Selfish Random Access: Equilibrium Conditions and Best-Response Learning*

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Abstract. This paper studies a class of random access games for wireless channels with multipacket reception. First, necessary and sufficient equilibrium conditions for a contention strategy profile to be a Nash equilibrium for general wireless channels are established. Then, applications of these equilibrium conditions for well-known channel models are illustrated. Various engineering insights and design ideas are provided. Finally, the results are extended to an incomplete information game setting, and best-response learning dynamics leading to Nash equilibria are investigated.

Keywords: Slotted ALOHA, multipacket reception, game theory, contention control, medium access control.

1 Introduction

1.1 Background and Contributions

Game theory and the related field of mechanism design have the potential to guide engineering efforts to overcome potential design challenges in fourth generation (4G) wireless networks by providing a bottom-up analytical and principled approach to design local operation rules and to verify resulting collective network behavior through equilibrium analyses. In particular, it is illustrated in recent works [1]-[3] that game theory provides new insights to reverse/forward engineer existing medium access control (MAC) protocols, better fairness and service differentiation, higher throughput and a mechanism to decouple contention control from handling failed packets for a class of multiple access networks. This paper also focuses on layer-2 MAC protocols for wireless networks, and provides new equilibrium results and design insights based on noncooperative game theory.

Wireless channels are broadcast channels by their nature. Therefore, transmissions through them must be coordinated to control multiple access interference (MAI). Contention based random access approaches, which we consider in this

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paper, include slotted ALOHA, CSMA/CA and IEEE 802.11 DCF, and do not require centralized scheduling. A key design degree of freedom in random access is channel access probability determination, or contention resolution, to mitigate MAI. Different protocols differ in how they implement contention resolution such as window based and persistence probability based approaches. This paper analyzes equilibrium channel access probabilities (equivalently called contention resolution strategies, or transmission probabilities) and the resulting network performance for a wide range of channel models and communication scenarios.

Our contributions can be summarized as follows. We focus on the contention resolution problem over wireless channels with multipacket reception capability, and characterize the set of Nash equilibria by providing necessary and sufficient conditions for a Nash equilibrium contention strategy profile. Multipacket reception capability is an important feature of our model to capture probabilistic receptions in wireless multiple access. We demonstrate applications of these results in practical communication scenarios, and provide engineering insights to achieve optimal throughput. Finally, random access games with incomplete information structure are analyzed by obtaining the form of equilibrium contention resolution strategies, establishing existence and uniqueness results and illustrating learning dynamics on the best-response path.

1.2 Related Work

There is a large and growing body of work applying game-theoretic techniques for contention resolution in wireless networks. Here, we mention the ones that are most relevant to this work.

We focus on a class of random access games, first introduced in [4] and then further improved in [5]. When compared to these works, we provide more detailed equilibrium conditions for a contention strategy profile to be a Nash equilibirum for more general wireless channels.

In [1]-[3] and [6], the main focus is on the dynamics of iterative strategy update mechanisms such as best-response, gradient, or Jacobi play, achieving a desired equilibrium point over collision channels. Unlike these works, the physical layer model considered in this paper is more general, including the collision channel model as a special case. Except for the incomplete game formulation, our main focus here is on necessary and sufficient conditions to be satisfied by steady-state equilibrium contention resolution strategies, rather than on the transient network behavior. These equilibrium conditions can be solved either analytically or numerically to obtain equilibrium transmission probabilities, and then the network can be readily stabilized to a desired equilibrium by broadcasting these probabilities to users. For the incomplete game formulation, even though the existence and uniqueness of the equilibrium can be established, such closed-form or numerical solutions are not readily available, and therefore the transient network behavior is also investigated by studying learning dynamics on the best-response path.

Similar to our problem set-up, multipacket reception capability in the random access setting is also considered in [7] and [8]. As compared to the existence results, which provide limited information about the structure of equilibrium strategies, appearing in these works, we obtain more detailed necessary and sufficient equilibrium conditions, and solve them, either analytically or numerically, to derive the shape of equilibrium contention resolution strategies.

2 System Model

Consider a wireless multiple access communication network in which N selfish mobile users, indexed by $\mathcal{I} = \{1, 2, \cdots, N\}$, are contending for channel access to communicate with a common base station (BS). The wireless channel is characterized by the stochastic reception matrix

$$\boldsymbol{R} = (r_{n,k})_{\substack{1\leq n\leq N,\\ 0\leq k\leq N}}, \tag{1}$$

where $r_{n,k}$ represents the probability that k packets are received correctly given n of them are transmitted. By default, $r_{n,k}$ is set to zero if $k > n$. If $r_{1,0} > 0$, we say that the channel is *imperfect* (or, noisy). All users have identical packet success probabilities given by

$$\gamma_n = \frac{1}{n}\sum_{k=1}^{n} k r_{n,k}. \tag{2}$$

It is assumed that $\gamma_{n+1} \leq \gamma_n$ to model destructive effects of MAI on packet receptions. It is also assumed that γ_{n+1} is strictly smaller than γ_n for at least one n in $\{1, 2, \cdots, N-1\}$ in order to avoid trivialities.

