Analyzing the Dynamics of Evolutionary Prisoner's Dilemma on Structured Networks

Ahmet Yasin Yazicioglu, Xiaoli Ma, and Yucel Altunbasak

School of Electrical and Computer Engineering,
Georgia Institute of Technology, Atlanta, GA 30332, USA
{yasin,xiaoli,yucel}@ece.gatech.edu

Abstract. The spread of cooperation in the evolutionary dynamics of social dilemma games such as Prisoner's Dilemma can be facilitated by various means such as topological heterogeneities, a high benefit-to-cost ratio, or asymmetric interactions. In evolutionary dynamics, the agents adopt the strategies of neighbors with higher payoffs with a probability proportional to the payoff difference. In this study, we analyze evolutionary dynamics of mixed strategies in the Prisoner's Dilemma game through the expected value of the payoff difference for arbitrary neighbors and the evolutionary advantage/disadvantage of nodes due to the degree distributions within their neighborhoods. Simulation results for various networks and game parameters are also presented.

1 Introduction

Social systems consist of various individuals who try to benefit from interactions among themselves. In such systems, the emergence of situations in which collective interests contradict private interests are inevitable. These situations are known as social dilemmas. When a social dilemma occurs, related individuals usually have two options: they either cooperate or defect, and their decisions affect the overall outcome. In this manner, while the cooperators represent the people who contribute to the collective behavior at personal expense, whereas the defectors represent the ones who do not. In game theory, social dilemmas are analyzed through widely-used metaphors such as the Prisoner's Dilemma, Stag-Hunt, and Hawk-Dove (also known as Chicken or Snowdrift) games [1,2,3]. While the payoffs taken from possible cooperate-defect combinations differ among the models, in all of these games, agents get higher payoffs (rewarded) when they both cooperate rather than both defect.

When the game models are considered for very large populations, the rationality assumption is relatively controversial [4]. For such populations, evolutionary game theory is used in the analysis of population dynamics. In evolutionary games, instead of being rational maximizers, players enter the game with particular strategies attached to them. Players then confront other players who are programmed to play their own strategies and accumulate payoffs from these interactions. At the end of each time period (generation), strategies that provide higher payoffs may be adopted by neighboring agents. However, the number of

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players who utilize strategies that provide lower payoffs decline. This process is similar to evolution in biology. While biological evolution may occur through natural selection, for socio-economic phenomena evolution mostly arises as a consequence of imitation or learning. Eventually, a population attains an equilibrium state, namely an *evolutionary stable configuration*. A strategy is called an "evolutionary stable strategy" (ESS) if a whole population using that strategy cannot be invaded by a small group with a mutant genotype that uses a different strategy. In this context, ESS is a refined form of the Nash equilibrium, which provides the main solution basis for classical analysis. While all evolutionary stable strategies are also Nash equilibria, the converse is not necessarily true [5].

Conventional evolutionary games treat a population as an infinite and homogeneous structure, in which every agent has an equivalent position on a corresponding network. However, this is not true for real populations [8,9] in which interactions and connections among the agents are determined by social and spatial constraints. Hence, most real populations have heterogeneous network topologies that may significantly affect the overall behavior of the corresponding population. For instance, in the Prisoner's Dilemma game, since cooperation is a strictly dominated strategy, it can never invade a population in the evolutionary sense for infinite homogeneous societies [1]. However, recent studies in the literature show that the topological properties of structured networks and game parameters [10] - [15], and heterogeneities and asymmetries in the interactions of agents [16,17] can facilitate the spread of cooperation through evolutionary dynamics. In analyzing the cooperative behavior of a network, we utilize mixed strategies as they provide a higher resolution in the quantitive comparison of different cases and reveal certain information that can not be extracted via pure strategy analysis [18].

In this work, we analyze the evolutionary dynamics on structured networks for the Prisoner's Dilemma game. Such an analysis may be used for various purposes. For instance, biological, economic, political studies and many others may benefit from estimating the level and the strength of cooperative behavior in a particular population. Moreover, when it is possible to manipulate the topology or game parameters, this analysis can be used for design purposes. In problems such as designing an organizational structure of a company, transportation planning or the design of many other systems in which a large number of autonomous agents will participate, it is desirable to attain a topology that implies a more cooperative behavior since it increases the overall utility of the population. The expected probability of cooperation at steady state provides a measure for quantifying the evolutionary favorability of cooperation. Due to the complex nature of dynamics on heterogeneous, large-scale networks, this value can be estimated through simulation results rather than analytical or numerical methods. However, particularly for design purposes, it is crucial to explicitly represent the influence of topological parameters and game parameters on the evolutionary favorability of cooperation. To this end, we use analytical methods for analysis on the micro level. In [18] we present the initial steps of our approach by examining the expected value of payoff difference for arbitrary neighbors. In

