Dynamic Spectrum Negotiation with Asymmetric Information

Yang Yan, Jianwei Huang, Xiaofeng Zhong, and Jing Wang

[1] Department of Electronic Engineering, Tsinghua University, Beijing, China
[2] Department of Information Engineering, CUHK, Hong Kong, China
yanyang07@mails.tsinghua.edu.cn,jwhuang@ie.cuhk.edu.hk
{zhongxf,wangj}@tsinghua.edu.cn

Abstract. Spectrum scarcity is becoming a serious issue due to the rapid development of wireless communication technology. Dynamic spectrum sharing can effectively improve the spectrum usage by allowing secondary unlicensed users (SUs) to dynamically and opportunistically share the spectrum with primary licensed users (PUs). In this paper, we investigate a spectrum negotiation mechanism under incomplete information in a dynamic environment, where both the PU and the SU can obtain rate increases through cooperative communications. Specifically, an SU relays traffics for a PU in exchange for dedicated transmission time for the SU's own communication needs. We model the bargaining process as dynamic Bayesian games and characterize the Perfect Bayesian Equilibria under different system model parameters. Analysis and numerical results indicate that both PU and SU obtain performance improvements compared with no cooperation, and thus achieve a win-win situation via the spectrum negotiation.

Keywords: dynamic spectrum negotiation, incomplete information, game theory, perfect bayesian equilibrium.

1 Introduction

Wireless spectrum is generally regarded as a scarce resource, and has been tightly controlled through licensing. Recent field measurements showed that, however, most licensed spectrum bands are heavily underutilized even in densely populated urban areas [1]. This indicates that the current fixed license-based spectrum allocation policy is not efficient. As one promising technology to address this issue, cognitive radio technology [2] enables efficient dynamic spectrum sharing among secondary *unlicensed* users (SUs) and primary *licensed* users (PUs). One way to achieve dynamic spectrum sharing is through market-driven spectrum negotiation/bargaining[1], which leads to a win-win situation by improving the payoffs of both PUs and SUs. In spectrum negotiation, PUs and SUs jointly decide (i) how PUs allocate resource to SUs, and (ii) how SUs compensate PUs either by offering monetary payments or providing performance improvements.

[1] In this paper, we use "negotiation" and "bargaining" interchangeably.

R. Jain and R. Kannan (Eds.): GameNets 2011, LNICST 75, pp. 516–529, 2012.

There are two key challenges for implementing spectrum negotiation: *asymmetric information* and *dynamics*. Spectrum negotiation often involves incomplete and asymmetric information, such as transmission power, energy cost, and channel state information of the users. This often complicates the decisions. Moreover, the negotiation can involve multiple periods, during which PUs and SUs need to update beliefs about its opponent's information and adjust their own strategies accordingly. In this paper, we present a first step on understanding how to address the above two issues for spectrum negotiation.

There are two types of spectrum negotiation mechanisms: *monetary-exchange* and *resource-exchange*. In a monetary-exchange mechanism, SUs provide (virtual) money to the PUs in exchange of spectrum resource. Reference [3] studied a sequential second-price auction mechanism for sharing a wireless resource (bandwidth or power) among competing transmitters, where a spectrum manager is responsible for allocating spectrum among non-cooperative secondary users. Reference [4] addressed spectrum pricing in a cognitive radio network, where multiple PUs compete to offer spectrum access opportunities to SUs. Reference [5] further investigated price competition with multiple PUs and multiple SUs by taking into account both bandwidth uncertainty and spatial reuse. The monetary-exchange model is suitable when PUs have temporarily idle resource. However, when PUs' own transmission demands cannot be satisfied (e.g., when PUs suffer from poor channel conditions), there will be no resource left for allocating to SUs. In this case, the resource-exchange model can be helpful.

In a resource-exchange model, SUs provide communication resources (e.g., the transmission power) to assist PUs' transmissions in exchange for spectrum usage. In this paper, we consider an resource-exchange model, where PU has a poor channel link to its receiver (e.g., base station). Our study is motivated by [6], where the remuneration of spectrum negotiation is realized by the cooperative transmission of SUs. In [6], a single PU knows utility functions of the SUs, and thus there is no asymmetric or incomplete information. The PU optimizes the resource splitting based on either instantaneous or long-term network channel state information of the whole system. Due to complete information, SUs simply follow the PU's spectrum leasing decision and there is no bargaining. Reference [7] considered a similar cooperative spectrum sharing scheme between multiple PUs and multiple SUs, again based on complete information. The only recent publication dealing with incomplete information of spectrum negotiation is [8], which proposed a contract mechanism between one PU and multiple SUs in a static network environment.

Compared with [6] and [7], we study a *dynamic* spectrum bargaining process with *incomplete information*, where both the PU and the SU have the negotiation power. The incomplete information model better captures the reality of wireless communications, but so far has received little attention in the literature due to its high analysis complexity. The most related research result in terms of methodology is the incomplete information game of wireless power control [9], where the authors studied a completely different application scenario (not related to cognitive radio).

