of packets is set to $L_i = 1$ packet for all $i \in \mathcal{M}$.

In Figure 2, we show a snapshot of a tree structure resulting from the proposed algorithm for a network with $M = 10$ randomly deployed smart sensors. This figure shows how a tree structure can form as a result of the distributed decisions of the smart sensors. In this snapshot, we can see that the smart sensors select their path not only based on distance but also on the offered channels. For instance, although smart sensor 6 is closer to smart sensor 4 than to smart sensor 5, it prefers to connect to 5. This is due to the fact that smart sensor 5 offers smart sensor 6 a communication over a channel k such that $F_{k,1} = 137.5$ kHz while smart sensor 4 offers a channel k such that $F_{k,1} = 37.5$ kHz. Hence, smart sensor 6 prefers to operate at a higher frequency as this ensures a higher capacity, and, eventually a better delay. Further, due to Property 1, smart sensor 7 decides to connect smart sensor 8 instead of smart sensor 10 since the latter can offer only two frequencies and has already assigned these frequencies to smart sensors 2 and 8. The strategies of all other nodes in Figure 2 are chosen by the smart sensors using a somewhat similar reasoning. Moreover, the network in Figure 2 is a Nash network as no smart sensor has an incentive to unilaterally change its current link. For example, consider smart sensor 6 whose feasible strategies are all other smart sensors and the CAP. If smart sensor 6 decides to disconnect from smart sensor 5 and connect to:

- Smart sensors $1, 2, 7, 8, 9$, or 10 its delay increase from 102.5 ms to about 382 seconds.
- Smart sensor 3, its delay increase from 10 ms to 153 ms.
- Smart sensor 4, its delay increase from 102.5 ms to 120.1 ms.

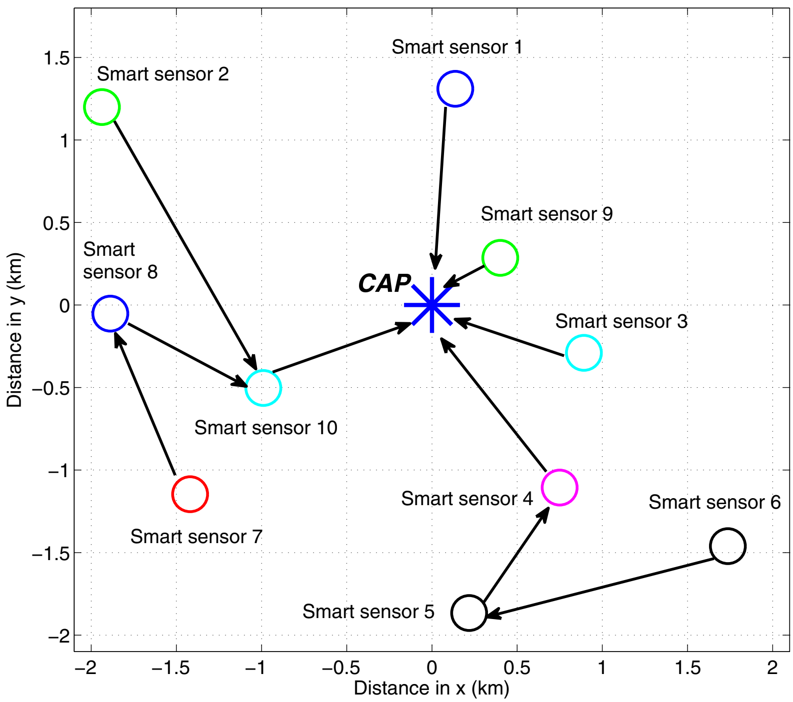


Fig. 2. Snapshot of a tree graph formed using the proposed algorithm for a network with $M = 10$ randomly deployed smart sensors

Hence, clearly, smart sensor 6 has no incentive to change its current strategy. Similar results can be seen for all other smart sensors in the Nash network of Figure 2. In a nutshell, Figure 2 shows how the smart sensors can self-organize into a Nash network while optimizing their delay given the offered frequencies and available potential partners.

Figure 3 shows the average achieved delay per smart sensor as the number of smart sensors M varies. The results are averaged over the random positions and channel selections of the smart sensors. The performance of the proposed network formation algorithm is compared with the direct transmission performance, i.e., the star network, as well as with a nearest neighbor algorithm in which each smart sensor selects the closest partner (in terms of distance) to connect to. In this figure, we can see that, as the number M of smart sensors in the network increases, the average achieved delay per smart sensor increases for the star network and the nearest neighbor algorithm. For the star network, this increase is due to the fact that, as the network size grows, it becomes more likely to have smart sensors that are far away from the CAP, and, thus achieving a poor capacity. Moreover, for the star network, the increase in the network size, constrains the channels that the CAP can offer. Hence, a larger star network will encompass smart sensors that are using channels in the lower part of the frequency band, and, thus, achieving a lower capacity as per (2)-(4).