Table 1. Game Table for Prisoner's Dilemma

		Player 2	
		Cooperate	*Defect*
Player 1	*Cooperate*	(*b-c*, *b-c*)	(-*c*, *b*)
	Defect	(*b*, -*c*)	(*0*, *0*)

this work, we extend our analysis and discussion related to the expected value of payoff difference and also investigate the evolutionary advantage/disadvantage of nodes due to the degree distributions within their neighborhoods. Based on the analysis, we discuss how topology and game parameters can influence the cooperative behavior of the network.

The organization of this paper is as follows: Section 2 presents the game model and the evolutionary dynamics. Section 3 analyzes the dynamics at the micro level along with the conditions for favorability of cooperation, and Section 4 presents the simulation results and discussions. Finally, Section 5 concludes the paper with some remarks and indicates possible future directions.

2 Game Model and Evolutionary Dynamics

Prisoner's Dilemma is perhaps the most-widely used metaphor representing the social dilemmas. In Prisoner's Dilemma, a cooperator pays a cost, c, for the other player to receive a benefit, b, where $b > c$. By contrast, a defector does not pay any cost and does not distribute any benefit. This scheme is depicted by the game table shown in Table 1.

Some social dilemmas in a population can be represented as rounds of Prisoner's Dilemma played among the people who interact with each other. The social group can be represented via a graph in which the individuals in the network occupy the vertices and links exist among the nodes that play the game against each other. At each round, the nodes play their current strategies against all of their neighbors and get accumulated payoffs. If mixed strategies are played, the payoffs are random rather than deterministic variables and the strategy of a node can be defined by the probability that it chooses to cooperate.

Analysis of this game on large populations can be obtained through evolutionary game theory [6,7]. Classical evolutionary game dynamics are defined for infinite well-mixed populations and known as "replicator dynamics". Replicator dynamics are defined by the following differential equation:

$$\dot{P}_i = P_i(U_i - \bar{U}) \tag{1}$$

where P_i is the fraction of phenotype i in a population, U_i is the fitness of this phenotype, defined as the average accumulated payoff for the members of this phenotype, and $\bar{U}$ is the average accumulated payoff in the entire population. For finite populations, replicator dynamics do not apply directly and analogous dynamics, which converge to the replicator dynamics in the limit of infinite complete graphs, should be used [3]. These dynamics involve the following events: At the beginning of each time step (generation), nodes play a single round of Prisoner's Dilemma against each of their neighbors and they accumulate the resulting payoffs. After that, each node, x, randomly picks one of its neighbors, y, and compares their accumulated payoffs U_x and U_y. Node x adopts the strategy of node y only if $U_y > U_x$ with the transition probability that increases monotonically to 1 as $U_y - U_x$ increases. Different functions can be used to define transition probabilities. One possible option is $1/(1 + exp[-(U_y - U_x)/K])$ where K characterizes possible noise effects [20]. Alternatively $(U_y - U_x)/(max(k_x, k_y)(b+c))$, where k_x and k_y are node degrees, can be used [12].

Let us assume that at the end of a time step an arbitrary node, x picks a random neighbor y to possibly adopt its strategy. As the essential property of any transition probability function is to equal zero when $U_y < U_x$ and monotonously increase as $U_y - U_x$ gets larger, in analyzing evolutionary dynamics we need to consider the difference of accumulated payoffs for these nodes. These accumulated payoffs can be represented as:

$$\begin{aligned} U_x &= \sum_{i=1}^{k_x-1} b_{xi} - \sum_{i=1}^{k_x-1} c_{xi} + \frac{b}{c} c_{yx} - c_{xy}, \\ U_y &= \sum_{i=1}^{k_y-1} b_{yi} - \sum_{i=1}^{k_y-1} c_{yi} + \frac{b}{c} c_{xy} - c_{yx}, \end{aligned} \tag{2}$$

where b_{xi} represents the benefit node x receives from its i^{th} neighbor, and c_{xi} represents the cost of cooperation that node x pays in its interaction with its i^{th} neighbor. Note that b_{xi}'s are independent variables which equals b with probability q_{xi} (cooperation probability of corresponding neighbor) and equals 0 with probability $1 - q_{xi}$. Random variables c_{xi} are also independent and each realization is either c (with probability p_x) or 0 (with probability $1 - p_x$). Random variables b_{yi} and c_{yi} similarly represent the interaction of node y with its neighbors. Furthermore, outcome of the round where x and y play against each other are depicted by the variables c_{xy}, the cost of cooperation node x pays, and c_{yx}, the cost of cooperation node x pays. Benefits received by the other player are deterministic functions of these variables since if one pays the cost the other receives the corresponding benefit. Note that $c_{xy} = c$ with probability p_x and $c_{xy} = 0$ with probability $1 - p_x$. Similarly we have $c_{yx} = c$ with probability p_y and $c_{yx} = 0$ with probability $1 - p_y$.

