

# Bayesian Filtering Methods for Target Tracking in Mixed Indoor/Outdoor Environments

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**Abstract.** We propose a stochastic filtering algorithm capable of integrating radio signal strength (RSS) data coming from a wireless sensor network (WSN) and location data coming from the global positioning system (GPS) in order to provide seamless tracking of a target that moves over mixed indoor and outdoor scenarios. We adopt the sequential Monte Carlo (SMC) methodology (also known as particle filtering) as a general framework, but also exploit the conventional Kalman filter in order to reduce the variance of the Monte Carlo estimates and to design an efficient importance sampling scheme when GPS data are available. The superior performance of the proposed technique, when compared to outdoor GPS-only trackers, is demonstrated using experimental data. Synthetic observations are also generated in order to study, by way of simulations, the performance in mixed indoor/outdoor environments.

**Keywords:** Bayesian filtering, indoor/outdoor tracking, Kalman filter, particle filter, switching models.

## 1 Introduction

The existing outdoor and indoor systems for target positioning and/or tracking have evolved in rather different ways. The global positioning system (GPS) is the most common technology in outdoor scenarios. It provides broad coverage, essentially ubiquitous except for a few “tough” environments, such as urban canyons [11], yet it has a poor accuracy, in the order of 10 meters [8,16]. Positioning based on cellular networks yields a similar precision and the coverage, even if not global [15], can include urban areas where GPS fails. Combinations of both technologies [15] are attractive but do not resolve the accuracy problem. During recent years, localization systems based on wireless sensor networks (WSNs) have gained momentum, specially for indoor applications [15,12]. In outdoors environments, WSNs providing radio signal strength (RSS), time of arrival (ToA) or angle of arrival (AoA) data can potentially beat GPS and cellular networks in terms of accuracy, but they are *ad hoc* systems to be deployed only in small areas [15].

Another major difference between positioning in outdoor and indoor environments is the need to model very different kind of signals. Let us focus hereafter in systems that use RSS observations, although similar arguments could be put forward for ToA and AoA measurements. In an “open” outdoor area, with no obstacles, it is relatively easy to extract range information (i.e., estimate the distance between the transmitter and the receiver of the signal) which can then be used for positioning. Unfortunately, such information is much harder to extract in indoor environments, due to the multipath propagation of the radio signals [15]. As a consequence the kind of models that are needed outdoors, with a direct line of sight (LOS) between the transmitter and receiver, and indoors, with strong multipath and non LOS transmission, can be very different.

In order to deal with the nonlinearities inherent to RSS (but also AoA and ToA) observations, sequential Monte Carlo (SMC) methods, also known as particle filters (PFs) [9,6,5], have been proposed as tools for positioning and tracking, both outdoors and (specially) indoors [10]. These methods rely on the simulation of candidate positions and tracks for the target of interest, which are later weighted and combined using a statistical procedure, and differ substantially from the (much simpler) Kalman filtering methods that are often used with GPS data [14].

In this paper, we tackle the design of a tracking algorithm that can work both indoors and outdoors, using GPS and/or RSS data collected from a WSN. The basic methodology that we adopt is particle filtering, which enables us to deal with variety of different models (representing various indoor and outdoor scenarios) both for the observations and for the target motion, possibly switching among them using the general scheme of [1]. However, we also exploit the availability of GPS data and the ability to process them using Kalman filtering in order to (a) simplify the complexity of the tracker (when GPS alone is available) and (b) design efficient particle filtering algorithms for the online fusion of GPS and RSS observations. The superior performance of the resulting methods, when compared to outdoor GPS-only trackers, is demonstrated using experimental data. Synthetic observations are also generated in order to study, by way of simulations, the performance in mixed indoor/outdoor environments.

The rest of the paper is organized as follows. In Section 2 we describe three environment-specific tracking models. The proposed algorithms are introduced in Section 3. In Section 4 we describe the experimental setup used to collect GPS and RSS data in an outdoor environment and we illustrate the performance of the proposed tracker, both with synthetic and experimental data.

## 2 System Model

### 2.1 Outdoor Linear Model

The dynamics of the target can be described using a linear and Gaussian state-space system [10]. Let  $\mathbf{x}_{4,t} = [\mathbf{r}_t^\top \mathbf{v}_t^\top]^\top \in \mathbb{R}^4$  be the state of the system. The state

vector contains the position,  $\mathbf{r}_t \in \mathbb{R}^2$  and the velocity,  $\mathbf{v}_t \in \mathbb{R}^2$ , of the object to be tracked in a 2-dimensional plane and the subscript  $t \in \mathbb{N}$  denotes discrete time. The state of the dynamic system evolves according to the stochastic model

