Jamming Game in a Dynamic Slotted ALOHA Network

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Abstract. In this paper we suggest a development of the channel capacity concept for a dynamic slotted ALOHA network. Our object is to find maxmin successful transmissions of an information over a dynamic communication channel. To do so, we analyze an ALOHA-type medium access control protocol performance in the presence of a jammer. The time is slotted and the system is described as a zero-sum multistage matrix game. Each player, the sender and the jammer, have different costs for respectively sending their packets and jamming, and the jammer wants to minimize the payoff function of the sender. For this case, we found explicit expression of the equilibrium strategies depending on the costs for sending packets and jamming. Properties of the equilibrium are investigated. In particular we have found a simple linear correlation between the probabilities to act for both players in different channel states which are independent on the number of packets left to send. This relation implies that increasing activity of the jammer leads to reducing activity of the user at each channel state. The obtained results are generalized for the case where the channel can be in different states and change according to a Markov rule. Numerical illustrations are performed.

1 Introduction

The first work related Game theory and Information theory through a max-min problem was proposed in 1952 by Mandelbrot in his PhD Thesis *Contribution a la theorie mathematique des jeux de communication*. He has studied the problem of communication through a noisy channel as a two-player zero-sum game where the sender maximizes mutual information and the noise minimizes it, subject to

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average power constraints. It has been shown that an i.i.d Gaussian signaling scheme and an i.i.d. Gaussian noise distribution are robust, in that any deviation of either the signal or noise distribution reduces or increases (respectively) the mutual information. Hence, the solution to this game-theoretic problem yields a rate of $\log(1 + \frac{P}{N_0})$ which is defined now as the Shannon capacity.

Recall that channel capacity is the tightest upper bound on the amount of information that can be reliably transmitted over a communication channel with noise. In this paper we suggest a development of this channel capacity concept for a dynamic slotted ALOHA network. Our object is to find maxmin successful transmissions of an information over a dynamic communication channel. Therefore, our work extends in a simple way the concept of Shannon capacity in a ALOHA network.

The ALOHA protocol proposed in[4], is a totally decentralized mechanism for defining a medium access protocol without carrier sense in a multi-user environment. The slotted-ALOHA has been proposed in [5] by introducing the synchronization between the devices. This distributed mechanism leads several extensions and is the base of several cellular networks protocols like GSM. There are several works on the the study of non-cooperation between users in an ALOHA network. For example in [6], the authors consider an ALOHA game which the users decides and advertises their transmission probability but keeps their desired throughput private. They study the existence of equilibrium points that could possibly be reached by the users for given user throughput demands. The users' convergence to equilibrium points is analyzed using a specified potential function that governs their dynamics. We can cite also the papers [8] and [9] in which the authors extend the previous model by incorporating channel state information as affecting the transmission policy. They have also shown that there exists particular configurations with several Nash equilibrium. Another model with partial information is proposed in [7] in which mobiles do not know the number of backlogged packets at other nodes. A Markov chain analysis is used to obtain optimal and equilibrium retransmission probabilities and throughput. Jamming in an ALOHA network has been first study in [2]. The performance of the system is defined as the minmax of a two-person constant sum game. The author considers the expected forward progress by taking into account geometrical considerations and routing protocols. In [3], the authors consider an ALOHA non-cooperative game in which one player is a jammer. The authors consider only probability of sending packet or jamming without an energy cost. In [10] jamming and transmission costs were employed for the plot of one step jamming game. Note that besides ALOHA network the jamming problem has been studied for a variety of wireless network settings including sensor networks [11] and other general wireless network models [12,13,14,15,16].

In this paper we assume that a user wants to transmit a message of N packets in a time smaller or equal to T. In fact, we assume that T is an exponentially distributed random variable with the mean $1/\lambda$. Why exponentially distributed? Delay-tolerant networks (DTN) are complex distributed systems that are composed of wireless mobile/fixed nodes, and they are typically assumed to experience frequent, long-duration partitioning, and intermittent node connection [18]. There have been various research works on the characteristics of the intercontact time between nodes [19]. Initial works typically assumed that the CCDF (complementary cumulative distribution function) of the inter-contact time decays exponentially over time and it is generally modeled using an exponential random variable [20]. This assumption is supported by numerical simulations conducted under most existing mobility models in the literature [21].

