Convergence Dynamics of Resource-Homogeneous Congestion Games*

Richard Southwell and Jianwei Huang

Information Engineering Department, The Chinese University of Hong Kong
Shatin, New Territories, Hong Kong
{richardsouthwell254,jianweihuang}@gmail.com

Abstract. Many resource sharing scenarios can be modeled as congestion games. A nice property of congestion games is that simple dynamics are guaranteed to converge to Nash equilibria. Loose bounds on the convergence time are known, but exact results are difficult to obtain in general. We investigate congestion games where the resources are homogeneous but can be player-specific. In these games, players always prefer less used resources. We derive exact conditions for the longest and shortest convergence times. We also extend the results to games on graphs, where individuals only cause congestions to their neighbors. As an example, we apply our results to study cognitive radio networks, where selfish users share wireless spectrum opportunities that are constantly changing. We demonstrate how fast the users need to be able to switch channels in order to track the time-variant channel availabilities.

Keywords: congestion game, resource allocation, cognitive radio, games on graphs, convergence time.

1 Introduction

Congestion games can be used to model a myriad of systems in biology, engineering, and the social sciences. In congestion games, players select resources to maximize their payoffs, while considering the congestion due to resource sharing. A key feature of congestion games is the *finite improvement property*: asynchronous player updating (where players switch resources to increase payoffs) always converges to a Nash equilibrium [1]. The finite improvement property is important due to the wide applicability of congestion games [2]. In many situations, such as drivers choosing routes or wireless users picking channels, it is useful to know that distributed and selfish behaviors always lead to a stable system state.

In this paper, we shall focus on the following question: *how long does convergence take in a congestion game?* Understanding this issue is critical for real-time resource allocation. Unfortunately, determining the convergence time

* This work is supported by the General Research Funds (Project Number 412509) established under the University Grant Committee of the Hong Kong Special Administrative Region, China.

R. Jain and R. Kannan (Eds.): GameNets 2011, LNICST 75, pp. 281–293, 2012.

of congestion games can be a computationally hard problem [3], [4]. The worst case convergence time can be exponential in the number of players n [5], although a polynomial convergence is possible for certain games [6], [7].

In this paper, we will analyze the convergence speed of a simple kind of congestion game, where *every resource has the same intrinsic value.* The players may have complicated and *different* payoff functions, but this does not affect the convergence dynamics. All that matters is that players prefer less congested resources. Despite the simplicity, this type of game can model a wide range of scenarios where selfish individuals share homogenous resources. In Section 5, we will illustrate why our games are particularly useful for analyzing cognitive radio networks.

To study the convergence of congestion games, the usual approach is to gain *loose bounds* by constructing a potential function. The special structure of our games allows us to take a more geometric approach, and derive many new (and sometimes *exact*) results about convergence (see Section 3). In Section 4, we will analyze more general congestion games, where the players are spatially distributed and can access different sets of resources.

Our main results include the following:

- An *exact* characterization of the *fastest* convergence time from *any* given initial state (Section 3.1);
- A *simple* updating mechanism, which *guarantees* the fastest convergence time (Section 3.1);
- *Exact* results and a *linear* bound on the slowest possible convergence time (Section 3.3);
- Characterizations of the convergence speed to Nash equilibrium on spatially extended congestion games (Section 4).

As a concrete application, in Section 5 we discuss how the analytical framework can be applied to spectrum sharing in cognitive radio networks. As the available resources (channels) come and go rapidly in cognitive radio networks (e.g., [8,9]), it is critical to understand how selfish users behave and whether they can adapt fast enough compared with the environment. **Proofs of our results are included in the online technical report [10].**

2 The Model

In an n-player congestion game, each player's strategy involves a set of resources. The payoff that a player p gets from using a resource i is described by a strictly decreasing function $f_i^p(x_i)$, where x_i is the number of players using i. In this paper, we are only concerned with *singleton congestion games*, where each player uses exactly one resource at any given time [6]. We further assume that resources are symmetric, i.e., $f_i^p = f_j^p = f^p$ for any resources i, j and any player p, so all resources are equivalent from a player's perspective. Note, however, that the payoffs are player-specific. This means players could have different tastes of the same resource, due to technological, psychological, or economic reasons.

