Hierarchical Coalition Formation Game of Relay Transmission in IEEE 802.16m

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Abstract. One of the main features of IEEE 802.16m is the relay transmission which could not only extend the service coverage, but also improve the quality-of-service (QoS) to the mobile stations. In this paper, we consider the *cooperation* among relay stations and mobile stations to improve the performance of relay transmission in IEEE 802.16m network. In particular, the relay stations and mobile stations are rational to maximize their own benefit by forming coalitions. A hierarchical coalition formation game is introduced which is similar to the Stackelberg game. At the upper level, relay stations are considered to be the leaders to cooperate with each other to relay the data from base station to the mobile station. At the lower level, mobile stations are considered to be the followers cooperating with each other to relay the data from base and relay stations. Given the coalition formed by the relay station (i.e., leaders), mobile stations (i.e., followers) form the coalitions such that their individual payoffs are maximized. Knowing this behavior of mobile station, relay stations form their coalitions to maximize their individual payoffs. The analysis based on Markov model is introduced to obtain the stable coalitional structures of both leaders and followers. This hierarchical coalition formation game model will be useful for jointly investigating the self-interest behaviors of relay stations and mobile stations in IEEE 802.16m relay network.

Keywords: Cooperative communications and networking, coalitional game theory, Markov model, IEEE 802.16m.

1 Introduction

IEEE 802.16m is proposed for mobile broadband access as an enhancement of the existing WiMAX systems. IEEE 802.16m will meet all requirements of 4G wireless system under the ITUs International Mobile Telecommunications Advanced (IMT-Advanced program) [1]. IEEE 802.16m will incorporate the relay transmission as a cost-effective approach to extend the coverage area and improve the capacity of the network as specified in the standard [2,3]. Relay transmission will be based on the cooperative communications in which the relay station (RS) forwards the data between base station (BS) and mobile station (MS). Radio resource management will remain the important and open issues in IEEE 802.16m

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relay network. One of them is the relay selection, especially, in the environment where RSs and MSs are rational to maximize their own benefits. In this case, RSs and MSs can cooperate to perform relay transmission to enhance the performance, but this will incur the cost (e.g., due to energy consumption). Therefore, with rational behavior, the analysis of the decision making for cooperation in IEEE 802.16m relay network will be required.

In this paper, we apply coalitional game theory to study the cooperation among the RSs and MSs in relay transmission. RSs in the same coalition will perform relay transmission for their subscribed MSs. In addition, if MSs form coalition, they will also perform relay transmission for each other to improve the transmission rate further. However, relay transmission incurs the cost (e.g., energy consumption), RSs and MSs have to form coalition such that their individual payoffs are maximized. As the main contribution, a hierarchical coalition formation game model is proposed to analyze this decision making process of RSs and MSs jointly. This game model is similar to the Stackelberg game, where RSs are considered to be the leaders forming coalition before MSs which are considered to be the followers. The stable coalitional structures (i.e., set of coalitions of all leaders and followers) are obtained using hierarchical Markov model. The performance evaluation clearly shows the impact of various parameters to the network performances (e.g., channel quality and cost of relay transmission). This hierarchical coalition formation game model will be useful for the implementation of the IEEE 802.16m relay network.

The rest of this paper is organized as follows: Related works are reviewed in Section 2. Section 3 describes the system model and assumptions. Section 4 presents the formulation of hierarchical coalition formation game for relay transmission. Section 5 presents the numerical results. The summary is given in Section 6.

2 Related Works

2.1 IEEE 802.16 Relay Networks

IEEE 802.16m is an enhancement of IEEE 802.16 standards which will support mobile multihop relay (MMR) networks [4]. Various issues were addressed for such networks. For example, in [5], a cooperative relay selection algorithm was proposed. In this algorithm, the signal intensity is used by base station to select the best relay station. In [6], a resource allocation algorithm for multicast service in IEEE 802.16j relay network was proposed. The objective is to maximize the total number of recipients constrained by the transmission budget. The resource is allocated for the base station and relay nodes to achieve this objective. In [7], the problem of relay station placement in IEEE 802.16 relay network was addressed. The objective is to minimize the number of relay stations required to meet all demand of users. An efficient heuristic algorithm was proposed to obtain the solution of placement.

2.2 Game Theory and Cooperative Communications

Due to the nature of the cooperative communications, game theory has been adopted to analyze various issues [8]. In [9], a game model was presented to investigate the cooperation among nodes using decode-and-forward (DF) cooperative transmission with Rayleigh fading channels. In [10], a bargaining game was formulated to study the bandwidth allocation problem between a source node and a number of relay nodes. Also, the conditions under which the source and relay nodes will cooperate were analyzed. The relay selection and power control problems in cooperative relay network were addressed in [11] as a two-level Stackelberg game. In this game, the source node, considered as a buyer, pays to the relay nodes to provide them with an incentive to cooperate and forward the signal to the destination. In [12], coalitional game theory was used, in combination with cooperative transmission, to solve the boundary node problem in an ad hoc packet forwarding network. A grouping algorithm for relay selection was proposed in [13] to minimize transmit power. Also, an optimal rate allocation scheme among the relay nodes was studied.

Although existing literature addressed various issues in cooperative communications using game theory, none of these works considered the problem of performing coalition formation among rational nodes (i.e., nodes are self-interested to maximize their individual payoffs). Especially, when a hierarchy exists in the relay network in which relay stations and mobile stations can form coalitions to achieve their goals. The closest work is [14] where a coalitional game framework was proposed for cooperative communications. However, no analysis on the stability of the resulting coalitional structure was considered. Also, the joint formations of relay stations and mobile stations were ignored.