Note that as T is a duration, we should have $\lambda < 1$. This parameter represents the average time between two transmission attempts. We consider a slotted model. In each slot, the user sends a packet with probability p and the jammer tries to jam with probability q. The user obtains one as the reward only if he sends successfully the file of N packets within the time frame T, otherwise the user gets the zero reward. We describe this plot using a multistage zero-sum game. The value of the game and the optimal strategies are found in closed form. In particular we show that if the transmission cost is too big then the game has a saddle point. At this equilibrium, for both players there is no sense to act i.e. to transmit and to jam. If the jamming cost is too big and the transmission cost is not too big then there is no sense for the jammer to jam. Of course, since there is no jamming threat and the transmission is not too costly then the user transmits packets safely. If both jamming and transmission costs are not too big, then mixed equilibrium arises where both players act equalizing chances for the opponent. We have established a conservation law for the activities of the user and the jammer. In particular, an increase of the jammer activity results in a decrease of the user activity. Furthermore, the conservation law is invariant with respect to the amount of data to send.

1.1 Organization of the Paper

The rest of this paper is organized as follows. In Section 2 and its subsection formulation and solution of the ALOHA game is given. Numerical modelling is performed in Section 3. In Section 4 the obtained results are generalized for the case where the channel can be in different states and change according to a Markov rule. Discussion of the obtain result and also a possible generalization of the game can be found in Section 5.

2 Model

We analyze an ALOHA-type medium access control protocol performance in the presence of a jammer with static channel state. We assume that a user wants to transmit a message of N packets in a time smaller or equal to T. In each slot the user sends a packet with probability p and the jammer tries to jam with probability q. The user obtains R as the reward only if he sends successfully the file of N packets within the time frame T, otherwise the user gets the zero reward.

For each transmission attempt, the sender will pay a cost C_T, and respectively, for each jamming attack, the jammer will pay C_J. Let V_i be the expected reward for total successful transmission when there are still i packets needed to be sent.

$$V_i = \max_p \min_q \left(-C_T p + C_J q + p(1-q)\lambda V_{i-1} + (1-p(1-q))\lambda V_i\right)$$

with $V_0 = R$, where R is the reward for successful transmission of all the packets.

Then the problem can be reformulated in the following multistage form:

$$V_i = \frac{1}{1-\lambda} \mathrm{val} \begin{array}{c} \\ \mathrm{T} \\ \mathrm{N} \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{J} & \mathrm{N} \end{array} \\ \begin{pmatrix} -C_T + C_J & \lambda(V_{i-1} - V_i) - C_T \\ C_J & 0 \end{pmatrix} \end{array}, \tag{1}$$

where *val* means either maxmin or the value of the game if maxmin coincides with minmax.

We study now the optimal strategies of the players and the value of the game. The results are collected in Theorems 1– 3 and their proofs are supplied in Appendix.

First, we will show that if the transmission cost is too big then the game has a saddle point telling that for both players there is no sense to act (to transmit and to jam).

Theorem 1. *Let there be still i packets needed to be sent. Then (N, N) is a saddle point if and only if*

$$\lambda V_{i-1} \le C_T,$$

then $V_i = 0$.
In particular, if the transmission cost C_T is too big, namely,

$$\lambda R \le C_T \tag{2}$$

then $V_i = 0$, $i \ge 1$ and for both players there is no sense to act (to transmit and to jam).

Second, we will show that if jamming cost C_J is too big then there is no sense for the jammer to jam. Of course, since there is no jamming thread and the transmission is not too costly then the user transmit packets safely.

Theorem 2. *Let there be still i packets needed to be sent. Let assume that*

$$C_T < \lambda R \text{ and } R\lambda(1-\lambda) + \lambda C_T \le C_J. \tag{3}$$

Then (T, N) is a saddle point for $i \le i_*$, (N, N) is a saddle point for $i > i_*$ and

$$V_i = \begin{cases} R\lambda^i - \dfrac{1-\lambda^i}{1-\lambda} C_T, & i \le i_* \\ 0, & i > i_*, \end{cases}$$

where i_* is given as follows:

$$i_* = \left\lceil \frac{\ln\left(\frac{C_T}{\lambda(R(1-\lambda)+C_T)}\right)}{\ln(\lambda)} \right\rceil . \tag{4}$$

Finally we will consider the case where jamming and transmission costs are not too big. Then mixed equilibrium arises in which both players act with some probabilities.