A central idea in congestion game dynamics is the **better response switch**, i.e., a player increases its payoff by switching to a better (less congested) resource. A common way to study congestion games is to imagine that players' strategies evolve through time via asynchronous better response switches (i.e., *one player switches to a better decision at each discrete time step*).[1] When no player can increase their payoff by switching, the system reaches a *Nash equilibrium.*

To summarize, our systems are defined by a set of n players, a set of r resources, and a strictly decreasing payoff function f^p for each player p. Every player uses one resource from the set $\{1, 2, ..., r\}$ at any time step. The system *state vector* is $\boldsymbol{x} = (x_1, x_2, ..., x_r)$, where x_i is the number of players using resource i. If a player using resource i switches to resource j, then x_i decreases by 1 and x_j increases by 1. We refer to this action as an $i \to j$ *switch.* We suppose that one and only one player switches to a better response every time step. A key fact about our games is Theorem 1.

Theorem 1 (Better Response Switch)
A switch $i \to j$ *is a better response switch if and only if* $x_j + 1 < x_i$.

Proof. Consider a player p using resource i with a payoff $f^p(x_i)$. Switching to j is a better response if and only if $f^p(x_j + 1) > f^p(x_i)$. Now, since f^p is strictly decreasing, we have $f^p(x_j + 1) > f^p(x_i)$ if and only if $x_j + 1 < x_i$.

Theorem 1 has powerful implications: the better response switches (through which our system evolves) are independent of the payoff functions and the identity of the players. All that matters (with respect to the dynamics) is that one player decreases its congestion level by switching. A state $\boldsymbol{x}$ is a Nash equilibrium if and only if no better response switches can be performed.

Theorem 2 (Nash Equilibrium). *A state vector* $\boldsymbol{x}$ *is a Nash equilibrium if and only if* $(n \bmod r)$ *of* $\boldsymbol{x}$*'s entries are equal to* $\left\lceil \frac{n}{r} \right\rceil$*, whilst the remaining* $r - (n \bmod r)$ *entries of* $\boldsymbol{x}$ *are equal to* $\left\lfloor \frac{n}{r} \right\rfloor$.

3 Convergence Time to a Nash Equilibrium

Starting from an arbitrary state $\boldsymbol{x}$, the players can reach a Nash equilibrium through several routes (sees figures 1, 2 and 3). This means the convergence time depends upon the ways players choose to switch. The convergence time is an important measure of how quickly the players organize themselves. Since players' identities and payoff functions are irrelevant to the convergence dynamics, we will only work with the number of players n, the number of resources r, and system state $\boldsymbol{x}$ from now on.

[1] We are subscribing to the *elementary step hypothesis* [5], that *one and only one* player improves their strategy at each time step. This is commonly used to model situations where simultaneous updating is unlikely.