Our paper is the first one that jointly considers incomplete information and dynamic bargaining in spectrum negotiation. The key results and contributions are summarized as follows:

- *Introduction of asymmetric information:* We model the realistic situation where the PU does not know the complete information of the SU. Specifically, we assume that PU is unaware of the exact value of SU's energy cost C, but has a belief about C's distribution. We compute the equilibrium behaviors for both PU and SU with such asymmetric information.
- *Dynamic negotiation process:* We investigate a multi-stage bargaining model, where both the PU and the SU have negotiation power. As the bargaining proceeds, PU and SU must adjust their beliefs about incomplete information and strategies accordingly. This can be modeled as a dynamic Bayesian game which is challenging to analyze. We explicitly compute and characterize multiple equilibria for a two-stage bargaining game. We demonstrate the existence of two types of equilibria, and show that one is more Pareto efficient than the other in the expected sense.

The rest of the paper is organized as follows. We introduce the system model and problem formulation in Section 2. In Section 3, we analyze the one-stage bargaining model, which serves as the basis for the two-stage bargaining model in Section 4. In Section 5, we discuss various insights obtained from the equilibrium analysis through numerical studies. We conclude in Section 6.

2 PU-SU Negotiation and Cooperation Model

Throughout this paper, we denote *primary user* and *secondary user* as PU and SU, respectively. We consider a time-slotted system, where one PU negotiates the spectrum allocation with one SU. Figure 1 introduces the key notations of the system model. Here, TP and RP represent the primary transmitter and receiver, and TS and RS represent the secondary transmitter and receiver. Let h_p, h_s, h_{ps} and h_{sp} denote the channel gains of the links between TP and RP, TS and RS, TP and TS, and TS and RP, respectively. We consider a block fading channel model, where the channel gains are fixed during one time slot and can change across time slots. We further assume that both PU and SU know the channel gains of all the links through a proper feedback mechanism[2]. The fixed transmission powers of the PU and SU are denoted as P_t and P_s, respectively. Our model can describe the situation where a cellular subscriber (PU) wants to send traffic to a base station which is far away, and a laptop user (SU) can help to relay the traffic for the cellular subscriber (see Fig. 1).

This stylized model enables us to analytically study the challenging issues of asymmetric information and dynamics. Such one-to-one model has been common in the economics literatures (e.g., [12–14]). In contrast, some recent economics

[2] In this paper, we focus on the study of incomplete information related to energy cost. The study of incomplete information related to channel conditions can be studied based a similar methodology and will be considered in the future work.

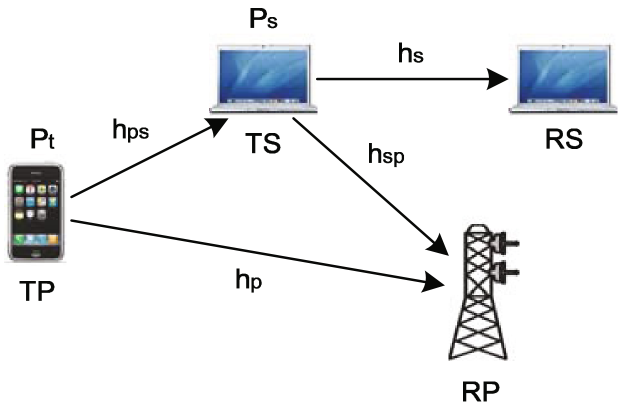

Fig. 1. PU-SU Cooperation Model

literatures (e.g., [15]) discussed more complicated models (e.g., one-to-many bargaining model) in a static or dynamic setting without information incompleteness. However, our focus in this paper is on dynamics in the bargaining with incomplete information, in which case the negotiation among multiple users is difficult to analyze even in the economics literature. We believe that a thorough understanding of the one-to-one model can help us to better tackle the one-to-many and many-to-many spectrum negotiation scenarios in the future.

Next we describe how the system works. Without spectrum negotiation, the PU directly transmits from TP to RP and achieves a data rate R_{dir} (determined by transmission power P_t and direct channel gain h_p). In that case, the SU cannot transmit and achieves a zero rate. However, spectrum negotiation becomes attractive to the PU if the direct channel h_p is poor but the relay channels h_{ps} and h_{sp} are good. In this case, the PU can increase its own data rate by using the SU transmitter TS as a relay. To attract the help from SU, the PU offers the SU α fraction of the transmission time for SU's own transmission (from TS to RS). Apparently, a larger α means a higher data for SU but a lower data rate for PU (due to the reduction of transmission time). PU and SU will bargain with each other to determine the value of α that is acceptable by both sides. Figure 2 illustrates three possibilities of the bargaining result. Without loss of generality, we normalize the time slot length T to 1, and denote $\tau \ll 1$ as the time "wasted" due to bargaining.

- Figure 2(a): At the beginning of the time slot, PU can choose to directly transmit during the whole time slot without bargaining, if it believes that SU cannot provide a performance improvement.
- Figure 2(b): If PU decides to bargain, then the unit time slot is divided into two phases: bargaining and transmission. During the bargaining phase, PU and SU negotiate on the fraction of the remaining time, α, to be allocated for the SU's own transmission. If no agreement is reached, PU proceeds with direct transmission for the remaining time without the cooperation of SU.