3 Micro Level Analysis

Our micro level analysis has two parts. First we consider the expected value of payoff difference, which determines, for arbitrary neighbors, whose strategy is more likely to fare better. This part of the analysis involves the game parameters and the topological parameters. Next we consider the evolutionary advantage of nodes. In that context we focus on the probability that a particular node is randomly picked by its neighbors to possibly adapt its strategy. This probability affects the chances of a node to spread its strategy and solely depends on the network topology.

3.1 Expected Value of Payoff Difference

For arbitrary neighbors x and y, transition probabilities are determined by the accumulated payoff difference of nodes, $U_y - U_x$. The strategy that provides its player a higher accumulated payoff will be evolutionary favored through the connection between x and y. Based on the expected value of accumulated payoff difference, $E[U_y - U_x]$, it is possible to say which strategy, on the average, will be favored [18]. If this value is negative, strategy of x provides a higher expected payoff and will be favored. On the other hand, strategy of y will be favored if $E[U_y - U_x]$ is positive. These two cases are separated by the condition where $E[U_y - U_x] = 0$. In light of Eq. 2, $E[U_y - U_x] = 0$ is attained when the following is satisfied:

$$c(k_x p_x - k_y p_y) + b(\bar{q}_y - \bar{q}_x + p_x - p_y) = 0, \tag{3}$$

where $\bar{q}_x = \sum_{i=1}^{k_x-1} q_{xi}$, $\bar{q}_y = \sum_{i=1}^{k_y-1} q_{yi}$. As b, c and k_y are positive, we can divide the inequality by $-(b + ck_y)$ and rearrange to obtain the condition as

$$p_y - \frac{b + ck_x}{b + ck_y} p_x + \frac{b}{b + ck_y} (\bar{q}_x - \bar{q}_y) = 0. \tag{4}$$

For the real variables p_x and p_y, Eq. (4) defines a line in two-dimensional space, $\mathbb{R}^2$. Note that for static network topology and constant game parameters this line has a constant slope but the intercept may change in time as the strategy of their neighbors, and consequently $\bar{q}_x$ and $\bar{q}_y$, may change during the evolutionary dynamics. For the points (p_x,p_y) which are located below this line we have p_y being favored as $E[U_y - U_x] > 0$. On the other hand, for the points (p_x,p_y) which are located above this line we have $E[U_y - U_x] < 0$ and p_x is favored. Note that as p_x and p_y are probabilities, they are bounded within interval $[0, 1]$. Depending on the parameters of the $E[U_y - U_x] = 0$ line, the feasible region where $p_x, p_y \in [0, 1]$ can at most be divided into four separate regions. Let us define these regions as follows: Region I is the region where $E[U_y - U_x] > 0$ and $p_y < p_x$. Region II is the region where $E[U_y - U_x] > 0$ and $p_y > p_x$. Region III is the region where $E[U_y - U_x] < 0$ and $p_y > p_x$. Finally, Region IV is the region where $E[U_y - U_x] < 0$ and $p_y < p_x$. In this context, Regions I and III are the regions where defective strategy has evolutionary advantage. Regions

II and IV, on the other hand, are the regions where cooperative strategy has evolutionary advantage. Existences and sizes of these regions are determined by the $E[U_y - U_x] = 0$ line. Possible cases can be classified under 9 major groups. Examples of cases in each group are depicted in Fig. 1.

Let us consider how the 9 groups shown in Fig. 1 are characterized. In light of Eq. (4) it can be seen that, as $(b + ck_x)/(b + ck_y)$ is always positive, the slope of the $E[U_y - U_x] = 0$ line is always positive. This constraint implies that either both x and y intercepts of the line are 0 or one of them is positive whereas the other one is negative. So initially we can separate the possible cases into 3 groups depending on the y intercept (or x intercept). As the $\frac{b}{b+ck_y}$ term is always positive, y intercept's being negative, positive or equal to zero is solely determined by $(\bar{q}_x - \bar{q}_y)$.