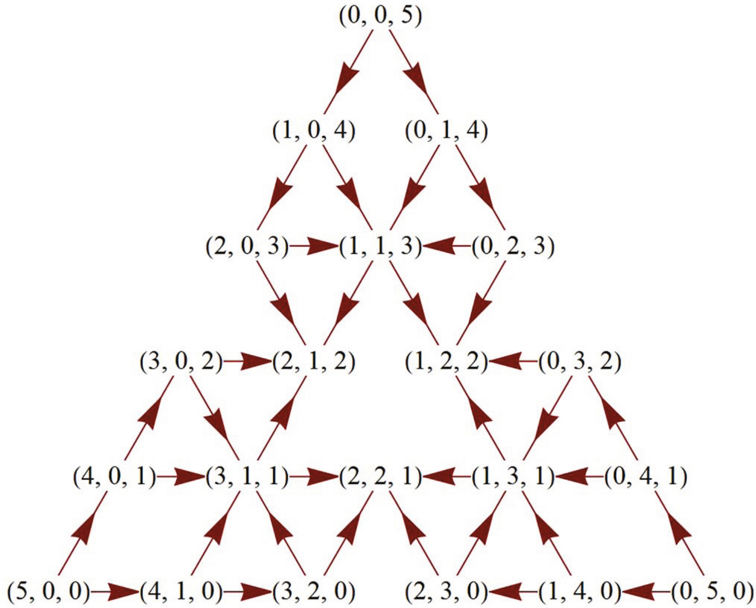

Fig. 1. The state space of a game with $n = 5$ players and $r = 3$ resources. The points represent state vectors and the arrows represent the *state transitions* which can be achieved through better response switches. For example, there is an arrow from $(5, 0, 0)$ to $(4, 1, 0)$ because the better response switch $1 \to 2$ converts $(5, 0, 0)$ into $(4, 1, 0)$. The Nash equilibria in this game are $(1, 2, 2)$, $(2, 1, 2)$, and $(2, 2, 1)$.

3.1 The Fastest Convergence

We want to determine the best way a group of players can switch their choices of resources to reach a Nash equilibrium. We will first study how many switches (*of any kind*) are required to convert one state into another.

Theorem 3 (The Switching Distance). *For any two states* $\boldsymbol{x}$ *and* $\boldsymbol{y}$*, the minimal number of switches (of any kind),* $d(\boldsymbol{x}, \boldsymbol{y})$*, required to convert* $\boldsymbol{x}$ *into* $\boldsymbol{y}$ *is*

$$d(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=1}^{r} \max\{0, y_i - x_i\} = \sum_{i=1}^{r} \max\{0, x_i - y_i\} = \frac{\sum_{i=1}^{r} |x_i - y_i|}{2}.$$

The **fastest convergence time**, $b(\boldsymbol{x})$, is the least number of better response switches that are required to convert state $\boldsymbol{x}$ into a Nash equilibrium.

Theorem 4 (Fastest Convergence Time). *For any state* $\boldsymbol{x}$*, the best (fastest) convergence time to a Nash equilibrium is*

$$b(\boldsymbol{x}) = \left(\sum_{i=1}^{r} \max\left\{0, x_i - \left\lfloor \frac{n}{r} \right\rfloor \right\} \right) - \min\left\{ \left| \left\{ i \in \{1, 2, ..., r\} : x_i \geq \left\lceil \frac{n}{r} \right\rceil \right\} \right|, n \bmod r \right\}.$$

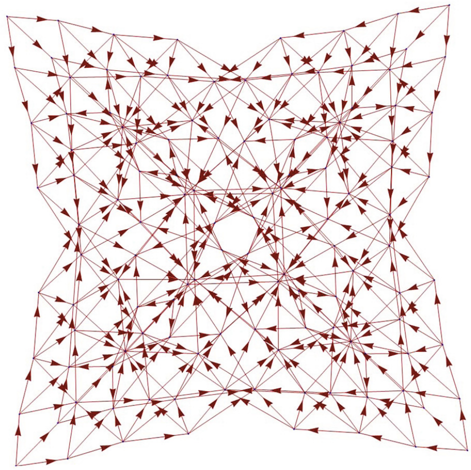

Fig. 2. The state space of a game with $n = 7$ players and $r = 4$ resources. The points represent state vectors and the arrows represent the *state transitions* which can be achieved through better response switches.

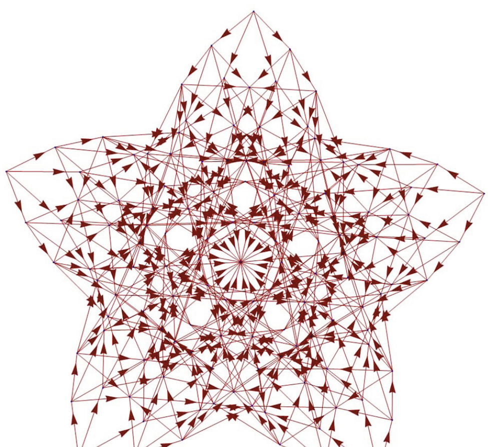

Fig. 3. The state space of a game with $n = 5$ players and $r = 5$ resources. The points represent state vectors and the arrows represent the *state transitions* which can be achieved through better response switches. Every path of sufficient length in this graph terminates at the central vertex -which represents the Nash equilibrium $(1, 1, 1, 1, 1)$.