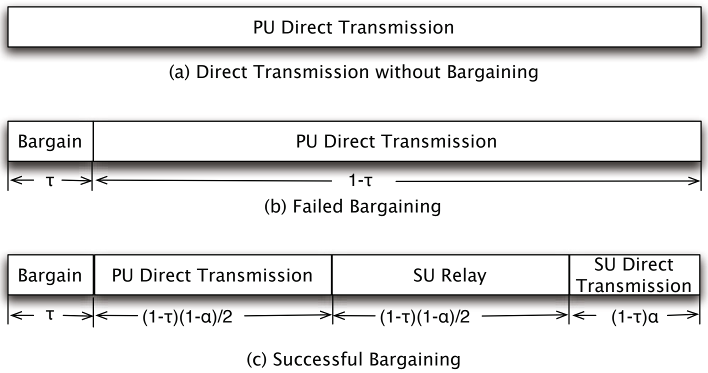

Fig. 2. Three Possibilities in Negotiation Process

– Figure 2(c): If an agreement is reached in the bargaining phase, then PU and SU transmit in an amplified-and-forward relay mode. The transmission can be further divided into three time periods. In the first period, primary transmitter TP broadcasts the data, and both primary receiver RP and secondary transmitter TS receive the information. In the second period, the secondary transmitter TS amplifies the received signal and forwards it to the primary receiver RP. The primary receiver RP combines the signal received from the first and second periods based on the maximum ratio combining scheme and decodes the signal. In the third period, SU utilizes the remaining third period (i.e., α fraction of the $(1-\tau)$ transmission time) for its own transmission.

In this paper, we assume that SU is an *energy-constrained* device (**e.g., wireless sensor or mobile device**). Parameter C is the SU's energy cost, which is related to SU's current battery status. We further assume that the precise value of C is known by SU but not by PU[3], thus the information is asymmetric. However, PU knows the distribution of C, and hence it is incomplete information. To simplify the analysis, we assume that C follows a uniform distribution[4].

Let us consider how both users make their decisions. SU chooses to accept or reject the offer α to maximize its utility function U_s, which is the difference between the achievable data rate and the energy cost. By taking the SU's response into consideration, PU chooses the optimal offer α to maximize its utility function U_p, which is the *expected data rate increase*. Details of U_s and U_p will be given shortly.

Finally, we note that the bargaining period with length τ in Fig. 2(b) and (c) can include multiple stages, i.e., users can bargain multiple times within the same

[3] If PU knows the value of C, it can simply calculate α so that SU gets zero payoff. This is a special case of the general model discussed here.

[4] The analysis for other distributions will be technically more involved but offers essentially the same engineering insights.

time slot. As a preliminary result, we first study the one-stage bargaining case in Section 3. This will help us to study the multi-stage bargaining in Section 4.

3 One-Stage Bargaining Game

In this section, we consider the case where there is *at most* one stage of bargaining in a time slot. The proportion of the time slot after bargaining is $\delta = (1 - \tau) < 1$, where τ is the duration of one-stage bargaining. PU needs to decide (i) whether to bargain, and (ii) the optimal offer α if it decides to bargain. SU should decide whether to accept the offer of α (if the PU offers one). **The SU's utility function** U_s is the difference between the achievable data rate and the energy cost,

$$U_s(\alpha) = \delta \left(\alpha R_s - \frac{1+\alpha}{2} P_s C \right),$$

where $R_s = \log(1 + P_s h_s)$ is the SU's own fixed data rate per unit time between its own transmitter and receiver (TS and RS). A good way to understand the utility function is to think C as data rate per *watt* the SU can get if it does not relay for PU. Therefore, $U_s(\alpha)$ is SU's *data rate increase* by accepting the offer α. Given PU's offer α, the optimal decision for SU is obvious: accept α if and only if $U_s(\alpha) > 0$.

Now let us consider PU's rate increase maximization problem. Without bargaining, the PU can always achieve a rate of R_{dir} through direct transmission as in Fig. 2(a). In that case, its *rate increase* is zero. Now let us calculate how much PU can gain by bargaining with SU. Without any prior knowledge, PU assumes that SU's energy cost C follows a uniform distribution in $[K_1, K_2]$, where $0 \le K_1 < K_2$. Due to such incomplete information, PU does not know whether the SU will accept or reject a particular choice of α before offering it. If the SU rejects the offer as in Fig. 2(b), PU can only directly transmit in the remaining δ time and achieve a *negative* rate improvement $(\delta R_{dir} - R_{dir}) < 0$. If SU accepts the offer as in Fig. 2(c), PU receives a rate increase of $\delta \frac{1-\alpha}{2} R_r - R_{dir}$, which can be either positive or negative. Here R_{dir} and R_r are the data rates achieved by PU's direct transmission and AF relay [10] respectively, i.e., $R_{dir} = \log(1+P_t h_p)$ and $R_r = \log\left(1 + P_t h_p + \frac{P_t P_s h_{ps} h_{sp}}{P_t h_{ps} + P_s h_{sp} + 1}\right)$. Therefore, if the PU decides to bargain with the SU, it will choose α to maximize the **PU's utility (expected rate increase)** defined as