If $(\bar{q}_x - \bar{q}_y) < 0$, then we have positive y intercept and we can further obtain 3 distinct groups based on p_x^*, the value of p_x on the $E[U_y - U_x] = 0$ line for $p_y = 1$: $p_x^* < 0$ as shown in Fig. 1 (a), $0 < p_x^* < 1$ as shown in Fig. 1 (b), and $1 < p_x^*$ as shown in Fig. 1 (c).

If $(\bar{q}_x - \bar{q}_y) = 0$, then we have both intercepts equal to zero and we can further obtain 3 distinct groups based on the value of the slope: slope is greater than 1 ($k_x > k_y$) as shown in Fig. 1 (d), slope is equal to 1 ($k_x = k_y$) as shown in Fig. 1 (e), and slope is smaller than 1 ($k_x < k_y$) as shown in Fig. 1 (f).

If $(\bar{q}_x - \bar{q}_y) > 0$, then we have positive x intercept and we can further obtain 3 distinct groups based on the value of p_y^*, the value of p_y on the $E[U_y - U_x] = 0$ line for $p_x = 1$: $p_y^* < 0$ as shown in Fig. 1 (g), $0 < p_y^* < 1$ as shown in Fig. 1 (h), and $1 < p_y^*$ as shown in Fig. 1 (i).

For arbitrary neighbors x and y, valuable information about their evolutionary interaction is encoded in the group to which the $E[U_y - U_x] = 0$ line belongs. For instance if this line belongs to the groups shown in Fig. 1 (a) or Fig. 1 (i), we have a significantly influential node, x or y, as it accumulates higher expected payoff than the other node for every possible value of p_x and p_y in $[0, 1]$. In this case the possible evolutionary interaction may favor the cooperation if the influential node is utilizing a cooperative strategy. Alternatively, if this line belongs to the group shown in Fig. 1 (e), which consists of a single line that is coincident with $p_x = p_y$ line, neither Region II nor Region IV exists hence it is not possible to expect an evolutionary interaction that will favor the cooperation. Once the particular $E[U_y - U_x] = 0$ line is classified, one can also quantify the favorability of cooperation through the interaction of x and y by using the areas of the regions. This quantification can easily be obtained through geometry and it can be used to compare the favorability of cooperation in different cases. To this end, one may compare the area of the regions where cooperative strategy is favored (areas of Regions II and IV) to the area of regions where defective strategy is favored (areas of Regions I and III). Since these areas can explicitly be represented in terms of game parameters and node degrees, one can relate these parameters to the expected evolutionary outcome of the interaction between nodes x and y.

Evolutionary interaction at any time instant between two neighbors, x and y, is determined by the current $E[U_y - U_x] = 0$ line. As shown in Eq. (4) parameters