Theorem 4 essentially states that $b(\boldsymbol{x})$ *is the minimal number of switches (of any kind) required to convert* $\boldsymbol{x}$ *into a Nash equilibrium.* To explain how a Nash equilibrium can be reached in this minimal number of switches, we define the strong switch. Suppose $\boldsymbol{x}$ is not a Nash equilibrium. We say an $i \to j$ switch is a **strong switch** of $\boldsymbol{x}$ if x_i is one of $\boldsymbol{x}$'s maximal entries and x_j is one of $\boldsymbol{x}$'s minimal entries. For example, the $2 \to 4$ switch is a strong switch of $\boldsymbol{x} = (5, 5, 3, 1)$, while the $2 \to 3$ switch is not. Theorem 4 is proved by showing that strong switching is *optimal*, in the sense that it leads to a Nash equilibrium within the minimum possible number of switches.

Corollary 1. *Starting from an arbitrary state* $\boldsymbol{x}$*, any sequence of* $b(\boldsymbol{x})$ *strong switches leads to a Nash equilibrium.*

Theorem 4 implies that the maximum value of $b(\boldsymbol{x})$ occurs when all the players use the same resource (e.g., $\boldsymbol{x} = (n, 0, 0, ..., 0)$). Hence $b((n, 0, 0, ..., 0)) = n - \lceil \frac{n}{r} \rceil$ is the largest number of better response switches ever *required* to reach a Nash equilibrium.

3.2 Average Fastest Convergence from Random Initial Conditions

Often we do not have a choice of the initial system state. Thus it is useful to understand how fast we can reach a Nash equilibrium from random initial conditions. This gives a *global average* measure of the best case performance of our systems. Suppose the players select their initial resources from $\{1, 2, ..., r\}$ uniformly at random. In this case, we let $\beta_r(n)$ denote the expected number of strong switches required to reach Nash equilibrium (for a game with r resources and n players). In other words, $\beta_r(n)$ is the expected value of $b(\boldsymbol{x})$, when $\boldsymbol{x}$ is generated by allocating resources randomly and uniformly.

Theorem 5 (Average Fastest Convergence Time)
Suppose n *is divisibly by* r*, in this case*[2] *we have,*

$$\beta_r(n) = \left(1 - \frac{1}{r}\right)^{n - \frac{n}{r}} \left(\frac{1}{r}\right)^{\frac{n}{r}} \left(n - \frac{n}{r}\right) \frac{n!}{(n - \frac{n}{r})! \frac{n}{r}!},$$

$$\lim_{n \to \infty} \beta_r(n) = \sqrt{\frac{n(r-1)}{2\pi}}.$$

3.3 The Slowest Convergence

In reality, players may not switch in the optimal way. Often it is more likely that random players perform random better response switches. In this case, there are many ways the system can evolve. Here we discuss the **slowest convergence time** $w_r(n)$ for a system with r resources and n players. We define $w_r(n)$ to be largest number of better response switches that drives some initial state into a Nash equilibrium.

[2] There is a strong evidence that this asymptotic form is also accurate when n is not divisible by r (see technical report [10]).

Theorem 6 (Slowest Convergence Time). *The worst (slowest) convergence time, $w_r(n)$, has the following properties;*

1. $\frac{n(r-1)}{2} - \left(\frac{2.r^3-3.r^2+r}{6}\right) \leq w_r(n) \leq \frac{n(r-1)}{2}$.
2. $\lim_{n\to\infty} w_r(n) = \frac{n(r-1)}{2}$.
3. $w_2(n) = \lfloor \frac{n}{2} \rfloor$.
4. $w_3(n) = n - 1$.
5. $w_4(n) = \delta_{n,1} + \delta_{n,2} + \frac{3.n}{2} - (n \mod 2)\frac{5}{2} - (n+1 \mod 2)\left(2 + \left(\lfloor \frac{n}{2} \rfloor \mod 2\right)\right)$ *where $\delta_{i,j}$ is the Kronecker delta.*
6. *If $n \leq r$ then $w_r(n) = \sum_{k=0}^{n-1} \left\lfloor \frac{\sqrt{1+8k}-1}{2} \right\rfloor$.*