$$(\delta R_{dir} - R_{dir}) \operatorname{Prob}(U_s(\alpha) \le 0) + \left(\delta \frac{1-\alpha}{2} R_r - R_{dir} \right) \operatorname{Prob}(U_s(\alpha) > 0) \quad (1)$$

where $\operatorname{Prob}(U_s(\alpha) \le 0)$ and $\operatorname{Prob}(U_s(\alpha) > 0)$ are the probabilities for SU to reject and accept offer, respectively. Denote the optimal time fraction that maximizes (1) as α^*. The PU will choose to bargain if and only if $U_p(\alpha^*) \ge 0$. Otherwise, it will simply choose direct transmission as in Fig. 2(a). The optimal choice α^* that maximizes (1) is given in the following theorem:

Theorem 1. *When* $K_1 > R_s/P_s$*,* $U_p(\alpha) = -(1-\delta)R_{dir} < 0$ *for any* $\alpha \in [0,1]$*. When* $K_1 \leq R_s/P_s$*, then*

$$\alpha^* = \min\left(\max\left(\bar{\alpha}_p, \frac{K_1}{2R_s/P_s - K_1}\right), \min\left(\left|\frac{K_2}{2R_s/P_s - K_2}\right|, 1\right)\right) \tag{2}$$

where

$$\bar{\alpha}_p = 2\sqrt{\frac{(R_s/P_s)(R_r - R_{dir})}{R_r(2R_s/P_s - K_1)}} - \frac{1}{2}.$$

The proof of Theorem 1 can be found in our online technical report [16]. When K_1 (the minimum possible value of the SU's energy cost C) is larger than R_s/P_s, the SU will not accept any offer α from the PU. In this case, the PU knows that the bargaining will fail and thus will choose direct transmission. When $K_1 \leq R_s/P_s$, PU will choose the optimal offer α^* in (2) to achieve the best tradeoff of rate increase and performance loss. We want to emphasize again that PU will compare $U_p(\alpha^*)$ with zero and decides whether it is worth trying to bargain or not.

The one-stage bargaining game is a subgame for the multi-stage bargaining game in Section 4, and we will use Theorem 1 in the later analysis.

4 Two-Stage Bargaining Game

Now we return to the dynamic bargaining case, where the bargaining within a time slot can happen over more than one stage. For the ease of illustration, we will focus on the two-stage bargaining case. Similar to the one-stage game, here the utility functions are PU's *expected data rate increase* and SU's *date rate increase*. We assume that PU's belief about SU's energy cost C *at the beginning of stage one of bargaining* is a uniform distribution over $[0, K]$. We denote δ_1 and δ_2 as the proportions of the slot after bargaining in the first and second stage. By setting different values of δ_1 and δ_2, we can model different bargaining overhead. For the two-stage bargaining game, PU's strategy involves (1) whether to bargain at the beginning of the first stage, (2) if yes, what the first stage offer α_1 is, and (3) if SU rejects α_1, what the second stage offer α_2 is.

Figure 3 illustrates the sequential decisions and possible scenarios of this two-stage bargaining game. PU and SU make decisions alternatively at the non-leaf nodes. PU first makes the decision on whether to bargain. If it selects direct transmission (D), the game ends. Otherwise, PU offers α_1 to SU. If SU accepts this offer, then the game ends. If SU rejects the offer, then PU makes a second offer α_2 to SU. Finally, SU either accepts or rejects α_2. The game ends in both cases. Every possible ending of the game is denoted by a black solid square together with the corresponding utilities (*data rate increases*) of PU (upper value) and SU (lower value).

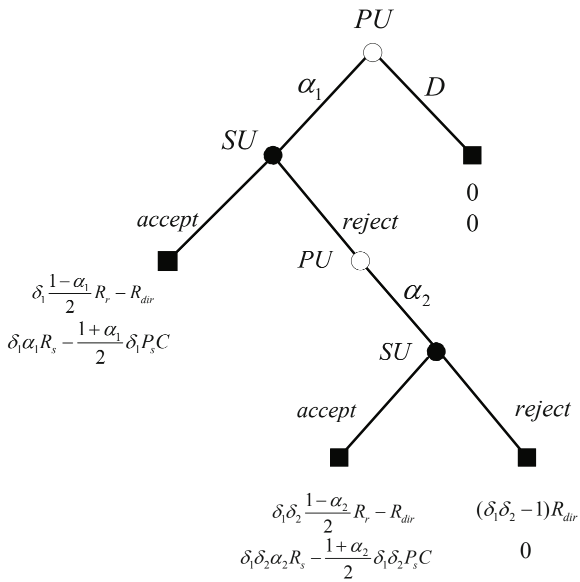

Fig. 3. Game Tree of the Two-stage Bargaining

The two-stage bargaining game is a dynamic Bayesian game (DBG)[5], and is more complex than the one-stage bargaining model for two reasons: (i) The SU may reject the first stage offer α_1 even though its utility is positive, if it believes that the second stage offer α_2 is much better; (ii) The PU needs to update its belief on SU's energy cost C at the end of the first stage by taking the SU's strategic decision process into consideration.