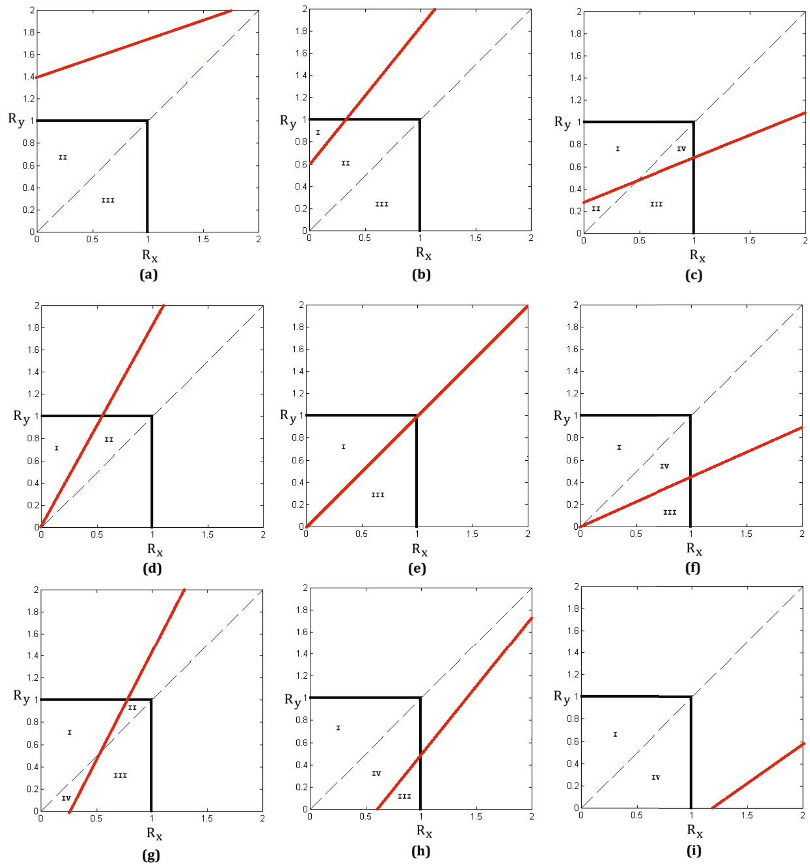

Fig. 1. Examples for each of the 9 major groups of possible cases determined by the $E[U_y - U_x] = 0$ line for two meeting agents, x and y. Upper bound of 1 for p_x and p_y on the axes of $\mathbb{R}^2$ (R_x and R_y) are marked with solid lines. Each group is characterized by the existence and structure of Region I ($E[U_y - U_x] > 0$, $p_y < p_x$), Region II ($E[U_y - U_x] > 0$, $p_y > p_x$), Region III ($E[U_y - U_x] < 0$, $p_y > p_x$) and Region IV ($E[U_y - U_x] < 0$, $p_y < p_x$). The $p_x = p_y$ line is also shown as dashed, whereas the line corresponding to $E[U_y] = E[U_x]$ is solid.

of this line depends on the topology and game parameters. To interpret Eq. (4) and the effect of different parameters on the cooperation let us first consider the example of a well-mixed homogeneous network where the cooperative behavior is known to be eliminated through evolution. For such a population, every node has the same degree (k_x=k_y) and every node is connected to all other nodes ($\bar{q}_x$=$\bar{q}_y$), which makes the $E[U_y - U_x] = 0$ line coincident with $p_y = p_x$ line, as depicted in Fig. 1 (e), for every neighbor x and y throughout the whole evolutionary dynamics. Since neither Region II nor Region IV exists for this case, it is impossible to expect an evolutionary interaction that is likely to favor the cooperation. Since $E[U_y] > E[U_x]$ can only occur when $p_y < p_x$, cooperation is strongly opposed by evolution. Note that the situation where the $E[U_y] = E[U_x]$ line coincident with $p_y = p_x$ line is the only case where the area of the portion where cooperative strategy is favored is zero and there is no combination of p_x and p_y where the cooperative strategy provides a higher expected payoff. In order to promote the cooperative behavior, the $E[U_y - U_x] = 0$ line needs to deviate from the $p_x = p_y$ line. This deviation can be achieved through the game parameters and heterogeneity in neighbor degrees. This way, for arbitrary neighbors it is possible to have some cases where more cooperative strategy provides a higher expected payoff and possibly be adopted by the other node.

3.2 Evolutionary Advantage of Nodes

In the previous section we presented the analysis and discussions on the evolutionary dynamics of cooperation through the expected value of accumulated payoff differences for arbitrary neighbors. However, the spread of a particular strategy from a node depends on the possibility of having a neighbor node considering to adopt its strategy. For a node y to have its strategy adopted by node x, first of all node x should pick node y to possibly adopt its strategy. Hence, in addition to the ares of different regions as proposed in previous part, the probabilities that one node picks the other node should also be considered. If the interaction of two nodes at any time instant is considered, this can be incorporated into the analysis by weighing the regions I and III with probability of node y picking node x and weighing the regions II and IV with probability of node x picking node y. Based on the assumption that nodes pick any of their neighbors with equal probability these probabilities are $1/k_y$ and $1/k_x$ respectively.

When we consider the overall network, although the final step in adoption of a strategy depends on the accumulated payoffs, clearly some nodes have higher chance than the others as being considered by more nodes at each generation. Intuitively, one may expect that the nodes with higher degrees are favored in this context and it is partly true. However, only having large degree is not enough for a node to have this topological advantage. Consider a scenario, where a particular node has a very large degree but its neighbors also have very large degrees. For this particular node the probability of being chosen by any of its neighbors is quite small and it may not acquire a significant advantage from the topology.

Let us consider the topological significance of a node for evolutionary dynamics as the expected number of nodes per generation which pick it and may adopt its strategy, and denote it as n. Based on the assumption that nodes pick any of their neighbors with equal probability, for a particular node x we have:

$$n_x = \sum_{i=1}^{k_x} \frac{1}{k_{yi}} \tag{5}$$

where k_{yi} is the degree of the i^{th} neighbor of x. For a complete graph with N nodes, nodes have equivalent positions and they are equally significant as all having $n = 1$. Any given graph can be considered as the outcome of a procedure where certain edges are removed from a complete graph. As these edges are removed, some nodes increase their chances to spread their strategies whereas others have their chances decreased. While the total significance (N) is distributed uniformly among the nodes for a complete graph, an arbitrary topology is likely to result in a different distribution.