If mobile user i transmits a packet successfully, it receives a normalized utility of 1 unit. If the transmission fails, it receives a utility of $-c_i$ units, where $c_i > 0$ is interpreted as the cost of packet failure for user i. If it waits, it receives a utility of 0 units. These utilities are necessary for setting up a selfish random access utility maximization problem as well as allowing us to model different battery levels, delay and quality-of-service requirements of different users. We let $\boldsymbol{u}_i = (1, 0, -c_i)$. The *random access game* $\mathcal{G}$ is defined to be the triple $\mathcal{G} = \left\langle \mathcal{I}, \{\mathcal{S}_i\}_{i\in\mathcal{I}}, \{\boldsymbol{u}_i\}_{i\in\mathcal{I}} \right\rangle$, where $\mathcal{S}_i = [\alpha_i, \beta_i] \subseteq [0, 1]$ is the set from which user i chooses a transmission probability (i.e., a contention resolution strategy) to access the wireless channel.

The average utility that a user receives is a function of her transmission probability and the transmission probabilities of other users. Let $\mathcal{S} = \prod_{i\in\mathcal{I}} \mathcal{S}_i$ be the product set of user strategies, $\boldsymbol{p}$ be a vector of transmission probabilities in $\mathcal{S}$, and $U_i(\boldsymbol{p})$ be the expected utility that the ith user receives as a function of $\boldsymbol{p}$. The selfish optimization problem to be solved by user i is to find $p_i^\star$ such that $U_i\left(p_i^\star, \boldsymbol{p}_{-i}\right) \geq U_i\left(p_i, \boldsymbol{p}_{-i}\right)$ for all $p_i \in \mathcal{S}_i$, where $\boldsymbol{p}_{-i}$ represents the vector of transmission probabilities of other users. We say that $\boldsymbol{p}^\star = (p_i^\star)_{i\in\mathcal{I}}$ is a *Nash equilibrium* if and only if $U_i\left(p_i^\star, \boldsymbol{p}_{-i}^\star\right) \geq U_i\left(p_i, \boldsymbol{p}_{-i}^\star\right)$ for all $i \in \mathcal{I}$ and $p_i \in \mathcal{S}_i$.

3 Equilibrium Contention Resolution Strategies: General Wireless Channels

In this section, we analyze equilibrium contention resolution strategies for a wireless channel with a general multipacket reception model, and obtain necessary and sufficient equilibrium conditions to be satisfied by a Nash equilibrium transmission probability vector. In the next section, we will illustrate the applications of our results in more specific communication scenarios by solving these equilibrium conditions.

The first critical issue to resolve is the existence of a Nash equilibrium. To this end, a positive existence result directly follows from the Glicksberg fixed point theorem [9]. (See also Theorem 1.2 in [10].) However, such existence results provide limited information about the structure of the Nash equilibria, and selfish transmission probabilities at these equilibria. Therefore, we establish necessary and sufficient conditions to be satisfied by a Nash equilibrium transmission probability vector in the following theorem.

Theorem 1. *For a given contention strategy profile $\boldsymbol{p}$, let*

$$\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}_{-i}\right) = \sum_{n=1}^{N} \sum_{\substack{\mathcal{I}_n \subseteq \mathcal{I}: \\ i \in \mathcal{I}_n, |\mathcal{I}_n| = n}} \gamma_n \prod_{j \in \mathcal{I}_n - \{i\}} p_j \prod_{j \in \mathcal{I} - \mathcal{I}_n} (1 - p_j). \tag{3}$$

Then, $\boldsymbol{p}^\star$ is a Nash equilibrium if and only if the following equilibrium conditions hold for all $i \in \mathcal{I}$.