Theorem 3. *Let there be still i packets needed to be sent. Let*

$$C_T < \lambda R \text{ and } C_J < R\lambda(1-\lambda) + \lambda C_T . \tag{5}$$

Then the game has mixed equilibrium for $i < i_*$ where i_* is the minimal integer such that

$$V_{i-1} - \frac{C_J}{1-\lambda} + \sqrt{\left(V_{i-1} - \frac{C_J}{1-\lambda}\right)^2 + 4\frac{C_J C_T}{(1-\lambda)\lambda}} \leq \frac{2\max\{C_T, C_J\}}{\lambda} .$$

The value of the game for $i < i_*$ is given by

$$V_i = \frac{C_J}{1-\lambda}\left(1 - \frac{C_T}{\lambda}\frac{1}{V_{i-1} - V_i}\right) . \tag{6}$$

The equilibrium mixed strategies $(p_i, 1-p_i)$ and $(q_i, 1-q_i)$ are given as follows:

$$p_i = \frac{C_J}{\lambda(V_{i-1} - V_i)}, \quad q_i = 1 - \frac{C_T}{\lambda(V_{i-1} - V_i)} .$$

For $i \geq i_*$ $V_i = 0$ and (N, N) is a saddle point.

It is interesting to note that there is a simple linear correlation between the probabilities to act for both players, namely

$$C_T p_i + C_J q_i = C_J . \tag{7}$$

This relation is independent of the number of number of packets left to send and moreover, this relation establishes a conservation law for the total activities of the user and the jammer in the regime of mixed strategies. In particular, an increase of the jammer activity results in a decrease of the user activity. Furthermore, this conservation law is invariant with respect to the amount of data to send.

3 Numerical Illustrations

As a numerical example consider situation with $\lambda = 0.9$ and $R = 1$. The value of the game in Figure 1 for transmission cost $C_T \in [0.001, 0.02]$, 2, 3 and 4 packets left to send and jamming cost $C_J = 0.005$ and $C_J = 0.01$. Also the optimal user strategy for transmission cost $C_T \in [0.001, 0.02]$, 2 and 4 packets left to send and jamming cost $C_J = 0.005$ and $C_J = 0.01$. The optimal jammer strategy and 2 and 4 packets left to send and jamming cost $C_J = 0.01$. We can restrict ourself mainly to the optimal use's strategies because a strong linear correlation between them and the jammer's strategies (7).

One can see that the value of the game and the optimal use's strategies are very sensitive to the changing of the environment. We observe that the activity of the user is decreasing with the cost of transmission, which is an intuitive result. Moreover, if jamming cost is decreasing then jammer activity arises (because the activity of the user decreases) and the value of the game goes down.

Finally, when transmission cost C_T increases, difference in user's payoff is increasing under different environment conditions.

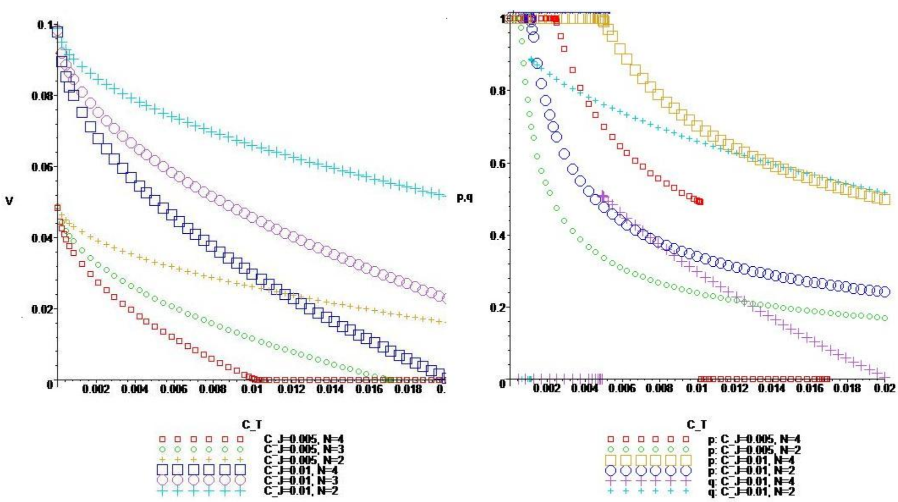

Fig. 1. The value of the game and the user's equilibrium strategy

4 Markov ALOHA Game

In this section we consider a variation of the game for the case where the channel can be in two states: good (1) and bad (0), and it can change its state according to a Markov rule. We denote by X_t the state of the channel at time slot t. Namely, with probability α_{xy}, $x, y = 0, 1$ the channel switches from state x to state y, i.e. $\text{Prob}(X_{t+1} = y | X_t = x)$. So, $\alpha_{x0} + \alpha_{x1} = 1$ and $\alpha_{0y} + \alpha_{1y} = 1$. We also assume that the probability of successful transmission in state x, if there is no jamming, is γ_x, where $\gamma_1 = 1$ and $\gamma_0 = \gamma$. So, if there is jamming then transmission is blocked with sure. If there is no jamming in the good channel state, then the transmission performs with sure and in the bad channel state it carries on with probability γ. Let $V_{i,x}$ be the expected reward for total successful transmission when there are still i packets needed to be sent and the channel is in state x. The action of the sender and the jammer depends on the state x of the channel. We define now by p_x (resp. q_x) the probability of transmission (resp. of jamming) when the channel is in state x. Then, in general case for $V_{i,x}$ the following maxmin equations hold:

$$\begin{aligned} V_{i,x} = \max_{p_x} \min_{q_x} \Big\{ & -C_T p_x + C_J q_x \\ & + \lambda \Big(p_x q_x (\alpha_{xx} V_{i,x} + \alpha_{xy} V_{i,y}) \\ & + p_x (1 - q_x)(\gamma_x \alpha_{xx} V_{i-1,x} + \gamma_x \alpha_{xy} V_{i-1,y}) \\ & + p_x (1 - q_x)((1 - \gamma_x)\alpha_{xx} V_{i,x} + (1 - \gamma_x)\alpha_{xy} V_{i,y}) \\ & + (1 - p_x)(\alpha_{xx} V_{i,x} + \alpha_{xy} V_{i,y}) \Big) \Big\} \end{aligned} \tag{8}$$

with

$$V_{0,x} = R \tag{9}$$

and $\{x, y\} = \{0, 1\}$.

Then the problem can be reformulated in the following multistage form:

$$\begin{aligned} & V_{i,1} - \lambda(\alpha_{11} V_{i,1} + \alpha_{10} V_{i,0}) \\ & = \operatorname{val} \begin{matrix} \\ \mathrm{T} \\ \mathrm{N} \end{matrix} \begin{pmatrix} \mathrm{J} & \mathrm{N} \\ -C_T + C_J & \lambda(\alpha_{11} W_{i,1} + \alpha_{10} W_{i,0}) - C_T \\ C_J & 0 \end{pmatrix}, \end{aligned} \tag{10}$$

and

$$\begin{aligned} & V_{i,0} - \lambda(\alpha_{00} V_{i,0} + \alpha_{01} V_{i,1}) \\ & = \operatorname{val} \begin{matrix} \\ \mathrm{T} \\ \mathrm{N} \end{matrix} \begin{pmatrix} \mathrm{J} & \mathrm{N} \\ -C_T + C_J & \lambda\gamma(\alpha_{00} W_{i,0} + \alpha_{01} W_{i,1}) - C_T \\ C_J & 0 \end{pmatrix}, \end{aligned} \tag{11}$$

with

$$W_{i,x} = V_{i-1,x} - V_{i,x} \text{ for } x = 1, 2,$$

where *val* means either maxmin or the value of the game if maxmin coincides with minmax.

4.1 Solution of Markov ALOHA Game

In this Section we will find solution of the Markov ALOHA game. First note that straightforward from (10) and (11) and Theorem 8 we have the following result.

Theorem 4. (N, N) *is a saddle point for both state if and only if*

$$\alpha_{11}V_{i-1,1} + \alpha_{10}V_{i-1,0} \leq C_T/\lambda,$$
$$\alpha_{01}V_{i-1,1} + \alpha_{00}V_{i-1,0} \leq C_T/(\gamma\lambda).$$

In particular, if $V_{i,1} = 0$ *then* $V_{i,0} = 0$. *Also, if the transmission cost* C_T *is too big, namely,*

$$C_T \geq \lambda R$$

then there is no sense in transmission at all and so in jamming, then

$$V_{i0} = V_{i1} = 0, \quad i \geq 1.$$

So, we can assume now that

$$C_T < \lambda R$$

Then we have only for situation left to deal with:

(a) the jamming cost is too big that jammer does not jam in both state, so users can send packets safely,
(b) the jamming cost is too big for bad channel state and not to big for good channel state, so in bad channel state users stick to pure equilibrium strategies (T, N) meanwhile in the bad channel state users employ mixed equilibrium strategies,
(c) the jamming cost is not big and then the users acts according to mixed equilibrium strategies.

These three situations are described in the following theorems.