Having a significance value greater than 1 provides an evolutionary advantage for the particular node. Note that regardless of the topology every node definitely picks one neighbor to possibly adopt its strategy, which can be considered as the measure of the local influence on the evolution of a node. Hence for a network with N nodes we have:

$$\sum_{i=1}^{N} n_i = N \tag{6}$$

Every node is equally influenced from its neighborhood in the evolution process, as each of them considers to adopt exactly one of its neighbors at each generation. However, nodes do not influence their neighborhoods equally. Nodes with significance values greater than 1 can have more influence on their neighborhoods than their neighborhoods have on them. On the other hand nodes with significance values smaller than 1 can not influence the evolution in their neighborhoods as much as they are influenced by that.

To see how this topological effect influences the evolutionary spread of a node's strategy, let us consider a simple scenario as shown in Figure 2. In this figure a small graph with 15 nodes is given. The nodes initially have pure cooperation or pure defection strategies assigned to them. Given that game parameters satisfy $b > 6c$ the evolution on this graph definitely ends up at a state where all nodes cooperate as cooperators accumulate higher payoffs than their defecting neighbors. Throughout the evolutionary process, nodes 1 and 2 spread their strategies. Both nodes have degree of 6, and their clustering coefficients are also equal. However, when we consider their chances to spread their strategies it can be seen that node 1 spreads its strategy much faster. If we consider the first generation in evolutionary dynamics and pick the transition probability as in [12], neighbors of node 1 and node 2 adopt their strategies with equal probability of transition, $p_t = (b-3c)/(3b+3c)$. However, expected number of neighbors that pick node 1 to adopt its strategy is larger than the expected number of neighbors that pick node 2. It can easily be shown that at the first time step, expected number of defectors that adopt cooperation from node 1 is $4p_t/3$ whereas expected number of defectors that adopt cooperation from node 2 is $3p_t/4$.

Nodes with large n values are much efficient in spreading their strategies and have an important role in spreading cooperative behavior as cooperators exploit having large n value better than defectors. This is due to the fact that as a cooperator converts a defecting neighbor it increases its fitness, its chance to survive and spread. As long as all of the defector neighbors of a cooperator are doing better, it will eventually adopt the defection strategy. However the interesting condition occurs when a portion of defecting neighbors are doing worse. In this case if the cooperator can convert any of those defectors, it increases its fitness

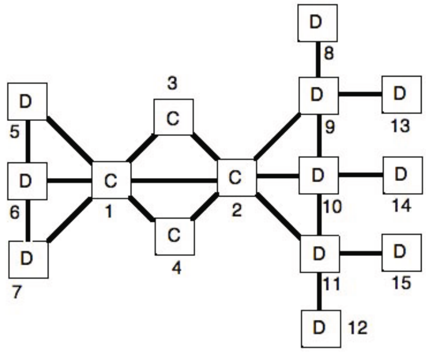

Fig. 2. A simple example to see how topology provides an evolutionary advantage to certain nodes. While for the game parameters $b > 6c$ this system converges to all cooperators, node 1 spreads cooperation faster than node 2.

and the portion of its defecting neighbors that are doing worse increases, as long as all other defecting neighbors are not also neighbors of the converted node. As n increases, the chance of being picked by such a defecting neighbor significantly increases and so does the survival potential of a cooperator. For defectors, on the other hand, this process works the opposite way. When the defector converts any of its cooperating neighbors, it decreases its fitness and the portion of its cooperating neighbors that are doing worse decreases, as long as all other cooperating neighbors are not also neighbors of this new defecting node. Eventually, the defector may end up at a state where it has a certain amount of cooperating neighbors doing better than him and adopt their strategy. Moreover when a defector has low n value, its chance of converting cooperating neighbor decreases hence it does not cause a significant decrease in its own fitness. However, this time it is more likely to adopt the strategy of one of its cooperating neighbors, especially if some of them have large n values and are able to spread cooperation in their neighborhood. Low n values have even worse effect on the cooperators as their survival chances are greatly attenuated when their spreading probability decreases. Based on this discussion we can say that when there is a heterogeneous distribution of n values in the network, this works in favor of the cooperators. Although, they have their reduced survival chances for small n values they can benefit greatly from large n values. Defectors, on the other hand, do not obtain a great survival advantage from neither small nor large n values.

4 Simulation Results

Simulations were carried out for various networks and different game parameters. First, we consider mixed strategy evolutions for various cases on Erdös-Rényi random, Watts-Strogatz small world and Barabási-Albert scale-free topologies with 1000 nodes. We check for the expected probability of cooperation at steady state for varying average degree and game parameters. Networks with average degrees (k) 4,6,8,10 and 12 are generated for each topology. Game parameters are normalized by setting $b = 1$ and c is varied in $0.02 - 0.2$ interval with 0.02 increments. For each combination of k and c, 10 simulations are run and averaged result is reported. Steady state values are obtained through averaging of 1000 time steps after a warm up period of $10,000$ time steps, starting from a uniform distribution of strategies among the nodes. Transition probabilities are computed as presented in [12].