(i) *If $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}^\star_{-i}\right) > \frac{c_i}{1+c_i}$, then $p_i^\star = \beta_i$.*
(ii) *If $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}^\star_{-i}\right) = \frac{c_i}{1+c_i}$, then $\alpha_i \leq p_i^\star \leq \beta_i$.*
(iii) *If $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}^\star_{-i}\right) < \frac{c_i}{1+c_i}$, then $p_i^\star = \alpha_i$.*

Proof. For a given $\boldsymbol{p}$, it is easy to see that $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}_{-i}\right)$ is the probability that a packet transmission from user i becomes successful given the channel reception matrix $\boldsymbol{R}$ and other users' transmission probabilities $\boldsymbol{p}_{-i}$. Then, the expected utility that user i receives when the random access game is played according to $\boldsymbol{p}$ is equal to

$$U_i\left(p_i, \boldsymbol{p}_{-i}\right) = p_i\left((1 + c_i)\,\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}_{-i}\right) - c_i\right). \tag{4}$$

Now, suppose that $\boldsymbol{p}^\star$ is a Nash equilibrium. Firstly, $p_i^\star$ must be β_i if $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}^\star_{-i}\right) > \frac{c_i}{1+c_i}$ since $U_i\left(p_i, \boldsymbol{p}^\star_{-i}\right)$ is a linear function of p_i when $\boldsymbol{p}^\star_{-i}$ is fixed. Secondly, $p_i^\star$ must be α_i if $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}^\star_{-i}\right) < \frac{c_i}{1+c_i}$. Finally, $p_i^\star$ can be set to any value in $[\alpha_i, \beta_i]$ if $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}^\star_{-i}\right) = \frac{c_i}{1+c_i}$. This completes the proof for the *only if* part of the theorem. The other direction also follows from similar arguments.

For a given $\boldsymbol{R}$, $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}_{-i}\right)$ can be interpreted as the contention signal that user i receives when the contention resolution strategies of other users are given by $\boldsymbol{p}_{-i}$. A higher contention signal received by user i means that less MAI is generated by other users, and therefore the higher the channel access and the resulting packet success probabilities of user i are.

In the next theorem, we establish a symmetry property for equilibrium contention resolution strategies in symmetric random access games.

Theorem 2. *Assume all users have the same strategy set* $[\alpha, \beta] \subseteq [0,1]$*, and the same cost of packet failure* $c > 0$*. If* $\boldsymbol{p}^\star$ *is a Nash equilibrium, then* $p_i^\star = p_j^\star$ *for all* $p_i^\star$ *and* $p_j^\star$ *in* (α, β)*. In particular, if* $p_i^\star \in (\alpha, \beta)$ *for all* $i \in \mathcal{I}$*, then all users access the channel with the same transmission probability* $p^\star$ *solving*

$$J\left(p^\star\right) = \frac{c}{1+c}, \tag{5}$$

where $J(p) = \sum_{n=0}^{N-1} \gamma_{n+1} \binom{N-1}{n} p^n \left(1-p\right)^{N-1-n}$.

Proof. Let $\mathcal{I}_{-\{i,j\}} = \mathcal{I} - \{i,j\}$ and $\boldsymbol{p}_{-\{i,j\}} = (p_k)_{k \in \mathcal{I}_{-\{i,j\}}}$. Let $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}_{-i}\right)$ be defined as in the proof of Theorem 1. We can expand $\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}_{-i}\right)$ as a function of p_j as

$$\begin{aligned}
\Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}_{-i}\right) = \Gamma_i\left(\boldsymbol{R}, p_j, \boldsymbol{p}_{-\{i,j\}}\right) = \\
p_j \sum_{n=0}^{N-2} \sum_{\substack{\mathcal{I}_n \subseteq \mathcal{I}_{-\{i,j\}}: \\ |\mathcal{I}_n| = n}} \gamma_{n+2} \prod_{k \in \mathcal{I}_n} p_k \prod_{k \in \mathcal{I}_{-\{i,j\}} - \mathcal{I}_n} \left(1 - p_k\right) \\
+ \left(1 - p_j\right) \sum_{n=0}^{N-2} \sum_{\substack{\mathcal{I}_n \subseteq \mathcal{I}_{-\{i,j\}}: \\ |\mathcal{I}_n| = n}} \gamma_{n+1} \prod_{k \in \mathcal{I}_n} p_k \prod_{k \in \mathcal{I}_{-\{i,j\}} - \mathcal{I}_n} \left(1 - p_k\right).
\end{aligned}$$

The last equation implies the relation $\Gamma_i\left(\boldsymbol{R}, p_i, \boldsymbol{p}_{-\{i,j\}}\right) = \Gamma_j\left(\boldsymbol{R}, \boldsymbol{p}_{-j}\right)$. We also have

$$\frac{\partial \Gamma_i\left(\boldsymbol{R}, \boldsymbol{p}_{-i}\right)}{\partial p_j} = -\sum_{n=0}^{N-2} \sum_{\substack{\mathcal{I}_n \subseteq \mathcal{I}_{-\{i,j\}}: \\ |\mathcal{I}_n| = n}} \left(\gamma_{n+1} - \gamma_{n+2}\right) \prod_{k \in \mathcal{I}_n} p_k \prod_{k \in \mathcal{I}_{-\{i,j\}} - \mathcal{I}_n} \left(1 - p_k\right),$$