Our state space can be regarded as a directed graph $\mathcal{D}$, with directed edges representing the state transitions that can be achieved through better response

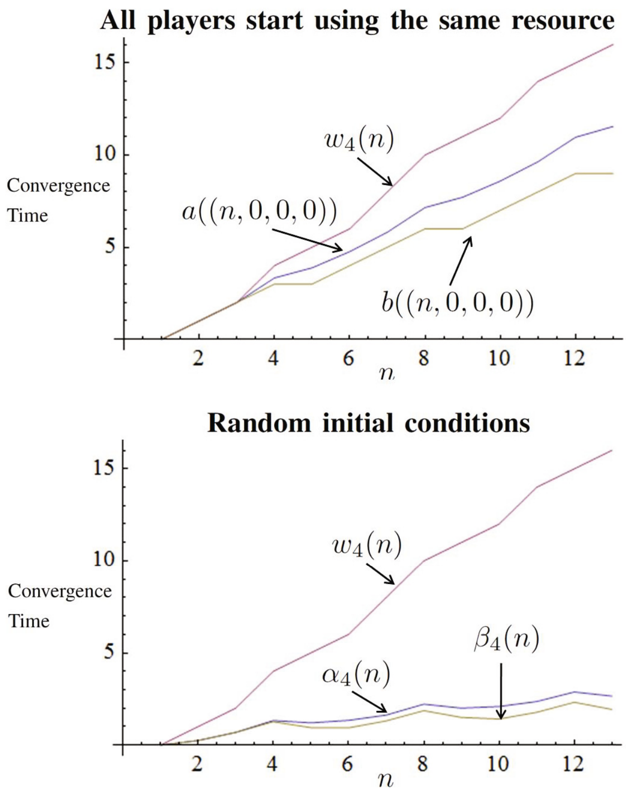

Fig. 4. Convergence times with $r = 4$ resources. The top line in both figures depicts the slowest run time $w_4(n)$. In the upper figure, the bottom line depicts the fastest run time $b((n, 0, 0, 0))$, whilst the middle line depicts the expected run time, $a((n, 0, 0, 0))$, of the random better response system. In the lower figure, the bottom line depicts the average fastest run time, $\beta_4(n)$, whilst the middle line depicts expected run time, $\alpha_4(n)$, of the random better response system (starting from a random resource allocation).

switches (see figures 1, 2 and 3). Here $w_r(n)$ is equal to the length of the longest path in $\mathcal{D}$. We proved Theorem 6 by considering the *reduced state space*, $\mathcal{D}^*$, which is subgraph of $\mathcal{D}$ induced upon the states with their entries in descending order. Here $w_r(n)$ is equal to the longest path from $(n, 0, 0, ...0)$ to $\left(\left\lceil \frac{n}{r} \right\rceil, ..., \left\lceil \frac{n}{r} \right\rceil, \left\lfloor \frac{n}{r} \right\rfloor, ..., \left\lfloor \frac{n}{r} \right\rfloor\right)$ in $\mathcal{D}^*$. Also $\mathcal{D}^*$ is equivalent to the r-part partition lattice [11]. The full proof of Theorem 6 can be found in the technical report [10].

3.4 Average Convergence, with Random Better Response Switches

In many scenarios, the players will update in some sort of random order. Understanding of random cases gives an insight into more realistic systems, and tests the relevance of our performance bounds. We consider a simple random updating model, where at each time step a single random "unsatisfied" player (i.e, one that *can* perform a better switch) performs a random better switch. Let $a(\boldsymbol{x})$ denote the **expected value of the convergence time of $\boldsymbol{x}$ under this "random better response system"**. We simulate and then compare with our other convergence time measures in Figure 4. Simulations suggest that the random convergence time $a(\boldsymbol{x})$ is often close to the fastest convergence time $b(\boldsymbol{x})$, and is much smaller than the worst case convergence time $w_r(n)$ (when the number of players n is reasonable large). This implies that $b(\boldsymbol{x})$ is a good estimation of the real convergence time and $w_r(n)$ is too pessimistic.