Theorem 5. (T, N) *is the saddle point for both states if and only if*

$$\begin{aligned} V_{i,1} &= \lambda(\alpha_1 V_{i,1} + \alpha_{10}V_{i,0}) - C_T, \\ V_{i,0} &= \lambda\frac{\gamma\alpha_{00} + \lambda(1-\gamma)\alpha_{01}\alpha_{10}}{1-\lambda(1-\gamma)\alpha_{00}}V_{i-1,0} \\ &\quad + \lambda\frac{\gamma\alpha_{01} + \lambda(1-\gamma)\alpha_{01}\alpha_{11}}{1-\lambda(1-\gamma)\alpha_{00}}V_{i-1,1} \\ &\quad - \frac{1+\lambda(1-\gamma)\alpha_{01}}{1-\lambda(1-\gamma)\alpha_{00}}C_T \end{aligned} \tag{12}$$

and

$$\frac{C_T}{\lambda} \leq \alpha_{11}(V_{i-1,1} - V_{i,1}) + \alpha_{10}(V_{i-1,0} - V_{i,0}) \leq \frac{C_J}{\lambda},$$
$$\frac{C_T}{\lambda\gamma} \leq \alpha_{01}(V_{i-1,1} - V_{i,1}) + \alpha_{00}(V_{i-1,0} - V_{i,0}) \leq \frac{C_J}{\lambda\gamma}$$

In particular for $i = 1$*:*

$$\begin{aligned} V_{1,1} &= \lambda R - C_T, \\ V_{1,0} &= \lambda R\frac{\gamma + \alpha_{01}\lambda(1-\gamma)}{1-\alpha_{00}\lambda(1-\gamma)} - \frac{1+\alpha_{01}\lambda(1-\gamma)}{1-\alpha_{00}\lambda(1-\gamma)}C_T \end{aligned} \tag{13}$$

and

$$R - \frac{C_J}{\lambda} \le \alpha_{11}V_{1,1} + \alpha_{10}V_{1,0} \le R - \frac{C_T}{\lambda},$$
$$R - \frac{C_J}{\lambda\gamma} \le \alpha_{00}V_{1,0} + \alpha_{01}V_{1,1} \le R - \frac{C_T}{\lambda\gamma}.$$

Now we consider the situation where both players in both states according to equilibrium apply mixed strategies.

Theorem 6. $(p_{i,x}, q_{i,x})$, $x = 0, 1$ *be the equilibrium in mixed strategy if and only if*

$$p_{i,1} = \frac{C_J}{\lambda(\alpha_{11}V_{i-1,1} + \alpha_{10}V_{i-1,0} - \alpha_{11}V_{i,1} - \alpha_{10}V_{i,0})},$$
$$q_{i,1} = 1 - \frac{C_T}{\lambda(\alpha_{11}V_{i-1,1} + \alpha_{10}V_{i-1,0} - \alpha_{11}V_{i,1} - \alpha_{10}V_{i,0})},$$
$$p_{i,0} = \frac{C_J}{\lambda\gamma(\alpha_{01}V_{i-1,1} + \alpha_{00}V_{i-1,0} - \alpha_{01}V_{i,1} - \alpha_{00}V_{i,0})},$$
$$q_{i,0} = 1 - \frac{C_T}{\lambda\gamma(\alpha_{01}V_{i-1,1} + \alpha_{00}V_{i-1,0} - \alpha_{01}V_{i,1} - \alpha_{00}V_{i,0})},$$

and $V_{i,1}$ *and* $V_{i,0}$ *are solutions of equations*

$$\begin{aligned} &V_{i,1} - \lambda(\alpha_{i1}V_{i,1} + \alpha_{10}V_{i,0}) \\ &= C_J - \frac{C_T C_J}{\lambda(\alpha_{11}V_{i-1,1} + \alpha_{10}V_{i-1,0} - \alpha_{11}V_{i,1} - \alpha_{10}V_{i,0})}, \\ &V_{i,0} - \lambda(\alpha_{01}V_{i,1} + \alpha_{00}V_{i,0}) \\ &= C_J - \frac{C_T C_J}{\lambda\gamma(\alpha_{01}V_{i-1,1} + \alpha_{00}V_{i-1,0} - \alpha_{01}V_{i,1}) - \alpha_{00}V_{i,0})}, \end{aligned} \tag{14}$$

where the following conditions have to hold:

$$\max\{C_T, C_J\} \le \lambda[\alpha_{11}(V_{i-1,1} - V_{i,1}) + \alpha_{10}(V_{i-1,0} - V_{i,0})],$$

$$\max\{C_T, C_J\} \le \lambda\gamma[\alpha_{01}(V_{i-1,1} - V_{i,1}) + \alpha_{00}(V_{i-1,0} - V_{i,0})].$$

It is interesting to note that there is a simple linear correlation independent on the number packets left to send between the probabilities to act for both players in different channel states, namely

$$C_T p_{i,x} + C_J q_{i,x} = C_J, \quad x = 0, 1$$

which implies the fact that increasing activity of the jammer leads to reducing activity of the user at each channel state.