which is strictly smaller than zero. Thus, $\Gamma_i\left(\boldsymbol{R}, p_j, \boldsymbol{p}_{-\{i,j\}}\right)$ is strictly decreasing in p_j for any given fixed $\boldsymbol{p}_{-\{i,j\}}$. Let $\boldsymbol{p}^\star$ be a Nash equilibrium such that there exist $p_i^\star$ and $p_j^\star$ in (α, β) and $p_i^\star \neq p_j^\star$. By Theorem 1, this can happen only if $\Gamma_i\left(\boldsymbol{R}, p_j, \boldsymbol{p}^\star_{-\{i,j\}}\right)$ crosses $\frac{c}{1+c}$ at two points $p_j^\star$ and $p_i^\star$; but this contradicts the strictly decreasing nature of $\Gamma_i\left(\boldsymbol{R}, p_j, \boldsymbol{p}^\star_{-\{i,j\}}\right)$ as a function of p_j. Equation (5) follows after some simplifications.

4 Applications and Discussion

We will now demonstrate some applications of the above general equilibrium results in two specific communication scenarios. Further applications are also possible. We start our discussion with equilibrium contention resolution strategies for imperfect collision channels.

4.1 Selfish Random Access over Imperfect Collision Channels

In the collision channel model, a packet transmission is assumed to be successful only if there is no other user attempting to transmit simultaneously. Hence, $r_{n,k} = \delta_{0,k}$ if $n \geq 2$, where $\delta_{i,j} = 1$ if $i = j$, and zero otherwise. We let $r_{1,0} = \theta$ and $r_{1,1} = 1 - \theta$ for some $\theta \in [0, 1]$. Here, the parameter θ can be interpreted as a measure of the noise level summarizing all random factors such as background noise, fading and path-loss affecting packet receptions. The smaller the θ is, the less noise is present in the system, and a packet transmission is more likely to be successful if there is no other transmission attempt. On the other hand, if θ is large, it is more likely that a packet fails even if there is no other user transmitting simultaneously.

By setting the strategy sets to $[0, 1]$, we can simplify the equilibrium conditions in Theorem 1 as follows: $\boldsymbol{p}^\star$ is a Nash equilibrium if and only if, for all $i \in \mathcal{I}$, it satisfies (i) $p_i^\star = 1$ if $(1 - \theta) \prod_{j \neq i} \left(1 - p_j^\star\right) > \frac{c_i}{1+c_i}$, (ii) $p_i^\star \in [0, 1]$ if $(1 - \theta) \prod_{j \neq i} \left(1 - p_j^\star\right) = \frac{c_i}{1+c_i}$, and (iii) $p_i^\star = 0$ if $(1 - \theta) \prod_{j \neq i} \left(1 - p_j^\star\right) < \frac{c_i}{1+c_i}$.

To simplify these conditions further, we will focus on the most interesting case in which $c_i < \frac{1-\theta}{\theta}$ for all $i \in \mathcal{I}$. Other cases can be analyzed similarly. Let $\boldsymbol{p}^\star$ be a Nash equilibrium such that a subset $\mathcal{I}_0$ of users in $\mathcal{I}$ transmit with positive probability, while others exercise zero transmission probability as their contention resolution strategies. Then, contention resolution strategies of users in $\mathcal{I}_0$ must satisfy the second equilibrium condition, which leads to closed form expressions

$$p_i^\star = 1 - \frac{1 + c_i}{c_i} \left(\frac{1}{1 - \theta} \varphi\left(\mathcal{I}_0\right) \right)^{\frac{1}{|\mathcal{I}_0| - 1}}, \forall i \in \mathcal{I}_0, \tag{6}$$

where the set function $\varphi : 2^{\mathcal{I}} - \{\emptyset\} \mapsto \mathbb{R}_+$ is defined as $\varphi\left(\mathcal{I}_0\right) = \prod_{i \in \mathcal{I}_0} \frac{c_i}{1+c_i}$ for all non-empty subsets $\mathcal{I}_0$ of $\mathcal{I}$. Note that such a solution is feasible only if $\left(\frac{1}{1-\theta} \varphi\left(\mathcal{I}_0\right) \right)^{\frac{1}{|\mathcal{I}_0| - 1}} \leq \frac{c_i}{1+c_i} < 1$, which further implies $\varphi\left(\mathcal{I}_0\right) < 1 - \theta$. Therefore, when $|\mathcal{I}_0| = 1$, we set $p_i^\star$ to 1 for $i \in \mathcal{I}_0$ without causing any ambiguity. Since transmission probabilities for users with different cost values are different, these transmission probabilities also indicate how different services are provisioned to different users.