We want to point out the assumption that once the PU decides to bargain, it cannot choose direct transmission before the two-stage process finishes. Such restriction leads to a simplified model. Allowing the PU to choose direct transmission after the first stage will introduce more possibilities and make the analysis much more complicated. On the other hand, our focus of this paper is to consider the impact of dynamic bargaining on the choices of PU and SU. The key insights of PU's belief updates and SU's ideas on the sequential strategy for the second stage are unlikely to change with the more complicated model.

One of the commonly used solution concepts for a DBG is the Perfect Bayesian Equilibrium (PBE), which is a strategy profile and belief system in a dynamic game of incomplete information so that they satisfy the following three requirements [12].

Requirement 1. *The player with the move must have a* ***belief*** *(probability distribution) about incomplete information.*

[5] The precise definition of DBG is beyond the scope of this paper. See [11, 12] for details.

Requirement 2. *Given their beliefs, the players'* ***strategies*** *must be sequentially rational. That is, the action taken by the player must be optimal given the player's belief and the other player's subsequent strategies.*

Requirement 3. *A player's belief is determined by the Bayes' rule and the players' equilibrium strategies.*

Back to our model, the strategies and beliefs of the PU and SU are as follows.

- PU's strategy: whether to bargain at the beginning of the game, the first-stage offer α_1 if decides to bargain, and the second-stage offer $\alpha_2(\alpha_1)$ (i.e., as a function of α_1) after SU rejects α_1.
- PU's belief: $\mu_1(C)$ denotes PU's belief on C at the beginning of the first stage, and $\mu_2(C|\alpha_1)$ denotes PU's *updated* belief about C at the beginning of the second stage after SU rejects α_1.
- SU's strategy: $[\mathcal{A}_1(\alpha_1|C), \mathcal{A}_2(\alpha_2|C, \alpha_1)]$. When its energy cost is C, $\mathcal{A}_1(\alpha_1|C)$ $= 1$ if SU accepts α_1, and $\mathcal{A}_1(\alpha_1|C) = 0$ otherwise. $\mathcal{A}_2(\alpha_2|C, \alpha_1) = 1$ if SU accepts α_2 after rejecting α_1, and $\mathcal{A}_2(\alpha_2|C, \alpha_1) = 0$ otherwise.
- SU's belief: since SU knows C, its belief is a singleton set (i.e., no uncertainty).

Before presenting our PBE analysis on the two-stage bargaining game, we want to discuss more about PBE and the method used in this subsection. First, according to Requirement 3, PBE requires the *consistency* between one player's belief and players' equilibrium strategies. Second, such an equilibrium often cannot be constructed by the traditional *backward induction method* through the game tree, as we need to construct a subgame-perfect Nash equilibrium[6]. Thus, the analysis of PBE is typically problem specific.

We will start the PBE analysis with the second stage. Since this is the last stage of the game, the analysis is similar as the one-stage game in Section III. The key difference is that PU does not have the option of direct transmission at the beginning of the second stage, and it needs to optimize the choice of α_2.

Specifically, we can apply Requirement 2 to solve SU's strategy $\mathcal{A}_2(\alpha_2|C, \alpha_1)$. Since this is the *last* move of the game, the optimal strategy for SU in the second stage is to accept α_2 if and only if the SU's utility $\delta_1\delta_2\alpha_2 R_s - \frac{1+\alpha_2}{2}\delta_1\delta_2 P_s C > 0$. Such decision is independent of α_1.

Given SU's optimal strategy in the second stage $\mathcal{A}_2(\alpha_2|C, \alpha_1)$, we can apply Requirement 2 to compute the PU's optimal strategy in the second stage. The PU will calculate the optimal α_2 that maximizes PU's expected utility function U_p, given PU's *updated* belief $\mu_2(C|\alpha_1)$ and SU's *subsequent* strategy $\mathcal{A}_2(\alpha_2|C, \alpha_1)$.

[6] Requirement 2 explains that a player's strategy at a given information set is based in part on its belief. However, the player's belief is from the players' strategies higher up the game tree according to Requirement 3. But, Requirement 2 (*sequential rationality*) means that these strategies higher up the game tree are based in part on the players' subsequent strategies, including the strategies at the given information set. This *circularity* implies that a single pass backward induction through the game tree typically will not suffice to compute a PBE. See [12] for details.

The tricky part is how to compute the belief $\mu_2(C|\alpha_1)$, which depends on the interaction in the first stage (Requirement 3). For it, we need to understand the SU's equilibrium strategy in the first stage in order to update PU's belief in the second stage.