Finally we consider the situation with jamming cost which is to high to jam in the good channel state and at the same time it allows to jam in the good channel state.

Theorem 7. *$(p_{i,1}, q_{i,1})$ and (T, N) be the equilibrium for good and bad channel states if and only if*

$$V_{i,0} = \frac{\lambda\gamma(\alpha_{00}V_{i-1,0} + \alpha_{01}V_{i-1,1}) - C_T}{1 - \lambda(1-\gamma)\alpha_{00}} + \frac{\lambda(1-\gamma)\alpha_{01}V_{i,01}}{1 - \lambda(1-\gamma)\alpha_{00}}$$

and

$$\begin{aligned} &V_{i,1} - \lambda(\alpha_{i1}V_{i,1} + \alpha_{10}V_{i,0}) \\ &= C_J - \frac{C_T C_J}{\lambda(\alpha_{11}V_{i-1,1} + \alpha_{10}V_{i-1,0} - \alpha_{11}V_{i,1} - \alpha_{10}V_{i,0})}, \end{aligned}$$

where the following conditions have to hold:

$$\begin{aligned} &\frac{C_T}{\lambda} \le \alpha_{11}(V_{i-1,1} - V_{i,1}) + \alpha_{10}(V_{i-1,0} - V_{i,0}) \le \frac{C_J}{\lambda}, \\ &\frac{\max\{C_T, C_J\}}{\lambda} \le \alpha_{11}(V_{i-1,1} - V_{i,1}) + \alpha_{10}(V_{i-1,0} - V_{i,0}). \end{aligned}$$

Then, we have obtained, in a general framework, where the channel can be in good or bad state, the existence of different equilibrium even in pure or in mixed strategy. In the next section, we explore a particular asymmetric case for the transition probabilities.

4.2 A Particular Case: The Asymmetric Case

In this Section we consider in detail the asymmetric case $\alpha_{11} = \alpha_{01} = \alpha$ and $\alpha_{00} = \alpha_{01} = 1 - \alpha$. Then in the situation with mixed strategies in both states by Theorem 6 we have that

$$\begin{aligned} &V_{i,1} - \lambda(\alpha V_{i,1} + (1-\alpha)V_{i,0}) = C_J \\ &- \frac{C_T C_J}{\lambda(\alpha V_{i-1,1} + (1-\alpha)V_{i-1,0} - \alpha V_{i,1} - (1-\alpha)V_{i,0})}, \\ &V_{i,0} - \lambda(\alpha V_{i,1} + (1-\alpha)V_{i,0}) = C_J \\ &- \frac{C_T C_J}{\lambda\gamma(\alpha V_{i-1,1} + (1-\alpha)V_{i-1,0} - \alpha V_{i,1} - (1-\alpha)V_{i,0})}, \end{aligned}$$

Summing up the last two equations multiply by α and $1 - \alpha$ respectively, and substracting from the first equation the second one multiplied by γ we obtain the following two relations first of them give a recurrent formula for finding the expected value of payoff $\alpha V_{i,1} + (1-\alpha)V_{i,0}$ at different states, and the second one gives a strong linear correlation between payoffs:

$$\begin{aligned}(1-\lambda)(\alpha V_{i,1} + (1-\alpha)V_{i,0}) = C_J \\ - \frac{C_T C_J(\alpha + (1-\alpha)/\gamma)}{\lambda(\alpha V_{i-1,1} + (1-\alpha)V_{i-1,0} - \alpha V_{i,1} - (1-\alpha)V_{i,0})},\end{aligned} \tag{15}$$

$$\begin{aligned}&(1-\lambda(1-\gamma)\alpha)V_{i,1} \\ &= (\gamma + \lambda(1-\gamma)(1-\alpha))V_{i,0} + C_J(1-\gamma).\end{aligned} \tag{16}$$

Then, subtracting (16) from (15) implies:

$$\alpha V_{i,1} + (1-\alpha)V_{i,0} = B$$

with

$$\begin{aligned}B := \frac{\alpha V_{i-1,1} + (1-\alpha)V_{i-1,0} + \dfrac{C_J}{1-\lambda}}{2} \\ - \frac{1}{2}\left[\left(\alpha V_{i-1,1} + (1-\alpha)V_{i-1,0} - \frac{C_J}{1-\lambda}\right)^2 \right. \\ \left. + 4\frac{(\gamma + \lambda(1-\gamma)(1-\alpha))C_J C_T}{(1-\lambda)\lambda}\right]^{1/2}\end{aligned}$$