3 System Model and Assumptions

In this section, the network model of IEEE 802.16m relay network considered in this paper is presented (Section 3.1). Then, the details of the relay transmission in such a network are given (Section 3.2). After that, an overview of the hierarchical coalition formation game model, which is the main contribution of this paper is presented (Section 3.3).

3.1 Network Model and Coalition Formation

We consider IEEE 802.16m relay network. Without loss of generality, a service area with one base station is considered. There are R relay stations (RSs) providing the relay service in this service area in which the set of RSs is denoted by $\mathcal{R}$. There are N mobile stations (MSs) in this service area, each of which subscribes to one of the RSs. A set of MSs subscribed to RS j is denoted by $\mathcal{N}_j$ and the set of all MSs is denoted by $\mathcal{N} = \bigcup_{j \in \mathcal{R}} \mathcal{N}_j$. For MS i, the corresponding RS is denoted by $j = J(i)$. When BS transmits data to the target MS i, RS

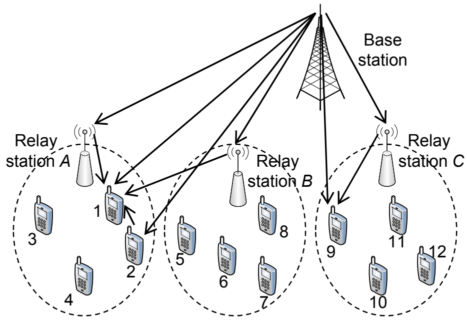

Fig. 1. System model for relay transmission with coalitions $\{A, B\}$ and $\{C\}$

$j = J(i)$ performs relay transmission to improve the transmission rate in the downlink direction of target MS i.[1]

RSs can cooperate by forming the coalitions (i.e., groups) to perform relay transmission for the MSs subscribed to the RSs in the same coalition. The coalition of RSs is denoted by $\mathcal{J}$.[2] In addition, we also assume that MSs can help each other by performing relay transmission. The coalition of MSs is denoted by $\mathcal{I}$.

Fig. 1 shows example of network model with three RSs (i.e., $\mathcal{R} = \{A, B, C\}$) and twelve MSs (i.e., $\mathcal{N} = \{1, \ldots, 12\}$) where $\mathcal{N}_1 = \{1, \ldots, 4\}$, $\mathcal{N}_2 = \{5, \ldots, 8\}$, and $\mathcal{N}_3 = \{9, \ldots, 12\}$. RSs A and B form a coalition denoted by $\mathcal{J} = \{A, B\}$, while RS C acts alone. Also, MSs 1 and 2 form a coalition denoted by $\mathcal{I} = \{1, 2\}$. In this case, when BS transmits data to MS 1 which subscribes to RS A, RSs A and B as well as MS 2 will perform the relay transmission for MS 1 to improve the performance (Fig. 1). Similarly, when BS transmits data to MS 2, RSs A and B as well as MS 1 will perform the relay transmission. However, BS transmits data to MS 9 which subscribes to RS C. Since RS C does not form coalition with other RSs and MS 9 does not form coalition with other MSs, only RS C will perform the relay transmission for MS 9.

3.2 Relay Transmission

The frame structure for IEEE 802.16m cooperative relay transmission is divided into uplink and downlink subframes. For a subframe (e.g., downlink), it consists of access zone and relay zone to support transmission from base station to mobile station and from relay station to mobile station. We assume that the transmissions from BS to MS and from RS to MS use cooperative diversity based on a decode-and-forward strategy as in [15]. However, the approach that we propose in this paper can accommodate other strategies (e.g., amplify-and-forward). In the first phase of the cooperative diversity scheme (i.e., access zone

[1] The model is also applicable to the uplink transmission.

[2] In this paper, to simplify the presentation, notations of coalition and set are assumed to be the same.

in the frame structure), BS transmits using a particular adaptive modulation and coding (AMC) mode to a target MS. In this phase, RSs and MSs receive the signals. In the second phase, the RSs which are in the same coalition as the RS which the target MS is subscribed to as well as the MSs which are in the same coalition as the target MS repeat the transmission from BS with the same AMC mode. At the end of the second phase, the target MS achieves a gain in SNR by combining the space-time decoded signals with those received in the first phase.

Let $\mathcal{J}$ and $\mathcal{I}$ denote the coalitions of RSs and MSs to perform relay transmission for MS i, respectively. Let γ_i, γ_j, and $\gamma_{i'}$ denote the instantaneous SNR from BS, from RS j and MS i' to the target MS i, respectively. If there are more than one relay transmission from RS (or MS) with the perfect channel between BS to RS (or MS), the post-processing SNR at the target MS i can be expressed as follows:

$$\gamma_i^{\mathrm{post}} = \gamma_i + \sum_{j \in \mathcal{J}} \gamma_j + \sum_{i' \in \mathcal{I} \setminus \{i\}} \gamma_{i'}. \tag{1}$$

When considering Rayleigh fading channels, the cumulative distribution function (CDF) of the post-processing SNR at the MS with single RS j (i.e., no coalition of RS and MS) is given by [15].