In Fig. 1, we plot the equilibrium transmission probabilities and the equilibrium throughput for the homogenous case versus c by setting N to 5. We focus only on the equilibrium where all users transmit with the same positive probability, which corresponds to the fair allocation of communication resources. The equilibrium transmission probability is, then, given by $p^\star = 1 - \sqrt[N-1]{\frac{1}{1-\theta} \frac{c}{1+c}}$. As expected, when the noise level θ increases, transmission probabilities and the system throughput decrease. In all cases, small values of c lead to high transmission probabilities, which in turn results in excessive packet collisions and low

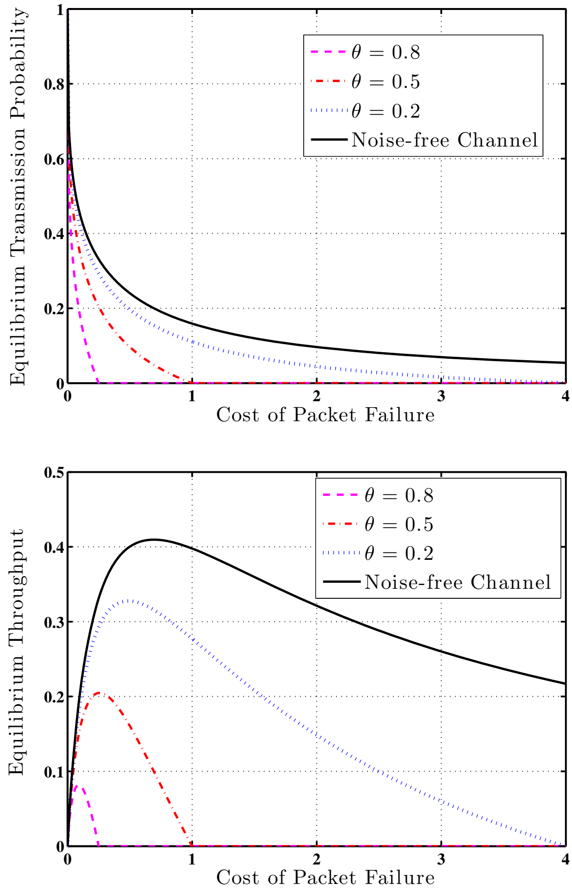

Fig. 1. Equilibrium contention resolution strategies (top figure) and the corresponding system throughput (bottom figure). Imperfect collision channels with $N = 5$.

throughput. Similarly, large values of c result in channel under-utilization, and therefore low throughput. In the middle-ground, there exists an optimal level of c maximizing the system throughput. It is easy to see that this maximum throughput is also the best that we can achieve via a central controller since transmission probabilities are continuous functions of costs. Therefore, there is no loss from selfish operation if selfish transmission probabilities can be manipulated to drive the system to the optimal operating point.

For example, Fig. 1 suggests that when c is small, a central controller can use the parameter θ as a signaling device to manipulate transmission probabilities, and drive the system to the optimal operating point, either by declaring a fictitious noise level to be greater than the true noise level, or by introducing artificial noise during the channel estimation phase. This approach will decrease

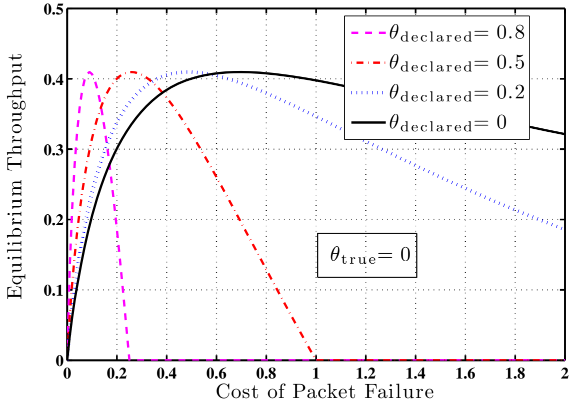

Fig. 2. System throughput when selfish users are manipulated by declaring fictitious noise levels. Perfect collision channel with $N = 5$.

users' greediness, eliminate excessive collisions and increase the system throughput. This operation can also be considered as a design process for user utility functions based on changing the effective value of c to achieve optimal performance. Figure 2 illustrates that the throughput increases significantly, and the same maximum throughput can be achieved by declaring fictitious noise levels $0.8, 0.5$ and 0.2 when c is around $0.1, 0.27$ and 0.5, respectively, for a noise-free channel.