Thus, the optimal payoffs are given as follows:

$$\begin{aligned}V_{i,0} &= \frac{1 - \lambda\alpha(1-\gamma)B - \alpha(1-\gamma)C_J}{1-\alpha(1-\gamma)}, \\ V_{i,1} &= \frac{(\gamma + \lambda(1-\gamma(1-\alpha)))B + (1-\alpha)(1-\gamma)C_J}{1-\alpha(1-\gamma)}\end{aligned}$$

5 Discussion and Extensions

In this paper we suggested a development of the channel capacity concept for a dynamic slotted ALOHA network. We found maxmin successful transmission of an information over a dynamic communication channel. To do so, we analyzed a simple ALOHA-type medium access control protocol performance in the presence of a jammer as a zero-sum dynamic game. The obtained results are generalized for the case where the channel can be in different states and change according to a Markov rule. We considered only the simplest case the channel can be in two states: good (1) and bad (0). If there is jamming then transmission is blocked with sure. If there is no jamming in the good channel state, then the transmission performs with sure and in the bad channel state it carries on with a probability γ. The probabilities with which the channel switches from one state to the other are known and fixed. For this game also the recurrent formulas for finding the optimal solution are obtained. As the other direction of the investigation we are planning to deal with the uncomplete information case, say, when jamming cost and transmission costs are unknown to the rival correspondingly.

6 Appendix

Before solving our game (1) let us remind the following result [1] which supplies all the equilibrium for 2×2 matrix zero-sum game.

Theorem 8. *Let A be the zero-sum game with the following matrix:*

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}.$$

This game

(a) *either has a saddle point (each saddle point can be found as the an element of this matrix which is the minimal one in its row and it is the maximal one in its column),*
(b) *or a couple of mixed equilibrium strategies $(x, 1-x)$, $(y, 1-y)$ where*

$$x = \frac{A_{22} - A_{21}}{A_{11} - A_{12} + A_{22} - A_{21}},$$
$$y = \frac{A_{22} - A_{12}}{A_{11} - A_{12} + A_{22} - A_{21}},$$
$$v = \frac{A_{11}A_{22} - A_{12}A_{21}}{A_{11} - A_{12} + A_{22} - A_{21}}.$$

Note that the mixed equilibrium exists if and only if either

$$A_{11} > A_{12}, A_{12} < A_{22}, A_{22} > A_{21}, A_{21} < A_{11} \tag{17}$$

or

$$A_{11} < A_{12}, A_{12} > A_{22}, A_{22} < A_{21}, A_{21} > A_{11}. \tag{18}$$

In our case

$$\begin{aligned} &A_{11} = -C_T + C_J,\ A_{12} = \lambda(V(i-1) - V(i)) - C_T, \\ &A_{21} = C_J,\ A_{22} = 0. \end{aligned} \tag{19}$$

Then, only two pairs of strategies (N, N) and (T, N) could be saddle points in our game under some circumstance. Theorems 1 and 2 supply the condition under which either (N, N) or (T, N) is saddle point. Theorem 3 deals with the rest case, namely, where the mixed equilibrium arises.

Proof of Theorem 1: By (19) (N, N) presents a saddle point if and only if

$$\lambda(V_{i-1} - V_i) - C_T \le 0 \text{ for any } i.$$

and the result follows.

Proof of Theorem 2: By (19) (T, N) presents a saddle point if and only if

$$\lambda(V_{i-1} - V_i) - C_T > 0$$

and

$$C_J - C_T \geq \lambda(V_{i-1} - V_i) - C_T$$

which is equivalent to

$$C_T \leq \lambda(V_{i-1} - V_i) \leq C_J. \tag{20}$$

Thus, by (20), (T, N) is a saddle point if the jamming cost has to be bigger than the transmission one, namely

$$C_T \leq C_J. \tag{21}$$

Also, since (T, N) is a saddle point, by (1), we have that

$$V_i = \frac{1}{1-\lambda}\left(\lambda(V_{i-1} - V_i) - C_T\right).$$

Thus,

$$V_i = \lambda V_{i-1} - C_T. \tag{22}$$

Substituting (22) into (20) turns (20) into the following equivalent form:

$$\frac{C_T}{\lambda} \leq V_{i-1} \leq \frac{1}{\lambda(1-\lambda)}(C_J - \lambda C_T). \tag{23}$$