$$F(\gamma) = \frac{\overline{\gamma}_i}{\overline{\gamma}_i - \overline{\gamma}_j}(1 - e^{-\frac{\gamma}{\overline{\gamma}_i}}) + \frac{\overline{\gamma}_j}{\overline{\gamma}_j - \overline{\gamma}_i}(1 - e^{-\frac{\gamma}{\overline{\gamma}_j}}), \tag{2}$$

where $\overline{\gamma}_i$ and $\overline{\gamma}_j$ are, respectively, the corresponding average SNRs from BS and RS j to the target MS i. For the multiple relay transmission case, (2) can be extended as in (3) [15], where $|\mathcal{J} \cup \mathcal{I}| \geq 3$, $I = 1 - e^{-\frac{\gamma}{\overline{\gamma}_i}}$, $B_j = 1 - e^{-\frac{\gamma}{\overline{\gamma}_j}}$,

$$F(\gamma) = \left(\prod_{j \in \mathcal{J}} \frac{\overline{\gamma}_i}{\overline{\gamma}_i - \overline{\gamma}_j} \right) \left(\prod_{i' \in \mathcal{I} \setminus \{i\}} \frac{\overline{\gamma}_i}{\overline{\gamma}_i - \overline{\gamma}_{i'}} \right) I + \sum_{j \in \mathcal{J}} \left(\frac{\overline{\gamma}_j}{\overline{\gamma}_j - \overline{\gamma}_i} \prod_{j' \in \mathcal{J} \setminus \{j\}} \frac{\overline{\gamma}_j}{\overline{\gamma}_j - \overline{\gamma}_{j'}} \right) B_j \times \sum_{i' \in \mathcal{I} \setminus \{i\}} \left(\frac{\overline{\gamma}_{i'}}{\overline{\gamma}_{i'} - \overline{\gamma}_i} \prod_{i'' \in \mathcal{I} \setminus \{i, i'\}} \frac{\overline{\gamma}_{i'}}{\overline{\gamma}_{i'} - \overline{\gamma}_{i''}} \right) B_{i'}. \tag{3}$$

and $B_{i'} = 1 - e^{-\frac{\gamma}{\overline{\gamma}_{i'}}}$. Note that for DF based relay transmission, the choice of an AMC mode only depends on the post-processing SNR at the corresponding destination. Given the available AMC modes as well as the minimum required SNR threshold Γ_c for each mode c, the probability of using mode c for target MS i can be calculated as $\alpha_{i,c}(\mathcal{J}, \mathcal{I}) = F(\Gamma_{c+1}) - F(\Gamma_c)$. The transmission rate of the target MS i can then be obtained from

$$R_i(\mathcal{J}, \mathcal{I}) = \sum_{c \in \mathcal{C}} \rho_c \alpha_{i,c}(\mathcal{J}, \mathcal{I}), \tag{4}$$

where $\mathcal{C}$ is a set of AMC modes, and ρ_c is the transmission rate of AMC mode c in packets/frame.

3.3 Hierarchical Coalition Formation Game

If RSs and MSs, in a service area of IEEE 802.16m relay network, are rational, they can cooperate by forming coalitions such that their individual payoffs are maximized. For MS i, the payoff is defined as a function of transmission rate and cost of relay transmission as follows:

$$U_i(\mathcal{J}, \mathcal{I}) = R_i(\mathcal{J}, \mathcal{I}) - \beta_i(|\mathcal{I}| - 1), \tag{5}$$

for $i \in \mathcal{I}$ and $J(i) \in \mathcal{J}$, where β_i is the cost factor of MS i. Note that the cost of relay transmission increases as the number of members in the same coalition increases. Similarly, for RS j, the payoff is defined as the total transmission rate of all subscribed MSs and cost of relay transmission as follows:

$$V_j(\mathcal{J}) = \sum_{i \in \mathcal{N}_j} \overline{R}_i(\mathcal{J}) - \beta_j \sum_{j' \in \mathcal{J} \setminus j} |\mathcal{N}_{j'}|, \tag{6}$$

where $\overline{R}_i(\mathcal{J})$ is the average transmission rate of MS i given coalition $\mathcal{J}$ of RSs[3]. Given the payoff functions of MSs and RSs defined in (5) and (6), respectively, there is a tradeoff for them as the rational entities to form the coalition. In particular, forming coalition can help the target MS to gain higher transmission rate from relay transmission. However, it may incur too high cost if the coalition is composed of many members. In addition, MSs and RSs form different coalitions. To address this coalition formation in IEEE 802.16m relay network, the hierarchical coalition formation game model is proposed to analyze the stability of cooperation behavior of RSs and MSs. The hierarchical coalition formation game model is composed of two coalitional games, i.e., coalition formation of RSs and MSs at the upper and lower levels, respectively. These two coalitional games are interrelated in which the coalition of RSs will affect the performance of MSs, and, hence, the coalition formation of MSs. Also, the coalition formation of MSs affects the transmission rate of each other in which the RSs have to take this effect into account. The details of the hierarchical coalition formation game model is presented in the next section.

4 Hierarchical Coalition Formation Game

In this section, first the definition of the coalition game of RSs and MSs is given (Section 4.1). The stable coalitional structure is analyzed (Section 4.2).

4.1 Game Definition

RSs and MSs will form coalitions separately, but their resulting coalitions can influence the decision of each other. We assume that RSs and MSs are rational

[3] This average transmission rate $\overline{R}_i(\mathcal{J})$ will be obtained later in this paper, specifically in (20).