4.2 Selfish Random Access for *T*-Out-of-*N* Channels

The second application of our results will be to a special type of multipacket reception channel in which all packets can be reconstructed successfully with probability $1-\theta_n$ if the collision size n is smaller than or equal to $T \in \{1, 2, \cdots, N\}$. On the other hand, if $n > T$, all packets are destroyed together. If the noise parameter θ_n is 0 for all n, then this is the channel model studied in [11] and [12]. Such channels can be implemented by using T-out-of-N codes [13].

We will focus only on the homogenous case and the Nash equilibrium at which all users access the channel with positive probability for illustrative purposes, but a similar analysis can be conducted for the heterogenous case and other equilibria as in the collision channel model above. We set the strategy sets to $[0, 1]$. In this case, $J(p)$ in (5) is given by $J(p) = \sum_{n=0}^{T-1} (1-\theta_{n+1}) \binom{N-1}{n} p^n (1-p)^{N-1-n}$. The common equilibrium transmission probability $p^\star$ is obtained by solving $J(p^\star) = \frac{c}{1+c}$. If $T < N$, then $J(1) = 0$, and it is enough to have $J(0) = 1 - \theta_1 \geq \frac{c}{1+c}$ for the existence of $p^\star$ solving $J(p^\star) = \frac{c}{1+c}$. Otherwise, $J(1) = 1 - \theta_N$, and we require $1 - \theta_1 \geq \frac{c}{1+c} \geq 1 - \theta_N$. Note also that if $1 - \theta_N > \frac{c}{1+c}$ (for $T = N$), then users transmit with probability one, and if $1 - \theta_1 < \frac{c}{1+c}$ (for $T \leq N$), they never transmit.

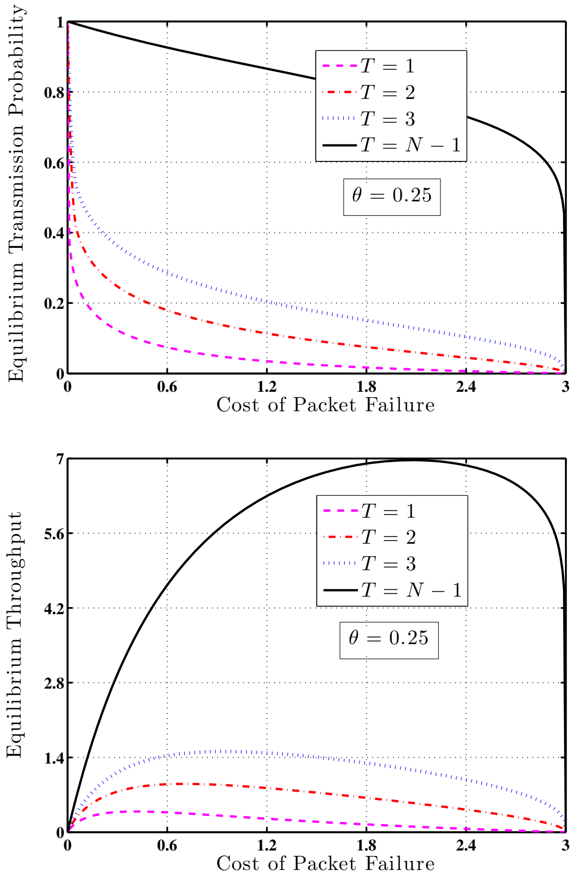

Fig. 3. Equilibrium contention resolution strategies (top figure) and the corresponding system throughput (bottom figure) for T-out-of-N channels. $\theta = 0.25$ and $N = 10$.

In Fig. 3, we plot the equilibrium strategies and the corresponding system throughput for the channels with the common noise parameter $\theta = 0.25$ when $N = 10$. Similar conclusions continue to hold for other values of N and θ_n varying with n. As expected, equilibrium transmission probabilities and the corresponding throughput increase with T. More importantly, maximum achievable throughput increases more than linearly with T. We have this maximum throughput to be around 0.39, 0.9, 1.51 and 6.97 for $T = 1, 2, 3$ and 9, respectively. For T large, we also observe a severe cut-off in transmission probabilities and a corresponding sharp decrease in the equilibrium throughput when the cost of packet failure comes close to the critical level $\frac{1-\theta}{\theta}$. On the other hand, it can be shown that throughput does not exhibit such an abrupt decrease with increasing cost for the noise-free channel. This indicates the importance of the calibration of costs and noise levels in order to avoid high penalty in equilibrium throughput for noisy T-out-of-N channels with large multipacket reception capability.

5 Imperfect Information Random Access Games

Now, we turn our attention to imperfect information random access games in which c_i is randomly distributed according to a cost distribution F_i but is perfectly known by user i before the start of a transmission. On the other hand, user i does not know the cost values of other users exactly but only has a set of belief distributions $\{F_j\}_{j\in\mathcal{I}-\{i\}}$ to predict them.