4 Spatial Variations on Our Models

So far we have assumed that each player interacts with all other players in the system. In many situations, the players are distributed over space, and can only cause congestion to their neighbors. This is the case in wireless networks, where only users close-by may cause significant mutual interferences. In this section, we look at the convergence of the *spatial congestion games* proposed in our previous work [9], where a graph structure $\mathcal{G}$ is used to represent the spatial relationship between players. This spatial game model is a generalization of the one defined in Section 2 with the added dimension of space.

Figure 5 shows an example of graph $\mathcal{G}$. Each vertex in $\mathcal{G}$ represents a player. A pair of players are connected by an edge in $\mathcal{G}$ when they are close enough to *potentially* cause congestion for one another. Each player p has a strictly decreasing payoff function, $f^p(N_i^p + 1)$, where N_i^p is the number of player p's neighbors that use the same resource i as player p. Just like before, the details of the payoff functions are irrelevant with respect to the convergence dynamics. All that matters is that players always prefer resources that are used by less of their neighbors.

In this general model, a *state* is an assignment of resources on the graph, one resource to each vertex (player). A *conflict* happens when a vertex uses the same resource as one of its neighbors. We still assume the resources are homogenous, so every vertex simply wishes to minimize the number of conflicts they incur. As before, the system evolves via asynchronous better response switches. Every

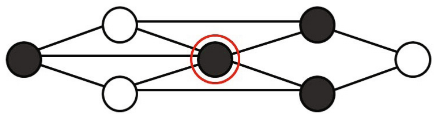

Fig. 5. An example of a state (resource allocation) of a spatial congestion game. Every vertex can access two resources: "white" and "black". The only unsatisfied player (vertex) is the circled one in the middle. This player currently suffers 3 conflicts. After this player does a better response switch (changing from black to white), the system will be in a Nash equilibrium.

time step, one vertex switches to a resource shared by less of its neighbors. A Nash equilibrium is a system state where no vertex can make a better response switch. We further generalize the model by assuming that different players may have *different sets of resources* available to them due to spatial variations. Formally, each player p has a specific resource set $R(p)$. Despite the generality of the model, convergence to a Nash equilibrium through better response switches is still guaranteed. Next we provide characterize the convergence speed and equilibrium characteristics. Theorem 7 is an extension of a result from [9].

Theorem 7. *Consider a general spatial congestion game with heterogeneous resource availabilities defined on a graph $\mathcal{G}$. Any sequence of $||\mathcal{G}||$ asynchronous better response switches leads to a Nash equilibrium, where $||\mathcal{G}|| \leq \frac{n(n-1)}{2}$ is the number of edges in the graph $\mathcal{G}$.*

Proof. The worst case initial state is where every vertex uses the same resource. In this case there are $||\mathcal{G}||$ conflicts. The number of conflicts in a given state forms a potential function, which decreases with better response switches. It follows that the system will converge to Nash equilibrium in at most $||\mathcal{G}||$ time steps.

Our next result bounds the worst case performance of a player at Nash equilibrium.

Theorem 8. *At a Nash equilibrium of a general spatial congestion game with heterogeneous resource availabilities, a player p will not suffer more than $\left\lfloor \frac{d(u)}{|R(p)|} \right\rfloor$ conflicts, where $d(p)$ is p's degree (number of neighbors) and $|R(p)|$ is the number of resources available to p.*

Proof. Let us prove by contradiction. Suppose player p suffers $K > \left\lfloor \frac{d(p)}{|R(p)|} \right\rfloor$ conflicts. Since the system is in a Nash equilibrium, player p cannot benefit by switching to a different resource. This means for any resource $i \in R(p)$, player p has at least K neighbors using resource i. It follows that p has degree greater than or equal to $|R(p)|K > d(p)$, which contradicts our assumption that player p has a degree $d(p)$.

Theorem 8 implies that when every player can access more resources than its degree, the system will always converge to a Nash equilibrium which involves no conflicts whatsoever.