to maximize their individual payoffs. Also, RSs can form coalition before MSs, and MSs can fully observe the coalition of RSs (i.e., by checking the relayed signal). By adopting the concept of Stackelberg game, in the hierarchical coalition formation game model, the leaders are RSs while the followers are MSs. In this case, first, MSs will form the coalition according to the coalition of RSs such that their individual payoffs are maximized. The RSs (i.e., the leaders) having this knowledge will form their coalition accordingly. Note that the proposed hierarchical coalitional game has a *non-transferable utility (NTU)*, since the value (i.e., transmission rate minus cost) of any coalition of RSs and MSs cannot be transferred (divided) arbitrarily among the members of a given coalition. Let $\mathcal{J}_x$ and $\mathcal{I}_y$ denote the coalitions of RSs and MSs, respectively, where x and y are the indexes of coalitions. We can define the coalitional structures of RSs and MSs as follows: r_u denotes the coalitional structure of RSs defined as $r_u = \{\ldots, \mathcal{J}_x, \ldots\}$, such that $\mathcal{R} = \bigcup_{\mathcal{J}_x \in r_u}$, where u is an index of coalitional structure. Similarly, m_v denotes the coalitional structure of MSs defined as $m_v = \{\ldots, \mathcal{I}_y, \ldots\}$, such that $\mathcal{N} = \bigcup_{\mathcal{I}_y \in m_v} \mathcal{I}_y$, where v is the index.

Consider example in Fig. 1, the coalitional structures of three RSs (i.e., A, B, and C) are defined as follows: $r_1 = \{\{A\},\{B\},\{C\}\}$, $r_2 = \{\{A,B\},\{C\}\}$, $r_3 = \{\{A,C\},\{B\}\}$, $r_4 = \{\{A\},\{B,C\}\}$, and $r_5 = \{\{A,B,C\}\}$. The coalitional structures of four MSs (i.e., 1, 2, 3, and 4) subscribed to RS A are defined as follows: $m_1 = \{\{1\},\{2\},\{3\},\{4\}\}, m_2 = \{\{1,2\},\{3\},\{4\}\}, m_3 = \{\{1\},\{2\},\{3,4\}\}$, $m_4 = \{\{1,3\},\{2\},\{4\}\}$, $m_5 = \{\{1\},\{3\},\{2,4\}\}$, $m_6 = \{\{1,4\},\{2\},\{3\}\}$, $m_7 = \{\{1\},\{4\},\{2,3\}\}$, $m_8 = \{\{1,2\},\{3,4\}\}$, $m_9 = \{\{1,3\},\{2,4\}\}$, $m_{10} = \{\{1,4\},$ $\{2,3\}\}$, $m_{11} = \{\{1,2,3\},\{4\}\}$, $m_{1,2} = \{\{1,2,4\},\{3\}\}$, $m_{13} = \{\{1,3,4\},\{2\}\}$, $m_{14} = \{\{1\},\{2,3,4\}\}$, and $m_{15} = \{\{1,2,3,4\}\}$.

Given coalition $\mathcal{J}$ of RS $j \in \mathcal{J}$ such that $j = J(i)$, the actions of MS i in forming coalition are as follows [16]:

– *Joining:* Let $\mathbb{M}_{\mathrm{jn}}$ denote the set of candidate coalitions of MSs that can join together to form a new single coalition $\mathcal{I}_{y'}$. If all MSs $i \in \mathcal{I}_y \in \mathbb{M}_{\mathrm{jn}}$ can gain higher individual payoffs, i.e.,

$$U_i(\mathcal{J}, \mathcal{I}_{y'}) \geq U_i(\mathcal{J}, \mathcal{I}_y), \quad \forall i \in \mathcal{I}_y, \tag{7}$$

where $\mathcal{I}_{y'} = \bigcup_{\mathcal{I}_y \in \mathbb{M}_{\mathrm{jn}}} \mathcal{I}_y$, then the coalitions can decide to join together.

– *Splitting:* Given a coalition $\mathcal{I}_y$, the MSs in this coalition can split (i.e., be partitioned) into multiple new coalitions $\mathcal{I}_{y'}$, if all the MSs $i \in \mathcal{I}_y$ can gain higher individual payoffs, i.e.,

$$U_i(\mathcal{J}, \mathcal{I}_{y'}) \geq U_i(\mathcal{J}, \mathcal{I}_y), \quad \forall i \in \mathcal{I}_y, \tag{8}$$

where $\mathcal{I}_y = \bigcup_{\mathcal{I}_{y'} \in \mathbb{M}_{\mathrm{sp}}} \mathcal{I}_{y'}$ and $\mathbb{M}_{\mathrm{sp}}$ is the set of new coalitions of MSs.

If RSs observe the coalition formation of MSs at the steady state and then perform their own coalition formation, the payoff function of RS (i.e., previously defined in (6)) can be expressed as a function of its own coalition and a set of coalitional structure of MSs, i.e., $V_j(\mathcal{J}, \Xi^\dagger(r))$. $\Xi^\dagger(r) = \{\ldots, m^\dagger(r), \ldots\}$ is a set of coalitional structure $m^{\dagger}(r)$ of MSs at the steady state given coalitional structure r of RSs. The actions of RS j are as follows:

- *Joining:* Let $\mathbb{R}_{\mathrm{jn}}$ denote the set of candidate coalitions of RSs that can join together to form a new single coalition $\mathcal{J}_{x'}$. If all RSs $j \in \mathcal{J}_x \in \mathbb{R}_{\mathrm{jn}}$ can gain higher individual payoffs, i.e.,

$$V_j(\mathcal{J}_{x'} \in r', \Xi^{\dagger}(r')) \geq V_j(\mathcal{J}_x \in r, \Xi^{\dagger}(r)), \quad \forall j \in \mathcal{J}_x, \tag{9}$$

  where $\mathcal{J}_{x'} = \bigcup_{\mathcal{J}_x \in \mathbb{R}_{\mathrm{jn}}} \mathcal{J}_x$, then the coalitions can decide to join together.
- *Splitting:* Given a coalition $\mathcal{J}_x$, the RSs in this coalition can split (i.e., be partitioned) into multiple new coalitions $\mathcal{J}_{x'}$, if all the RSs $j \in \mathcal{J}_x$ can gain higher individual payoffs, i.e.,

$$V_j(\mathcal{J}_{x'} \in r', \Xi^{\dagger}(r')) \geq V_j(\mathcal{J}_x \in r, \Xi^{\dagger}(r)), \quad \forall j \in \mathcal{J}_x, \tag{10}$$

  where $\mathcal{J}_x = \bigcup_{\mathcal{J}_{x'} \in \mathbb{R}_{\mathrm{sp}}} \mathcal{J}_{x'}$ and $\mathbb{R}_{\mathrm{sp}}$ is the set of new coalitions of RSs.