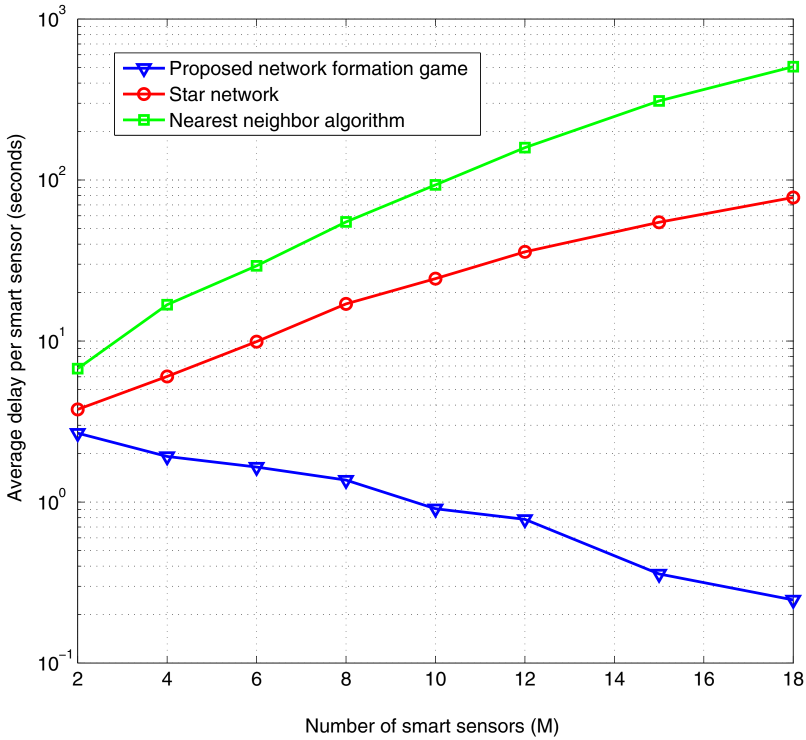


Fig. 3. Performance assessment of the proposed distributed network formation algorithm as the number of smart sensors M in the network (average over random positions and random channel choices of the smart sensors)

In the case of the nearest neighbor algorithm, the increase in the average delay with increasing M is due to the fact that, as more smart sensors are deployed in the network, the average delay resulting from a nearest neighbor-based multi-hop transmission increases due to the additional traffic. Moreover, for the nearest neighbor case, a smart sensor makes its selection solely based on distance and, hence, may connect to another smart sensor that is offering channels in the lower part of the band, hence, decreasing the potential capacity that can be achieved. Consequently, as the nearest neighbor algorithm yields, on the average, longer transmission paths with little capacity gains, as seen in Figure 3, its achieved average delays are larger than the star network.

In contrast, Figure 3 shows that, for the proposed network formation game, the average delay per smart sensor decreases with the network size. This result is interpreted by the fact that as the network size M grows, each smart sensor has a larger pool of partners from whom to select. Moreover, the increase in the number of smart sensors is accompanied by an increase in the number of possible transmission paths and channels that can be used. As a result, as more smart sensors are deployed, each smart sensor is able to exploit further the benefits of the proposed network formation algorithm in order to minimize its delay. In this respect, Figure 3 demonstrates that, at all network sizes M, the proposed network formation algorithm yields significant reductions of at least 28.7% and 60.2% in terms of the average delay per smart sensor, relative to the star network and the nearest neighbor algorithm, respectively.

6 Conclusions

In this paper, we have introduced a novel model for multi-hop communications in cyber-physical networks (such as the smart grid) that are bound to adopt narrowband power line communication for data transmission. In this respect, we have formulated a network formation game among a number of smart sensors (e.g., smart meters) that seek to transmit their data, using multi-hop, to a common control center or repeater. We have shown that the outcome of the formulated game is a tree structure that interconnects the smart sensors. To form this tree structure, we have developed a distributed myopic algorithm based on game theory. Using the proposed network formation algorithm, each smart sensor is able to decide, in a distributed manner, on its preferred path for data transmission in such way as to optimize a cost function that captures the overall transmission delay. We have shown that, using the proposed algorithm, the smart sensors self-organize into a Nash network in which no node has an incentive to change its current data transmission path. Simulation results have demonstrated that the proposed algorithm presents a significant advantage in terms of reducing average achieved delay per smart meter of at least 28.7% and 60.2%, relative to the star network and a nearest neighbor algorithm, respectively. Future extensions to this work can consider interference over the narrowband power line communication channel, advanced channel scheduling techniques, as well as network formation algorithms that can adapt to rapidly changing environments.

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