$$\underbrace{\begin{bmatrix} r_{1,t} \\ r_{2,t} \\ v_{1,t} \\ v_{2,t} \end{bmatrix}}_{\mathbf{x}_{4,t}} = \underbrace{\begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \\ v_{1,t-1} \\ v_{2,t-1} \end{bmatrix}}_{\mathbf{x}_{4,t-1}} + \underbrace{\begin{bmatrix} \frac{1}{2}T^2 & 0 \\ 0 & \frac{1}{2}T^2 \\ T & 0 \\ 0 & T \end{bmatrix}}_{\mathbf{Q}} \underbrace{\begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix}}_{\mathbf{u}_t}, \quad (1)$$

where  $\mathbf{A}$  is a transition matrix that depends on the period  $T$ ,  $\mathbf{x}_{4,t-1}$  is the state vector of the previous time instant and  $\mathbf{u}_t$  is a  $2 \times 1$  real Gaussian vector of zero mean and diagonal covariance matrix,  $\sigma_u^2 \mathbf{I}_2$ . As a result, the process noise is a  $4 \times 1$  real Gaussian vector,  $\mathbf{Q}\mathbf{u}_t$ , with zero mean and covariance matrix  $\sigma_u^2 \mathbf{Q}\mathbf{Q}^\top$ , which represents the effect of unknown accelerations. This model is often termed constant velocity model [10].

More often, tracking in a purely outdoor scenario can be carried out using GPS data. The GPS observations give the position of the object in geodetic coordinates (latitude, longitude and altitude) and are easily converted to cartesian local coordinates. Thus, mathematically, we model the GPS observations as a linear function of the state,  $\mathbf{x}_{4,t} \in \mathbb{R}^4$ , with an added noise term  $\boldsymbol{\varepsilon}_t$ , which accounts for errors in the measurements

$$\underbrace{\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} r_{1,t} \\ r_{2,t} \\ v_{1,t} \\ v_{2,t} \end{bmatrix}}_{\mathbf{x}_{4,t}} + \boldsymbol{\varepsilon}_t. \quad (2)$$

Note that  $\boldsymbol{\varepsilon}_t$  is modelled as Gaussian noise vector of zero mean and known covariance matrix  $\boldsymbol{\varepsilon}_t \sim N(\boldsymbol{\varepsilon}_t; 0, \Sigma_\varepsilon)$ . When experimental data are available, the covariance parameter,  $\Sigma_\varepsilon$ , can be adjusted from the sample (empirical) variance.

## 2.2 Indoor Non-linear System Model

For an indoor environment we seek a more flexible system model capable of capturing rapidly changing movements and capable of modeling measurements with a high variance. On one hand, the movements which we are going to track are closer to maneuvers than to a linear motion. On the other hand, we assume that the observations available for tracking the target are RSS measurements collected by a sensor network. Unfortunately, the severe multipath propagation effects in indoor scenarios make the modeling of RSS data a challenging task.

Let  $\omega_t \in \mathbb{R}$  be the change, in radians, of the angle of the velocity at time  $t+1$ , and redefine the state vector as  $\mathbf{x}_{5,t} = [\omega_t, \mathbf{r}_t, \mathbf{v}_t]^\top$ , that evolves according to

$$\omega_t \sim p(\omega_t|\omega_{t-1})$$

$$\underbrace{\begin{bmatrix} r_{1,t} \\ r_{2,t} \\ v_{1,t} \\ v_{2,t} \end{bmatrix}}_{\mathbf{x}_{4,t}} = \underbrace{\begin{bmatrix} 1 & 0 & \frac{\sin(\omega_{t-1}T)}{\omega_{t-1}} & -\frac{\cos(\omega_{t-1}T)-1}{\omega_{t-1}} \\ 0 & 1 & \frac{1-\cos(\omega_{t-1}T)}{\omega_{t-1}} & \frac{\sin(\omega_{t-1}T)}{\omega_{t-1}} \\ 0 & 0 & \cos(\omega_{t-1}T) & -\sin(\omega_{t-1}T) \\ 0 & 0 & \sin(\omega_{t-1}T) & \cos(\omega_{t-1}T) \end{bmatrix}}_{\mathbf{A}(\omega_{t-1})} \underbrace{\begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \\ v_{1,t-1} \\ v_{2,t-1} \end{bmatrix}}_{\mathbf{x}_{4,t-1}} + \mathbf{Q}\mathbf{u}_t, \quad (3)$$

where the transition matrix  $\mathbf{A}(\omega_{t-1})$  is now a function of the angle  $\omega_{t-1}$  as well as the period  $T$  and the conditional probability density function (pdf)  $p(\omega_t|\omega_{t-1})$  is known. If we select different distributions for  $\omega_t$ , we can create different motion models. Note also that in the extreme case where  $\omega_t = 0$  for all  $t$ , (3) becomes the constant velocity model (1).