Consider arbitrary first and second stage offers α_1 and α_2[7]. We further assume that K is reasonably large ($K > \frac{R_s}{P_s}$). We further assume that K (the upper bound of C) is reasonably large ($K > \frac{R_s}{P_s}$). Define

$$C^*(\alpha_1, \alpha_2) = \frac{2R_s(\alpha_1 - \delta_2\alpha_2)}{P_s((1+\alpha_1) - \delta_2(1+\alpha_2))}.$$

The following lemma provides SU's equilibrium strategy in the first stage for given α_1 and α_2.

Lemma 1. *SU* rejects α_1 *in the first stage if one of the following is true: (i)* $C \in [\frac{2\alpha_1 R_s}{P_s(1+\alpha_1)}, K]$ *and* $\alpha_1 > \alpha_2$*, (ii)* $C \in [C^*(\alpha_1, \alpha_2), K]$ *and* $\delta_2\alpha_2 < \alpha_1 \leq \alpha_2$*, or (iii)* $C \in [0, K]$ *and* $\alpha_1 \leq \delta_2\alpha_2$*. SU* accepts α_1 *otherwise.*

The proof of Lemma 1 can be found in [16]. With Lemma 1, we can characterize two types of PBEs of the two-stage bargaining game. We want to emphasize that PBEs in Theorem 2 and 3 are just *possible* ones. Whether they exist in a particular game requires further investigation. The proofs of Theorem 2 and 3 can be found in [16]. The physical meanings of the PBE will be further discussed in Section V.

4.1 Type I PBE

Let us look at the first type of PBE, where α_2^* is slight better than α_1^* (i.e., $\delta_2\alpha_2^* < \alpha_1^* \leq \alpha_2^*$). An SU with a small energy cost will accept α_1 in the first stage, so that it can start to benefit immediately. An SU with a medium or large energy cost will wait for the second stage hoping for a better offer. In the second stage, only an SU with a medium energy cost will accept α_2, and an SU with a high energy cost has to reject α_2.

Note that the SU does not know the value of α_2 in the first stage, and thus it needs to make the above decisions by *anticipating* the value of α_2. The PU needs to decide α_1 and α_2 by taking the SU's anticipation into consideration. A PBE exists if the SU's anticipation is *consistent* with what the PU offers. The first type of PBE is summarized in Theorem 2.

Theorem 2. *Given* α_1*, the beliefs and strategies for PU and SU are:*

– $\alpha_2^*(\alpha_1)$*: PU's second stage offer is the solution of the following fixed point equation of* α_2*:*

$$\alpha_2 = \min\left(\max\left(\alpha_p^*(K_1(\alpha_1,\alpha_2)), \frac{K_1(\alpha_1,\alpha_2)}{2R_s/P_s - K_1(\alpha_1,\alpha_2)}\right), \min\left(\left|\frac{K}{2R_s/P_s - K}\right|, 1\right)\right),$$

[7] Later on we will show that the optimal value of α_2 will depend on α_1.

where

$$\alpha_p^*(K_1(\alpha_1,\alpha_2)) = 2\sqrt{\frac{R_s(R_r-R_{dir})}{P_sR_r(2R_s/P_s-K_1(\alpha_1,\alpha_2))}}-\frac{1}{2},$$

and

$$K_1(\alpha_1,\alpha_2)=\frac{2R_s(\alpha_1-\delta_2\alpha_2)}{P_s((1+\alpha_1)-\delta_2(1+\alpha_2))}.$$

- $\mu_1(C)$: *PU believes* C *is uniformly distributed in* $[0,K]$.
- $\mu_2(C|\alpha_1)$: *PU updates its belief on* C *as uniformly distributed in* $[C^*(\alpha_1, \alpha_2^*(\alpha_1)), K]$.
- $\mathcal{A}_1(\alpha_1|C)$: *SU rejects* α_1 *if* $C \in [C^*(\alpha_1,\alpha_2^*(\alpha_1)),K]$.
- $\mathcal{A}_2(\alpha_2|C,\alpha_1)$: *SU accepts* α_2 *if and only if* $\delta_1\delta_2\alpha_2R_s-\frac{1+\alpha_2}{2}\delta_1\delta_2P_sC>0$.

Finally, PU computes first stage offer α_1^* *to maximize*

$$\left(\delta_1\frac{1-\alpha_1}{2}R_r-R_{dir}\right)P_1+\left(\delta_1\delta_2\frac{1-\alpha_2^*(\alpha_1)}{2}R_r-R_{dir}\right)P_2+(\delta_1\delta_2-1)R_{dir}P_3,$$

where $P_1=\frac{K_1(\alpha_1)}{K}$, $P_2=\frac{\frac{2\alpha_2}{1+\alpha_2}\frac{R_s}{P_s}-K_1(\alpha_1)}{K}$, *and* $P_3=\frac{K-\frac{2\alpha_2}{1+\alpha_2}\frac{R_s}{P_s}}{K}$. *PU chooses direct transmission if* $U_p(\alpha_1^*)<0$. *The above beliefs and strategies constitute a PBE if and only if* $\delta_2\alpha_2^*(\alpha_1^*)<\alpha_1^*\le\alpha_2^*$.