In this Bayesian game setting, the strategy of user i is a function s_i that maps $c_i \in (0,\infty)$ to a transmission probability $p_i \in [\alpha_i, \beta_i]$. With a slight abuse of notation, we will still represent the strategy set of user i by $\mathcal{S}_i$. As is standard, a strategy profile $\boldsymbol{s}^\star$ is said to be a Nash equilibrium if $s_i^\star$ is a solution of the selfish utility maximization problem $\max_{s_i\in\mathcal{S}_i} U_i\left(s_i, \boldsymbol{s}_{-i}^\star\right)$ for all $i \in \mathcal{I}$. In contrast to our analysis in Section 3, this optimization problem is now over the infinite dimensional function spaces. However, as established in the next theorem, equilibrium strategy profiles can be identified by using a threshold vector $\boldsymbol{\tau}^\star$ in $\mathbb{R}_+^N$. We will skip the proof of Theorem 3 since it is similar to the proofs given above for Theorems 1 and 2.

Theorem 3. *Let $\tilde{\Gamma}_i\left(\boldsymbol{R}, \boldsymbol{s}_{-i}\right)$ be given as*

$$\tilde{\Gamma}_i\left(\boldsymbol{R}, \boldsymbol{s}_{-i}\right) = \sum_{n=1}^{N} \sum_{\substack{\mathcal{I}_n\subseteq\mathcal{I}: \\ i\in\mathcal{I}_n, |\mathcal{I}_n|=n}} \gamma_n \prod_{j\in\mathcal{I}_n-\{i\}} \int_0^\infty s_j\left(c_j\right) dF_j\left(c_j\right) \prod_{j\in\mathcal{I}-\mathcal{I}_n} \left(1 - \int_0^\infty s_j\left(c_j\right) dF_j\left(c_j\right)\right)$$

for all $i \in \mathcal{I}$. Then, a strategy profile $\boldsymbol{s}^\star$ is a Nash equilibrium if and only if $s_i^\star$ is a threshold strategy in the form

$$s_i^\star\left(c_i\right) = \beta_i 1_{\left\{c_i<\tau_i^\star\right\}} + p_i 1_{\left\{c_i=\tau_i^\star\right\}} + \alpha_i 1_{\left\{c_i>\tau_i^\star\right\}} \tag{7}$$

almost surely (with respect to F_i) for all i, where $\tau_i^\star = \frac{\tilde{\Gamma}_i\left(\boldsymbol{R},\boldsymbol{s}_{-i}^\star\right)}{1-\tilde{\Gamma}_i\left(\boldsymbol{R},\boldsymbol{s}_{-i}^\star\right)}$ and $p_i \in [\alpha_i, \beta_i]$.

Theorem 3 allows us to restrict the search for equilibrium strategies to only threshold strategies. Therefore, by viewing the random access game as a game in which users choose a threshold, we can write the best-response function $\boldsymbol{B}$: $\mathbb{R}_+^N \mapsto \mathbb{R}_+^N$, with a slight abuse of notation, as

$$\boldsymbol{B}\left(\boldsymbol{\tau}\right) = \left(\frac{\tilde{\Gamma}_1\left(\boldsymbol{R}, \boldsymbol{\tau}_{-1}\right)}{1-\tilde{\Gamma}_1\left(\boldsymbol{R}, \boldsymbol{\tau}_{-1}\right)}, \frac{\tilde{\Gamma}_2\left(\boldsymbol{R}, \boldsymbol{\tau}_{-2}\right)}{1-\tilde{\Gamma}_2\left(\boldsymbol{R}, \boldsymbol{\tau}_{-2}\right)}, \cdots, \frac{\tilde{\Gamma}_N\left(\boldsymbol{R}, \boldsymbol{\tau}_{-N}\right)}{1-\tilde{\Gamma}_N\left(\boldsymbol{R}, \boldsymbol{\tau}_{-N}\right)}\right).$$

By restricting thresholds to take values only from $[0, \Delta]$ for some large but finite positive constant Δ and assuming all belief distributions are continuous, we can use the Brouwer fixed point theorem to conclude that $\boldsymbol{B}$ has at least one fixed point, which is the Nash equilibrium of the random access game. Moreover, by appealing to [14], we can also ensure the uniqueness of the Nash equilibrium if the Jacobian of $\boldsymbol{B}$ does not have an eigenvalue equal to 1 for all $\boldsymbol{\tau} \in [0, \Delta]^N$.