The stable coalitional structure (i.e., a set of coalitions) is considered to be the solution. For MSs as the followers, given the coalitional structure r of RSs, the stable coalitional structure $m^*(r)$ of MSs can be defined based on the following condition:

$$U_i(\mathcal{J} \in r, \mathcal{I}^*) \geq U_i(\mathcal{J} \in r, \mathcal{I}), \quad \forall i \in \mathcal{N}, \tag{11}$$

for all $\mathcal{I}^* \in m^*(r)$ and $\mathcal{I} \in m(r)$ where $m^*(r) \neq m(r)$. In this case, coalitional structures $m^*(r)$ and $m(r)$ of MSs are defined as the function of coalitional structure r of RSs. From (11), at the stable coalitional structure $m^*(r)$ of MSs given coalitional structure r of RSs, none of MS can join or split and result in the new coalitions which improve the payoff if other MSs keep the coalition unchanged.

Let $\Xi^*(r) = \{\ldots, m^*(r), \ldots\}$ denote the set of stable coalitional structure $m^*(r)$ of MSs at the steady state given coalitional structure r of RS. For RSs as the leaders, the stable coalitional structure r^* can be defined based on the following condition:

$$V_j(\mathcal{J}^* \in r^*, \Xi^*(r^*)) \geq V_j(\mathcal{J} \in r, \Xi^*(r)), \quad \forall j \in \mathcal{R}, \tag{12}$$

where $r^* \neq r$. From (12), at the stable coalitional structure r^* of RSs, none of RS can join or split and result in the new coalitions which improve the payoff if other RSs keep the coalition unchanged given that the MSs are at their stable coalitional structure.

To obtain this stable coalitional structures of RSs and MSs, the Markov model will be developed.

4.2 Stable Coalitional Structure

To analyze the stability of the coalition formation game, a Markov model can be used [17]. Since RSs and MSs can form coalitions separately (but can influence each other), we can develop the hierarchical Markov model which is composed of two models for MSs and RSs at the lower and upper levels, respectively. The state of each Markov is defined as the coalitional structure[4]. In this case, the MSs form coalition and reach the stable coalitional structure. Then, the RSs will observe the stable coalitional structure of MSs and form coalition accordingly until the stable coalitional structure of RSs is reached.

Markov Model of Coalition Formation of Mobile Stations. We first analyze the dynamics of coalition formation of MSs. The state space of Markov model for the coalition formation of MSs is defined as a function of coalitional structure r of RSs as follows:

$$\Omega(r) = \{m_v \mid v = \{1, \dots, D_{|\mathcal{N}|}\}\}, \tag{13}$$

where m_v represents a coalitional structure (spanning all MSs). $D_{|\mathcal{N}|}$ is the Bell number obtained from

$$D_i = \sum_{j=0}^{i-1} \binom{i-1}{j} D_j, \text{ for } i \geq 1, \text{ and } D_0 = 1. \tag{14}$$

The transition probability matrix of the Markov model for the coalition formation of MSs is expressed again as a function of coalitional structure r of RSs as follows:

$$\mathbf{P}(r) = \begin{bmatrix} P_{m_1,m_1}(r) & \cdots & P_{m_1,m_{D_{|\mathcal{N}|}}}(r) \\ P_{m_2,m_1}(r) & \cdots & P_{m_2,m_{D_{|\mathcal{N}|}}}(r) \\ \vdots & \ddots & \vdots \\ P_{m_{D_{|\mathcal{N}|}},m_1}(r) & \cdots & P_{m_{D_{|\mathcal{N}|}},m_{D_{|\mathcal{N}|}}}(r) \end{bmatrix}, \tag{15}$$

where $P_{m_v,m_{v'}}(r)$ is the probability of transition from state m_v to $m_{v'}$. Let $\mathbb{P}_{m_v,m_{v'}}$ denote the set of candidate MSs which are bound to make a coalition formation decision which will result in the change of the coalitional structure from m_v to $m_{v'}$. This transition probability $P_{m_v,m_{v'}}(r)$ can be obtained from

$$P_{m_v,m_{v'}}(r) = \begin{cases} \prod_{i \in \mathbb{P}_{m_v,m_{v'}}} \delta_{\text{ms}} \theta_i(m_{v'} | m_v, r), & \text{if } m_v \twoheadrightarrow m_{v'}, \\ 0, & \text{otherwise}, \end{cases} \tag{16}$$

where $m_v \twoheadrightarrow m_v'$ is a feasibility condition. In particular, if a coalitional structure $m_{v'}$ is reachable from m_v given the decision of all MSs, then the condition $m_v \twoheadrightarrow m_{v'}$ is true. Otherwise, condition $m_v \twoheadrightarrow m_{v'}$ becomes false. δ_{ms} is the probability that an MS makes a decision (e.g., $\delta = 0.5$). $\theta_i(m_{v'}|m_v, r)$ is the best-reply rule of MS i. That is, $\theta_i(m_{v'}|m_v, r)$ is the probability that MS i changes decision, and then the coalitional structure changes from m_v to $m_{v'}$