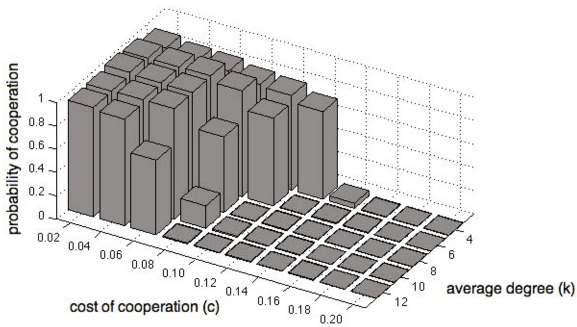

Fig. 3. Expected probability of cooperation at steady state on Erdös-Rényi random networks for various average degrees and costs of cooperation

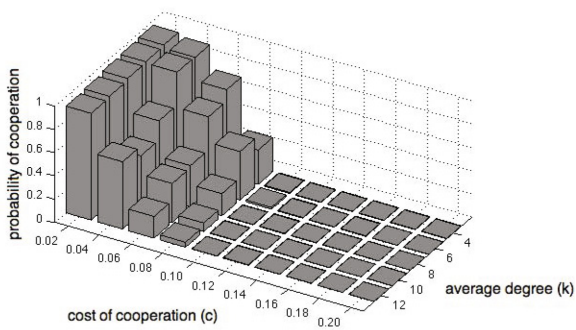

Fig. 4. Expected probability of cooperation on Watts-Strogatz small world networks for various average degrees and costs of cooperation

As it can be seen from the results, expected probability of cooperation is highly dependent on the network topology. For the same average degree and cost of cooperation the steady state behaviors are quite different among the three topologies. As Barabási-Albert scale-free being the most heterogeneous topology, it shows a more cooperative behavior in a wider range of k and c. In this topology, direct links among the hubs also help to promote a certain level cooperation. Note that, in this topology hubs mostly have high significance values (n) as a

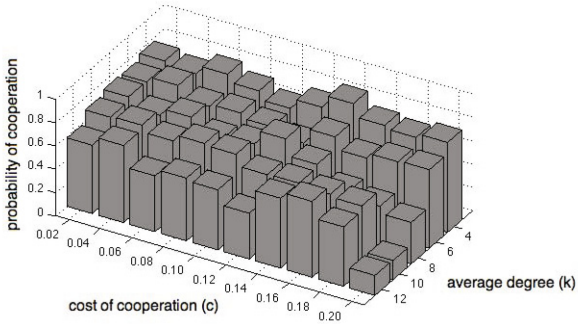

Fig. 5. Expected probability of cooperation on Barabási-Albert scale-free networks for various average degrees and costs of cooperation

significant amount of their neighbors have very low degrees compared to them. When two hubs are connected, the one with higher probability of cooperation gets an advantage. This is due to the discussion we presented about the effect of large n values for cooperators and defectors. Both hubs are likely to be imitated by most of their low degree neighbors through evolution, however as this happens the more cooperative hub creates itself a more cooperative neighborhood resulting in higher payoffs and whereas the less cooperative hub reduces its own payoff as it spreads it strategy. Ultimately the defective hub adopts the strategy of cooperative hub and its neighbors are also likely to adopt this strategy in the following generations. Note that in this fashion hubs can also convert their neighbors with higher probability of cooperation and this is the reason why at very low values of c and k, the other two topologies can display more cooperative behaviors. In a sense, the strategy with the highest probability of cooperation that is played by a hub and also is able to keep that hub resistive (depending on the parameters b and c) to imitate its low degree neighbors has high chance of spreading throughout the population for Barabási-Albert scale-free topology. While connections among the hubs favor this more cooperative strategy, high significance value differences between hubs and low degree neighbors can also cause the elimination of even more cooperative strategies which were initially utilized by low degree nodes.

Results also depict that the Watts-Strogatz small world networks result in the least cooperative behavior among the three topologies and cooperation easily dies out with increasing average degree or cost of cooperation. This is expected since the small world networks are obtained from rewiring (with a certain probability of rewiring) of regular networks, hence their heterogeneity lies somewhere between the two.