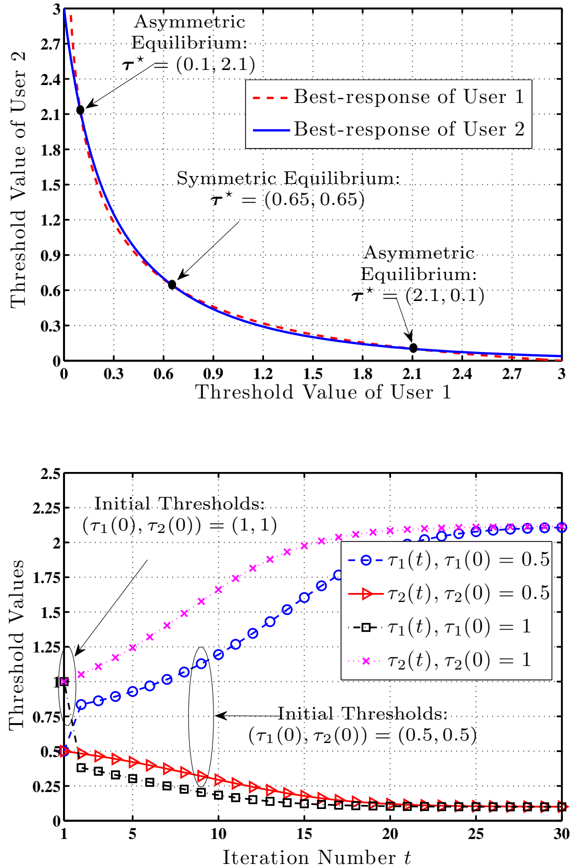

Fig. 4. For the imperfect collision channel with $r_{1,1} = 0.75$, the top figure illustrates the best-response functions of users 1 and 2, while the bottom figure illustrates the learning process leading to Nash equilibria. Belief distributions are exponential with the same parameter $\lambda = 1$.

Now, we focus on a numerical application of Theorem 3 to imperfect collision channels with two users having the same exponential belief distribution with parameter $\lambda > 0$, i.e., $F_1(c) = F_2(c) = 1 - \mathrm{e}^{-\lambda c}$. We set $r_{1,1}$ to 0.75.

The resulting network behavior can be quite complicated. To start with, depending on the value of λ, the equilibrium does not need to be unique or symmetric. For example, in Fig. 4, we show the communication scenario in which $\lambda = 1$ and the best-response functions of users intersect at three different points, two of which correspond to asymmetric equilibria at which one user sets its threshold to 0.1, while the other one sets it to 2.1. Moreover, the symmetric equilibrium is unstable, i.e., small perturbations to this equilibrium will lead the system to converge to an asymmetric equilibrium when there are multiple equilibria. That is, starting from any initial condition except for the symmetric equilibrium, the learning process on the best-response path converges to only one of these asymmetric equilibria. For example, at the bottom in Fig. 4, we show two learning curves with different initial conditions leading to these asymmetric equilibria. At iteration $t+1$, user 1 moves first and updates its threshold to $\tau_1(t+1) = \frac{\tilde{\Gamma}_1(\boldsymbol{R},\tau_2(t))}{1-\tilde{\Gamma}_1(\boldsymbol{R},\tau_2(t))}$ by estimating $\tilde{\Gamma}_1(\boldsymbol{R},\tau_2(t))$. Then, user 2 moves and updates its threshold to $\tau_2(t+1) = \frac{\tilde{\Gamma}_2(\boldsymbol{R},\tau_1(t+1))}{1-\tilde{\Gamma}_2(\boldsymbol{R},\tau_1(t+1))}$ by estimating $\tilde{\Gamma}_2(\boldsymbol{R},\tau_1(t+1))$. Iteration $t+1$ terminates after user 2 updates its threshold. As a result, the time-scale of each iteration corresponds to several tens of time-slots in a physical system allowing users to accurately estimate the congestion signals. Based on these observations, we conclude that the symmetric equilibrium, desirable for fairness purposes, exists but may never appear even in symmetric communication scenarios when there are multiple equilibria.

On the other hand, the Nash equilibrium is unique for small values of λ, e.g., $\lambda = 0.5$. Such an equilibrium is automatically symmetric due to the symmetry in the problem, and our analysis indicates that it is also the stable equilibrium for noisy collision channels. However, it should be noted that a Nash equilibrium may not be stable even if it is unique for other channel models, e.g., noise-free collision channels [15].

6 Conclusions

In this paper, we have focused on layer-2 contention resolution strategies for wireless networks with multipacket reception. We have obtained necessary and sufficient conditions for a Nash equilibrium strategy. Applications of these equilibrium conditions have been illustrated for specific channel models along with the resulting network performance analysis and various engineering insights. Finally, we have examined the contention resolution problem with imperfect information, derived the form of equilibrium strategies, their existence and uniqueness, and analyzed a strategy update mechanism based on best-response dynamics converging to an equilibrium.

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