[4] In the rest of this paper, the terms "state" of Markov chain and "coalitional structure" of coalition formation game are used interchangeably.

given coalitional structure r of RSs. This best-reply rule of MS i is defined as follows:

$$\theta_i(m_{v'}|m_v, r) = \begin{cases} \hat{\theta}, \text{ if } U_i(\mathcal{J}, \mathcal{I}_y \in m_{v'}) \geq U_i(\mathcal{J}, \mathcal{I}_y \in m_v), \\ \epsilon, \text{ otherwise,} \end{cases} \quad (17)$$

where $0 < \hat{\theta} \leq 1$ is a constant (e.g., $\hat{\theta} = 0.1$), and ϵ is a small probability that the MS makes an irrational decision. It is assumed that the MS can make an irrational coalition formation decision due to lack of information or the need for "exploration" in the learning process.

The diagonal element of matrix $\mathbf{P}(r)$ defined in (15) is obtained from

$$P_{m_v, m_v}(r) = 1 - \sum_{m_{v'} \in \Omega(r) \backslash \{m_v\}} P_{m_v, m_{v'}}(r). \quad (18)$$

Given the coalitional structure r of RSs, the stable coalitional structure $m^*(r)$ of MSs can be obtained. This stable coalitional structure exhibits internal and external stability [17]. Internal stability implies that, given a coalition, no MS in this coalition has an incentive to leave this coalition and act alone (non-cooperatively as a singleton), since the payoff any MS receives in the coalition is higher than that received when acting non-cooperatively. External stability implies that, in a given partition, no MS can improve its payoff by switching its current coalition and join another one. In particular, a coalitional structure $m^*(r)$ is said to be *stable*, if the conditions for internal and external stability are verified for all the coalitions in $m^*(r)$. A stable coalitional structure $m^*(r)$ of MSs can be identified from the stationary probability of the Markov model defined with state space in (13) and transition probability in (15). The stationary probability of the Markov model for the coalition formation of MSs can be obtained by solving

$$\boldsymbol{\pi}^T(r)\mathbf{P}(r) = \boldsymbol{\pi}^T(r), \quad \text{and} \quad \boldsymbol{\pi}^T(r)\mathbf{1} = 1, \quad (19)$$

where $\boldsymbol{\pi}(r) = \left[\pi_{m_1}(r) \cdots \pi_{m_v}(r) \cdots \pi_{m_{D_{|\mathcal{N}|}}} \right]^T$ is a vector of stationary probabilities and $\pi_{m_v}(r)$ is the probability that the coalitional structure m_v will be reached given the coalitional structure r of RSs. $\mathbf{1}$ is a vector of ones.

If the probability of irrational decisions approaches zero (i.e., $\epsilon \to 0^+$), there could be an ergodic set $\mathbb{E}_{\text{ms}}(r) \subseteq \Omega(r)$ of states m_v in the Markov model for coalition formation of MSs defined by the state space in (13) and the transition probability in (15). This ergodic set $\mathbb{E}_{\text{ms}}(r)$ exists if $P_{m_v, m_{v'}}(r) = 0$ for $m_v \in \mathbb{E}_{\text{ms}}(r)$ and $m_{v'} \notin \mathbb{E}_{\text{ms}}(r)$, and no nonempty proper subset of $\mathbb{E}_{\text{ms}}(r)$ has this property. In this regard, the singleton ergodic set is the set of absorbing states.

Once all MSs reach the state in an ergodic set, they will remain in this ergodic set forever. Therefore, the absorbing state is referred to as the stable coalitional structure $m^*(r)$, and $\mathbb{E}_{\text{ms}}(r)$ is the set of stable coalitional structures of MSs where $m^*(r) \in \mathbb{E}_{\text{ms}}(r)$. With this stable coalitional structure, no MS has an incentive to change the decision given the prevailing coalitional structure.

Given the coalitional structure r of RSs, the average transmission rate of MS i (i.e., used to calculate the payoff of RS i as defined in (6)) can be obtained from

$$\overline{R}_i(\mathcal{J}) = \sum_{m_v \in \Omega(r)} \pi_{m_v}(r) \left(\sum_{\mathcal{I}_y \in m_v} R_i(\mathcal{J}, \mathcal{I}_y) \right), \tag{20}$$

for $\mathcal{J} \in r$ where $R_i(\mathcal{J}, \mathcal{I})$ is the transmission rate of MS i which can be obtained from (4) given coalition $\mathcal{J}$ of RSs and coalition $\mathcal{I}_y$ of MSs.

Markov Model of Coalition Formation of Relay Stations. We then analyze the dynamics of coalition formation of RSs. The state space of Markov model for the coalition formation of RSs is defined as follows:

$$\Psi = \{r_u \mid u = \{1, \ldots, D_{|\mathcal{R}|}\}\}, \tag{21}$$

where r_u represents a coalitional structure (spanning all RSs), and $D_{|\mathcal{R}|}$ is the Bell number obtained from (14).