Next we consider the effect of significance values on the spread of cooperation. To this end we simulate the small network in Fig. 2 starting from initial condition as shown there. This network is specially generated to highlight the influence of significance values (n) on the spreading chance and speed of strategies. Network is structured so that for $b > 6c$ it converges to all cooperators starting from the particular initial condition. Nodes 1 and 2 are the main sources for the spread of cooperation among the other nodes. They have the same degree, clustering coefficients and almost identical except the degree of their neighbors, hence n values. We simulate this system for $b = 1$ and $c = 0.15$ for 100 repetitions and compute the average probability of cooperation for each node versus time. Note that there are only pure strategies in this simulation hence p values are either 1 or 0 for each node and computing the average probability of cooperation at a particular time step is equivalent to the percentage of simulations where that node acts as a cooperator at that time step. Each simulation is run for 500 time steps as it has been observed to be enough to have all nodes settled as cooperators throughout the 100 simulations. Results are shown in Fig. 6.

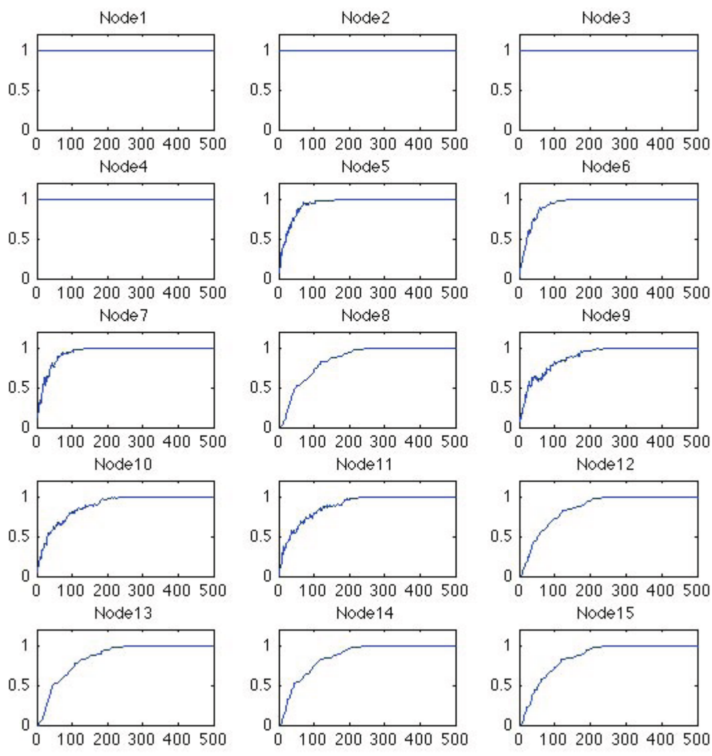

Fig. 6. Average probability of cooperation against time for 100 realization of the simple example with 15 nodes, starting from initial condition as shown in Fig. 2

Spreading speed and efficiency of nodes 1 and 2 can be compared through the comparison of average probability of cooperation of nodes 5, 6, 7 to average probability of cooperation of nodes 9, 10, 11 as these are the one hop neighbors or node 1 and node 2. Results depict that nodes 9, 10, 11 are quite slower than the nodes 5, 6, 7 in adopting and settling at the cooperator state.

5 Conclusion and Future Directions

In this paper, mixed strategy evolution in PD game on structured networks was considered. Evolutionary dynamics are analyzed on the micro level for arbitrary neighbors. The expected value of accumulated payoff differences and the expected number of neighbors that pick a particular node to possibly adopt its strategy were considered as significant factors in the evolutionary process and were analyzed. Evolution dynamics and influences of the network topology and game parameters were presented along with simulation results for various cases. Random, small world and scale-free topologies were simulated for different average degrees and costs of cooperation. Scale-free topology presented a higher robustness against changes in game and network parameters and more cooperative behaviors for a wide range of these parameters. By contrast, increases in the average degree or the cost of cooperation highly attenuate tendency to cooperate for small world topology due to the higher regularity of this topology. However, it was also shown that, for very small values of the cost of cooperation, it is possible to observe more cooperative behaviors with small world or random topologies as the robustness of scale-free topology may be limiting for such cases due to the the imitation of hubs by lower degree neighbors who initially utilize strategies with higher probability of cooperation. Furthermore, it was shown that the expected number of neighbors that pick a particular node to possibly adopt its strategy has an important role in that node's chance to spread its strategy. When a node has higher fitness, this value affects the speed of spread from that node. The spreading speed is important for the survival of cooperators since their survival greatly depend on the number of their cooperating neighbors.

As a future work, evolutionary dynamics for other widely used social dilemma games can be explored. Moreover, the dependency of evolutionary dynamics on the network topology and other variables can be analyzed in further details. Also similar analysis can be studied for dynamic topologies where edges and nodes are added/removed.

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