5 Application: Cognitive Radio Networking

A good application for the above analytical framework is spectrum sharing in cognitive radio networks. Most of the usable wireless spectrum is owned by *license holders* who possess exclusive transmission rights. However, measurements show that many wireless channels are heavily under-utilized most of the time. Cognitive radio technology allows unlicensed *users* to opportunistically access these channels when the license holders are absent. One of the central questions of such spectrum sharing is: *how should users access the channels in a distributed fashion?*

Congestion games are a natural way to model how users switch channels to minimize their interference. The models discussed in this paper are very useful here due to several reasons. (1) In most wireless communication standards, spectrum is divided into equal bandwidth channels. Interleaving techniques can further homogenize the qualities of the channels. This means that channels are homogeneous to the same user. Different users, however, can achieve different data rates due to different choices of coding and modulation schemes. (2) A wireless user only generates significant interferences to close-by users, so spatial information is important. (3) License holders are spatially located and may have different activities; unlicensed users at different locations may be able to access different channels. (4) License holders often have stochastic traffic, meaning the channel availability is time-varying.

5.1 The Significance of Previous Results

Fast convergence is essential in cognitive radio networks due to the time variability of channels. Corollary 1 shows that the fastest convergence is achieved with strong switches, in which case no user needs to switch channel more than once. This is desirable as switching channel causes a costly disruption to quality of service. Theorem 6 states the upper-bound of the worst case convergence time $w_r(n)$ is linear in r and n. This suggests that congestion games remains a viable method for spectrum sharing in large networks. Theorem 6 also suggests that a user does not have to scan every channel; it is enough to locate one better channel before switching. This is useful as channel scanning is often time consuming in wireless communications.

The spatial congestion game makes a more realistic model for cognitive radio networks. This fits well into the commonly used protocol interference model, where the interference relationships among users is modeled by a graph. Theorems 7 and 8 guarantee that users in the network quickly converge to an efficient Nash equilibria. Theorem 8 is especially encouraging, because it implies that the Nash equilibrium will be interference free if users have enough channels to choose from.

5.2 Modeling the Dynamic Radio Environment

As licence holders enter and leave the system, the set of available channels changes. As this happens the Nash equilibria of the system changes and the users must adapt to this. Figure 6 illustrates how the ability of users to adapt quickly influences their performance. Here we study how fast the (unlicensed) users should adapt in order to catch up with channel dynamics.

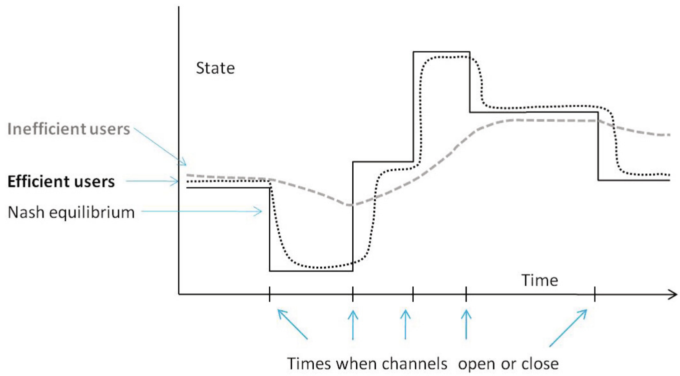

Fig. 6. An illustration of the dynamic spectrum allocation problem. As time goes by channels open and close. This changes the position of the Nash equilibrium, so the population of users must adapt their state accordingly. If the users converge quickly (dotted line) they will be able to stay close to the Nash equilibrium. If the users cannot adapt fast enough (dashed line) they will spend most of their time away from Nash equilibrium (note that in reality the Nash equilibrium will be moving around in high dimensional space).