The transition probability matrix of the Markov model for the coalition formation of RSs is expressed as follows:

$$\mathbf{Q} = \begin{bmatrix} Q_{r_1,r_1} & Q_{r_1,r_2} & \cdots & Q_{r_1,r_{D_{|\mathcal{R}|}}} \\ Q_{r_2,r_1} & Q_{r_2,r_2} & \cdots & Q_{r_2,r_{D_{|\mathcal{R}|}}} \\ \vdots & \ddots & \vdots & \\ Q_{r_{D_{|\mathcal{R}|}},r_1} & Q_{r_{D_{|\mathcal{R}|}},r_2} & \cdots & Q_{r_{D_{|\mathcal{R}|}},r_{D_{|\mathcal{R}|}}} \end{bmatrix}, \tag{22}$$

where $Q_{r_u,r_{u'}}$ is the probability of transition from state r_u to $r_{u'}$. Let $\mathbb{Q}_{r_u,r_{u'}}$ denote the set of candidate RSs which can make coalition formation decision and result in the change of the coalitional structure from r_u to $r_{u'}$. Similar to (16), this transition probability $Q_{r_u,r_{u'}}$ can be obtained from

$$Q_{r_u,r_{u'}} = \begin{cases} \prod_{j \in \mathbb{Q}_{r_u,r_{u'}}} \delta_{\mathrm{rs}} \phi_j(r_{u'}|r_u), \text{ if } r_u \twoheadrightarrow r_{u'}, \\ 0, \qquad \text{otherwise,} \end{cases} \tag{23}$$

where again $r_u \twoheadrightarrow r'_u$ is a feasibility condition. δ_{rs} is the probability that an RS makes a decision. $\phi_j(r_{u'}|r_u)$ is the best-reply rule of RS j. That is, $\phi_j(r_{u'}|r_u)$ is the probability that RS j changes decision, and, hence, the coalitional structure changes from r_u to $r_{u'}$. This best-reply rule of RS j is defined as follows:

$$\phi_j(r_{u'}|r_u) = \begin{cases} \hat{\phi}, \text{ if } V_j(\mathcal{J} \in r_{u'}) \geq V_j(\mathcal{J} \in r_u), \\ \epsilon, \text{ otherwise,} \end{cases} \tag{24}$$

where $0 < \hat{\phi} \leq 1$ is a constant (e.g., $\hat{\phi} = 0.1$), and ϵ is the probability of RS to make an irrational decision. Diagonal element of matrix $\mathbf{Q}$ defined in (22) can be obtained from

$$Q_{r_u,r_u} = 1 - \sum_{r_{u'} \in \Psi \backslash \{r_u\}} Q_{r_u,r_{u'}}. \tag{25}$$

Similar to that of MSs, the stable coalitional structure of RSs can be analyzed. Specifically, a coalitional structure r^* is said to be *stable*, if the conditions for internal and external stability are verified for all the coalitions in r^*. To obtain the stable coalitional structure r^* of RSs, the transition probability in (22) is used. The stationary probability of the coalitional structure of RSs can be obtained by solving

$$\boldsymbol{\mu}^T \mathbf{P} = \boldsymbol{\mu}^T, \quad \text{and} \quad \boldsymbol{\mu}^T \mathbf{1} = 1, \tag{26}$$

where $\boldsymbol{\mu} = \left[\mu_{r_1} \cdots \mu_{r_u} \cdots \mu_{r_{D_{|\mathcal{R}|}}} \right]^T$ is a vector of stationary probabilities and μ_{r_u} is the probability that the coalitional structure r_u will be reached.

For $\epsilon \to 0^+$, there could be an ergodic set $\mathbb{E}_{\text{rs}} \subseteq \Psi$ of states r_u in the Markov model defined by the state space in (21) and the transition probability in (22). This ergodic set $\mathbb{E}_{\text{rs}}$ exists if $Q_{r_u, r_{u'}} = 0$ for $r_u \in \mathbb{E}_{\text{rs}}$ and $r_{u'} \notin \mathbb{E}_{\text{rs}}$, and no nonempty proper subset of $\mathbb{E}_{\text{rs}}$ has this property. In this regard, the singleton ergodic set is the set of absorbing states. The absorbing state is referred to as the stable coalitional structure r^*, and $\mathbb{E}_{\text{rs}}$ is a set of stable coalitional structures of RSs, where $r^* \in \mathbb{E}_{\text{rs}}$. With this stable coalitional structure, no RS has an incentive to change the decision given the prevailing coalitional structure.

While (19) and (26) are used to obtain the stationary probabilities for the coalition formations of MSs and RSs, respectively, the joint stationary probability for coalitional structures m_v and r_u for MSs and RSs, respectively, can be obtained from

$$\psi_{r_u, m_v} = \pi_{m_v}(r_u) \mu_{r_u}. \tag{27}$$

The average payoff of MS i can be obtained

$$\overline{U}_i = \sum_{r_u \in \Psi} \sum_{m_v \in \Omega(r_u)} \psi_{r_u, m_v} U_i(\mathcal{J} \in r_u, \mathcal{I} \in m_v) \tag{28}$$

and the average payoff of RS j can be obtained from

$$\overline{V}_j = \sum_{r_u \in \Psi} \mu_{r_u} V_j(\mathcal{J} \in r_u). \tag{29}$$

5 Performance Evaluation

5.1 Parameter Setting

The relay transmission based on IEEE 802.16m is considered. There is single base station (BS) and three relay stations (RSs) whose set is denoted by $\mathcal{R} = \{A, B, C\}$. There are twelve advanced mobile stations (MSs) whose set is denoted by $\mathcal{N} = \{1, \ldots, 12\}$. We consider the case that only the MSs subscribed to the same RS can perform relay transmission for each other. The SNR from BS to all MSs is 5dB. The SNR from RSs to their subscribed MSs is 5dB. The SNR from RSs A, B, and C to MSs 5-8, 9-12, and 1-4 is 3dB, respectively. The SNR from RSs A, B, and C to MSs 9-12, 1-4, and 5-8 is 1dB, respectively. The SNR among MSs subscribed to the same RS is 3dB. The cost factor of MSs is 0.2, and that of RSs is 0.05.