4.2 Type II PBE

Next we examine the second type of PBE, where α_2^* is much larger than α_1^* (i.e., $\alpha_1^*\le\delta_2\alpha_2^*$). If an SU believes that the second stage offer α_2 is much better than the first stage offer α_1 (even after considering time discount δ_2), it will definitely reject the first stage offer. In the second stage, an SU with a small C will accept the offer and an SU with a high C will reject the offer. This is summarized in Theorem 3.

Theorem 3. *The following beliefs and strategies constitute infinitely many PBEs.*

- α_2^*: *PU's second stage offer equals to a constant independent of* α_1,

$$\min\left(\max\left(\alpha_p^*,0\right),\min\left(\left|\frac{K}{2R_s/P_s-K}\right|,1\right)\right),$$

where

$$\alpha_p^*=\sqrt{\frac{2(R_r-R_{dir})}{R_r}}-\frac{1}{2}.$$

- α_1^*: *any value satisfying* $\alpha_1^*\le\delta_2\alpha_2^*$.
- $\mu_1(C)=\mu_2(C|\alpha_1)$: *PU believes* C *is uniformly distributed in* $[0,K]$ *in the first and second stage.*
- $\mathcal{A}_1(\alpha_1|C)$: *SU never accepts* α_1.
- $\mathcal{A}_2(\alpha_2|C,\alpha_1)$: *SU accepts* α_2 *if and only if* $\delta_1\delta_2\alpha_2R_s-\frac{1+\alpha_2}{2}\delta_1\delta_2P_sC>0$.

5 Numerical Results

In this section, we simulate and compare the PU's and SU's data rate increases at the equilibria in the two-stage game. We set transmission power $P_t = P_s = 1$, SU's maximum energy cost $K = 1.5R_s/P_s$, and $\delta_1 = \delta_2 = \delta = 0.8$. The PU's direct transmission rate $R_{dir} = 1$ and rate achieved via relay $R_r = 500$. In this case, SU's cooperative transmission can bring a significant improvement to PU's data rate. The SU's own data rate $R_s = 10$.

Figure 4 and 5 show the equilibrium data rate increases of PU with different energy cost C. Two figures correspond to two different types of PEBs of the same game. The *mean payoffs* (dotted curves) denote the average value over all possible values of C. Since both PU and SU obtain positive mean payoffs, spectrum leasing leads to a win-win situation.

Figure 4 corresponds to the PBE in Theorem 2. We can classify PU's payoff (circle line) into three regions depending on the SU's energy cost C: (i) Small C (e.g., $C \leq 6$ in Fig. 4): SU accepts α_1 and thus PU and SU receive significant data rate increases. (ii) Medium C (e.g., $C \in [7, 9]$ in Fig. 4): SU rejects α_1 but accepts α_2. Compared to the small C case, PU's payoff dramatically decreases, since α_2 is larger than α_1 and more time is wasted in the bargaining. SU's payoff decreases smoothly between these two regions, since the larger offer α_2 mitigates the negative effect of additional bargaining overhead. (iii) Large C (e.g., $C \geq 10$ in Fig. 4): SU rejects both offers and PU receives a negative payoff.

Figure 5 corresponds to the PBE in Theorem 3. In this PBE, the SU never accepts the first stage offer α_1, as it expects that the second stage offer α_2 is much better. As a result, the two-stage game becomes similar to a one-stage game. We can classify the PU's payoff into two regions based on the value of