We assume that the license holders enter and leave channels randomly, so the availability of each channel can be described by a two-state Markov chain. On average there are c channels open to the users in the network. In any given time step, a new channel opens with a probability p_1, an old channel closes with a probability p_0, and nothing happens with a probability $1 - p_0 - p_1$. We assume the time scale is so small that multiple channels will not open or close simultaneously.[3]

Necessary Switching Rate. The users must be able to perform at least $A = p_1 \left\lfloor \frac{n}{c+1} \right\rfloor$ switches per time step in order to always stay at a Nash equilibrium. The reasoning is as follows. When a new channel opens, it requires at least $\left\lfloor \frac{n}{c+1} \right\rfloor$ switches to fill the new channel with the proper number of users to reach a Nash equilibrium. When an old channel closes, the disruption depends upon

[3] In this section we are considering a “time step” to be defined with respect to the channel dynamics, and we are considering how fast the users must be able to update with respect to *this* time scale.

how many users occupied it and how they evacuate. If the users do not have time to perform A switches per time step, then they will drift away from the new Nash equilibria.

Sufficient Switching Rate. If users can do $B = \max\{p_1 w_{c+1}(n), p_0 w_{c-1}(n)\} \leq \frac{n}{2} \max\{p_1 c, p_0(c-2)\}$ switches per time step, they will almost certainly be able to stay at the Nash equilibrium. To see this, note that every sequence of better response switches $w_c(n)$ will converge to Nash equilibrium. Then users can "track" the equilibria if they can do B switches every time step.

6 Conclusion

The resource-homogeneous nature of our models has allowed us to investigate the convergence dynamics at a deeper level. This is satisfying because, although simple, our systems can be used to model a wide range of phenomena. Exact results about convergence rates are rare, and we hope that the results presented here will aid the study of more general systems. Perhaps our most critical modeling assumption is the elementary step hypothesis [5], that one player updates their strategy every time step. This could be unrealistic in many scenarios with large numbers of players. In the future we will study the dynamics of other simple congestion games with different updating mechanisms. There are many other directions we wish to take in future research. One interesting direction, for cognitive radio, will be to incorporate *switching costs* into our models. **Proofs of our results are included in the online technical report [10].**

Acknowledgments. The authors wish to thank Jing Ji for her helpful input into this research.

References

1. Rosenthal, R.: A class of games possessing pure-strategy Nash equilibria. International Journal of Game Theory 2, 65–67 (1973)
2. Vöcking, B., Aachen, R.: Congestion games: Optimization in competition. In: Proceedings of the Second ACiD Workshop (2006)
3. Fabrikant, A., Papadimitriou, C., Talwar, K.: The Complexity of Pure Nash Equilibria. In: Proceedings of STOC 2004 (2005)
4. Fotakis, D., Kontogiannis, S., Spirakis, P.: Atomic Congestion Games Among Coalitions. In: Bugliesi, M., Preneel, B., Sassone, V., Wegener, I. (eds.) ICALP 2006, Part I. LNCS, vol. 4051, pp. 572–583. Springer, Heidelberg (2006)
5. Fotakis, D., Kaporis, A.C., Spirakis, P.G.: Atomic Congestion Games: Fast, Myopic and Concurrent. In: Monien, B., Schroeder, U.-P. (eds.) SAGT 2008. LNCS, vol. 4997, pp. 121–132. Springer, Heidelberg (2008)
6. Ieong, S., Mcgrew, R., Nudelman, E., Shoham, Y., Sun, Q.: Fast and Compact: A Simple Class of Congestion Games. In: Proceedings of AAAI 2005 (2005)

7. Ackermann, H., Röglin, H., Vöcking, B.: On the Impact of Combinatorial Structure on Congestion Games. In: Proceedings of FOCS 2006 (2006)
8. Law, L., Huang, J., Liu, M., Li, S.: Price of Anarchy for Cognitive MAC Games. In: Proceedings of GLOBECOM 2009 (2009)
9. Ahmad, S., Tekin, C., Liu, M., Southwell, R., Huang, J.: Spectrum Sharing as Spatial Congestion Games. arXiv:1011.5384v1 [cs.GT] (2010)
10. Southwell, R., Huang, J.: Online technical report, `http://richardsouthwell.wordpress.com/technicalreport/`
11. Wan, H., Wootton, J.: A global compositional complexity measure for biological sequences: AT-rich and GC-rich genomes encode less complex proteins. Computers and Chemistry 24, 71–94 (2000)