Now let have a look at (23) for $i = 1$. Since $V_0 = R$ then the left part of (23) is clear. The right part of (23) holds if

$$R\lambda(1-\lambda) + \lambda C_T \leq C_J. \tag{24}$$

Then by induction from (22) we can obtain that

$$V_i = R\lambda^i - \frac{1-\lambda^i}{1-\lambda}C_T \text{ while (23) holds.}$$

Also, since $\lambda C_T > R$ then (24) implies (21).
It is clear that V_i is decreasing function from $V_0 = R$ and $V_\infty = -C_T/(1-\lambda)$ and (4) holds, where i_* is the root of the equation

$$R\lambda^i - \frac{1-\lambda^i}{1-\lambda}C_T = \frac{C_T}{\lambda}$$

Finally note that by (3)

$$\max\{C_T, R\lambda(1-\lambda) + \lambda C_T\} = R\lambda(1-\lambda) + \lambda C_T.$$

This completes proof of Theorem 2.

Proof of Theorem 3: In this Theorem we want to find mixed strategies and the condition where they take place. Since by (19 $A_{21} = C_J > C_J - C_T = A_{11}$ then the situation (17) cannot hold. Also, $A_{22} = 0 < C_J = A_{21}$. Then conditions (18) are equivalent to the following two inequalities:

$$-C_T + C_J < \lambda(V_{i-1} - V_i) - C_T \text{ and } 0 < \lambda(V_{i-1} - V_i) - C_T.$$

Thus, we have the following condition for existence of equilibrium in mixed strategies:

$$\max\{C_T, C_J\} \le \lambda(V_{i-1} - V_i). \tag{25}$$

Then, by Theorem 8, we have that (6) holds. Introduce the following notation:

$$W_i = V_{i-1} - V_i.$$

In the new notation, (6) can be presented in the following way:

$$-W_i + V_{i-1} = \frac{C_J}{1-\lambda}\left(1 - \frac{C_T}{\lambda W_i}\right). \tag{26}$$

So,

$$W_i = \frac{V_{i-1} - \frac{C_J}{1-\lambda} \pm \sqrt{(V_{i-1} - \frac{C_J}{1-\lambda})^2 + 4\frac{C_J C_T}{(1-\lambda)\lambda}}}{2}.$$

Since, by (25), $W_i > 0$ from the last relation we have that

$$W_i = \frac{V_{i-1} - \frac{C_J}{1-\lambda} + \sqrt{(V_{i-1} - \frac{C_J}{1-\lambda})^2 + 4\frac{C_J C_T}{(1-\lambda)\lambda}}}{2}. \tag{27}$$

Then, substituting (27) into (25) implies the following equivalent presentation for (25) just in terms of V_{i-1}:

$$\frac{2C_T}{\lambda} \le V_{i-1} - \frac{C_J}{1-\lambda} + \sqrt{(V_{i-1} - \frac{C_J}{1-\lambda})^2 + 4\frac{C_J C_T}{(1-\lambda)\lambda}}. \tag{28}$$

Also, (27) yields that $V(i)$ has the form

$$V_i = \frac{V_{i-1} + \frac{C_J}{1-\lambda} - \sqrt{\left(V_{i-1} - \frac{C_J}{1-\lambda}\right)^2 + 4\frac{C_J C_T}{(1-\lambda)\lambda}}}{2} \tag{29}$$

and

$$V_{i-1} - V_i = \frac{V_{i-1} + \sqrt{\left(\frac{C_J}{1-\lambda} - V_{i-1}\right)^2 + 4\frac{C_J C_T}{(1-\lambda)\lambda}} - \frac{C_J}{1-\lambda}}{2} > 0. \tag{30}$$

Thus, V_i given by (29) is decreasing on i. Then by (27) W_i is also decreasing. Finally, we have to check whether (25) holds for $i = 1$. By (30) it is equivalent to

$$\sqrt{\left(\frac{C_J}{1-\lambda} - R\right)^2 + 4\frac{C_J C_T}{(1-\lambda)\lambda}} \ge \frac{C_J}{1-\lambda} - R + \frac{2\max\{C_T, C_J\}}{\lambda}. \tag{31}$$

Since for $C_T > C_J$ the inequality (31) is equivalent to $C_T \le \lambda R$, and for $C_T < C_J$ the inequality (31) is equivalent to $C_J \le \lambda C_T + \lambda R(1-\lambda)$ we have the following result supplying the value of the game. This completes proof of Theorem 3.

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