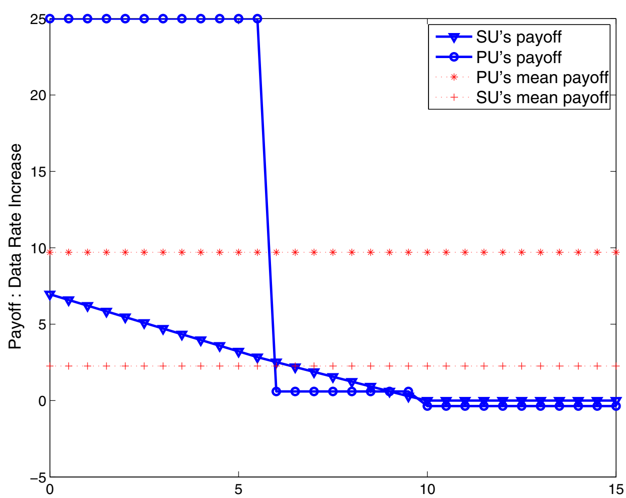

Fig. 4. First type PBE (Theorem 2) in the two-stage bargaining game

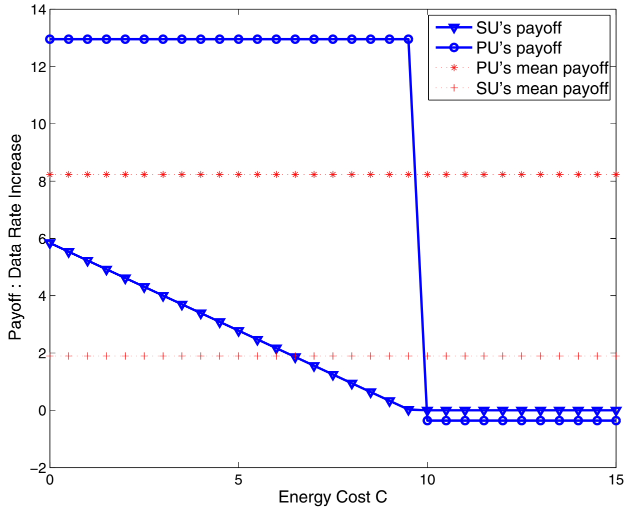

Fig. 5. Second type PBE (Theorem 3) in the two-stage bargaining game

C, following a similar argument as for Fig. 4. By comparing Figure 4 and 5, we notice that the PU's expected utility and SU's utility are both higher in Fig. 4 than in Fig. 5. The key reason is that the PBE in Fig. 5 always wastes the first stage bargaining opportunity. In other words, the PBE in Theorem 2 *Pareto dominates* the PBE in Theorem 3.

6 Conclusion

This paper studies a *dynamic* spectrum negotiation problem with *incomplete information*. We model the interactions between a PU and an SU as a dynamic Bayesian game, and derive two types of Perfect Bayesian Equilibria. Simulations show that both equilibria can coexist in the same game, and one type Pareto dominates the other in the expected sense. This paper represents an initial step of studying the resource allocation in cognitive radio networks with incomplete information and dynamics. Only one PU and one SU bargaining model has been studied, and it is certainly desirable although very challenging to consider dynamic bargaining among many PUs and SUs.

Acknowledgements. This work has been supported by CSC Postgraduate Scholarship, NSF of China No.60832008, National S&T Major Project 2010ZX03003-002-02, TNList Cross-discipline Foundation, China's 973 Project 2007CB310608, China's 863 Project 2009AA011501, PCSIRT, and National S&T Pillar Program 2008BAH30B09. This work is also supported by the General Research Funds (Project No. 412509 and 412710) established under the University Grant Committee of the Hong Kong Special Administrative Region, China. The authors express their gratitude to DoCoMo Beijing Communications Laboratories Co., Ltd for their helpful comments and criticisms.

References

1. McHenry, M.A., McCloskey, D.: Spectrum Occupancy Measurements: Chicago, Illinois, November 16-18, Shared Spectrum Company, Technical report (2005)
2. Haykin, S.: Cognitive Radio: Brain-Empowered Wireless Wommunications. IEEE Journal on Selected Areas in Communications 23(2), 201–220 (2005)
3. Bae, J., Beigman, E., Berry, R., Honig, M., Vohra, R.: Sequential Bandwidth and Power Auctions for Distributed Spectrum Sharing. IEEE Journal on Selected Areas in Communications 26(7), 1193–1203 (2008)
4. Niyato, D., Hossain, E.: Competitive Pricing for Spectrum Sharing in Cognitive Radio Networks: Dynamic Game, Inefficiency of Nash equilibrium, and Collusion. IEEE Journal on Selected Areas in Communications 26(1), 192–202 (2008)
5. Kasbekar, G., Sarkar, S.: Spectrum Pricing Games with Bandwidth Uncertainty and Spatial Reuse in Cognitive Radio Networks. In: ACM MobiHoc (2010)
6. Simeone, O., Stanojev, I., Savazzi, S., Bar-Ness, Y., Spagnolini, U., Pickholtz, R.: Spectrum Leasing to Cooperating Secondary ad hoc Networks. IEEE Journal on Selected Areas in Communications 26(1), 203–213 (2008)
7. Jing, Z., Qian, Z.: Stackelberg Game for Utility-Based Cooperative Cognitive Radio Networks. In: ACM MobiHoc, pp. 23–32 (2009)
8. Lingjie, D., Lin, G., Jianwei, H.: Contract-Based Cooperative Spectrum Sharing. In: IEEE International Dynamic Spectrum Access Networks (DySPAN) Symposium, Aachen, Germany (2011)
9. Adlakha, S., Johari, R., Goldsmith, A.: Competition in Wireless Systems via Bayesian Interference Games. arXiv, vol.cs.GT (2007)
10. Cover, T.M., El Gamaal, A.: Capacity Theorems for the Relay Channel. IEEE Transaction on Information Theory 25(5), 572–584 (1979)
11. Fudenberg, D., Tirole, J.: Game Theory. The MIT Press, Cambridge (1991)
12. Gibbons, R.: Game Theory for Applied Economists. Princeton University Press (1992)
13. Rubinstein, A.: A Bargaining Model with Incomplete Information about Time Preferences. In: Econometrica (1985)
14. Chatterjee, K., Samuelson, W.: Bargaining under Incomplete Information. Operations Research, 835–851 (1983)
15. Rubinstein, A., Osborne, M.: Bargaining and Markets. Academic Press (1990)
16. Yang, Y., Jianwei, H., Xiaofeng, Z., Jing, W.: Dynamic Spectrum Negotiation with Asymmetric Information. The Chinese University of Hong Kong, Technical report (2010), http://home.ie.cuhk.edu.hk/∼jwhuang/publication/DynamicBargainTechReport.pdf