5.2 Numerical Results

Payoff under Different Coalition Fig. 2 shows the payoff of MS 1 under different coalitions of RSs and MSs for which the cost factor of MS is 0.3. It is observed that when RS A which MS 1 is subscribed to does not form any coalition, the payoff is the lowest, and as there are more members in the coalition with MS 1, the payoff increases. This result is from the fact that without relay transmission from other RSs, MS 1 can gain higher transmission rate and higher payoff only by forming the coalition with other MSs. However, if RS A forms coalition $\{A, B, C\}$, the transmission rate of MS 1 is high (the top curve in Fig. 2). In this case, MS 1 may not achieve the highest payoff if it forms coalition $\{1, 2, 3, 4\}$ since the cost of relay transmission for MSs 2, 3, and 4 is higher than the transmission rate gained from them performing relay transmission for MS 1. As a result, MS 1 will split from coalition $\{1, 2, 3, 4\}$ to $\{1, 2, 3\}$ which yields higher payoff due to lower cost. To analyze this complex decision making process of RSs and MSs, an analytical model would be required.

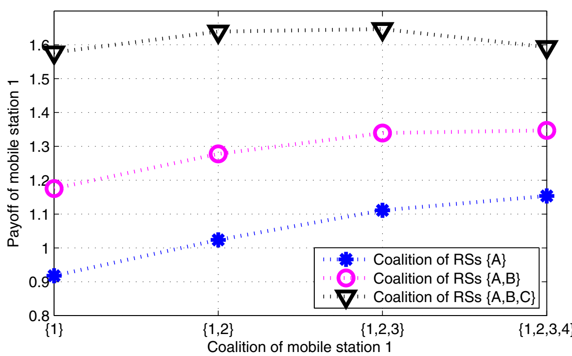

Fig. 2. Payoff of MS 1 under different coalitions of RSs and MSs

Performance under Varied SNR Fig. 3 shows the stable coalitional structure of RS under different SNR from BS to MSs. As expected, when the SNR from BS increases, the transmission rate from BS to the MS increases. Therefore, the cooperation among MSs and RSs is not necessary, and they split into smaller coalitions. The similar result is also observed when the SNR from RS and MS changes. For example, if the SNR from RS B to MSs subscribed to RS A increases, it is likely that RS A will form coalition with RS B. We omit this similar result for brevity of the paper.

Fig. 4 shows the minimum SNR between MS 4 and MSs 1, 2, 3 such that MS 4 will join coalition with MSs 1, 2, and 3. As the SNR among MSs 1, 2, and 3 increases, the minimum SNR between MS 4 and MSs 1, 2, 3 to maintain the stable coalition increases. Since the SNR among MSs 1, 2, and 3 increases, they gain high transmission rate without relay transmission from MS 4. Therefore, for MS 4 to join the coalition, the SNR between MS 4 and other MSs has to be

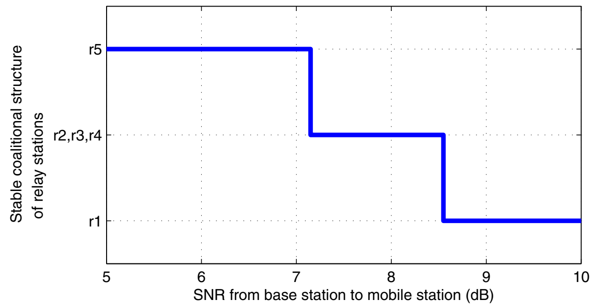

Fig. 3. Stable coalitional structure under SNR from base station

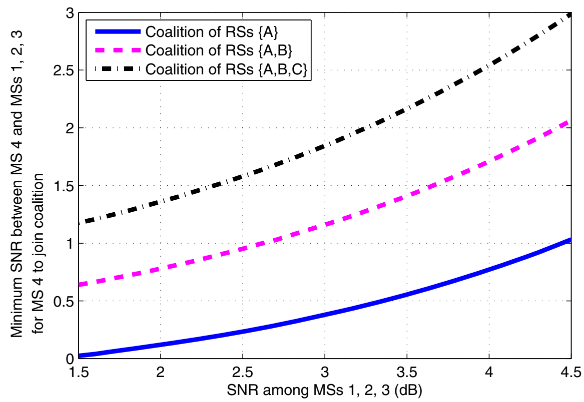

Fig. 4. Maximum SNR between MS 4 and MSs 1, 2, 3 such that the MS 4 will join coalition

high enough. Also, given different coalition of RSs (i.e., the bigger coalition of RSs, the higher minimum SNR between MS 4 and other MSs to maintain stable coalition).

Based on the above results, the coalition formation among RSs and MSs can be affected by various factors. The proposed hierarchical coalition formation game will be useful to analyze and obtain the stable coalition formation under complex environment of IEEE 802.16m relay network.

6 Summary

In this paper, the novel hierarchical coalition formation game has been proposed to model the cooperation among rational relay stations and mobile stations jointly. The cooperation is performed through the relay transmission which can improve the performance, but also incurs the cost. This hierarchical coalition formation game is similar to the Stackelberg game in which relay stations are considered to be the leaders making cooperation decision before mobile stations (i.e., the followers). To analyze the stable coalitional structure of the coalition formation, the hierarchical Markov model has been developed. The extensive performance evaluation has been performed. The numerical results show that cost of relay transmission and channel quality have the effect to the coalition formation of both RSs and MSs.

For the future work, the queueing dynamics of the mobile stations due to the relay transmission will be considered in the coalition formation.

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