Let us assume that, the objective can move according to one out of  $L$  models of motion, identified by the indices  $\{1, 2, \dots, L\}$ . Each one of the motion models corresponds to a different transition pdf for the Markovian process  $\{\omega_t\}_{t \in \mathbb{N}}$ . Thus, to identify the different densities, we introduce a new state variable, denoted  $a_t$ . This is a discrete random indicator,  $a_t \in \{1, \dots, L\}$ , so that  $a_{t-1} = l$  implies that  $\omega_t$  is generated according to the  $l$ -th model. Therefore we need to write  $\omega_t \sim p(\omega_t|\omega_{t-1}, a_{t-1})$  to make the dependence explicit. The probability mass function (pmf)  $p(a_t|a_{t-1})$  is part of the model and therefore is assumed known. This type of dynamic model description, where there are various sub-models to describe different type of motion, is often denoted interacting multiple models (IMM) [10]. Incorporating the indicator  $a_t$  to the state, we obtain a  $6 \times 1$  vector,  $\mathbf{x}_{6,t} = [a_t, \omega_t, \mathbf{r}_t, \mathbf{v}_t]^\top$  which evolves in time according to the equations

$$a_t \sim p(a_t|a_{t-1}), \quad \omega_t \sim p(\omega_t|\omega_{t-1}, a_{t-1}), \quad \mathbf{x}_{4,t} = \mathbf{A}(\omega_{t-1})\mathbf{x}_{4,t-1} + \mathbf{Q}\mathbf{u}_t. \quad (4)$$

For the observation model, we assume that at time  $t$  we obtain  $J$  RSS measurements. The datum obtained from sensor  $j$  at time  $t$  is denoted as  $y_{j,t}$ . The relationship between the received observation,  $y_{j,t}$ , and the position of the target,  $\mathbf{r}_t$ , depends on the environment in which the measurement is taken and can change with time [13]. In order to model this uncertainty in the observations we are going to use an interacting multiple model (IMM) approach the same as for the dynamic model. Specifically, we represent the observation  $y_{j,t}$  using one out of  $K$  different models that describe the different environments. Finally, we model the observations as

$$y_{j,t} = f_{m_{j,t}}(\mathbf{r}_t) + \varepsilon_{m_{j,t}}, \quad (5)$$

where  $m_{j,t} \in \{1, \dots, K\}$  is a random index with a known probability mass function (pmf),  $p(m_{j,t})$ , which identifies the observation model at time  $t$  for each sensor  $j$ ,  $f_{m_{j,t}}$  is the function that describes the propagation conditions in model  $m_{j,t}$ , and  $\varepsilon_{m_{j,t}} \sim N(\varepsilon_{m_j}; 0, \sigma_{m_{j,t}}^2)$  is Gaussian noise with zero mean and a known variance  $\sigma_{m_{j,t}}^2$ , which is also associated with the model  $m_{j,t}$ . The form of the functions  $\{f_1, f_2, \dots, f_K\}$  and variances  $\{\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2\}$  should be determined

from a bank of empirical observations collected in the scenarios in which the tracking will be performed. In [1] we give full details of the functions and variances obtained when the RSS is measured using a network of ZigBee models. We assume the same indoor environment and hardware setup in the present paper.

We write the measurement-model indicators together in a  $J \times 1$  vector  $\mathbf{m}_t = [m_{1,t}, \dots, m_{J,t}]^\top$ , hence the full target state has  $J + 6$  components,  $\mathbf{x}_{J+6,t} = [\mathbf{m}_t, a_t, \omega_t, \mathbf{r}_t, \mathbf{v}_t]^\top$ . The observations are put together in a  $J \times 1$  vector  $\mathbf{y}_t = [y_{1,t}, \dots, y_{J,t}]^\top$ . The indices in  $\mathbf{m}_t$  are assumed independent, but not necessarily identically distributed. The probability mass functions  $p(m_{j,t}), j = 1, \dots, J$  are assumed known and independent of time.

### 2.3 Outdoor Nonlinear Model

In this section we describe an outdoor system model that includes ZigBee observations as well as GPS observations. The dynamic model is the same as described in Section 2.1.

The observation vector is composed of  $J$  measurements received from ZigBee sensors and  $S$  GPS measurements, that is

$$\mathbf{y}_t = [y_{1,t}, \dots, y_{J,t}, y_{J+1,t}, \dots, y_{S+J,t}]. \quad (6)$$

The GPS observation model is the same as described in Section 2.1.

The outdoor ZigBee observation models we propose are based on the log distance path-loss model described in [13]

$$y_{j,t} = f_j(\mathbf{r}_t) + \varepsilon_j = L_{j,0} + \gamma_j 10 \log_{10} \left( \frac{d_{j,0}}{d_{j,t}} \right) + \varepsilon_j, \quad (7)$$

where  $d_{j,t}$  is the distance between sensor  $j$  and the target at time instant  $t$ ,  $d_{j,0}$  is a reference distance for sensor  $j$ ,  $L_{j,0}$  is the path loss corresponding to the reference distance,  $\gamma_j$  is the path loss exponent and  $\varepsilon_j \sim N(\varepsilon_j; 0, \sigma_{\varepsilon_j}^2)$  has a normal distribution with zero mean and variance  $\sigma_{\varepsilon_j}^2$ . The parameters  $L_{j,0}$ ,  $\gamma_j$ ,  $d_{j,0}$  and  $\sigma_{\varepsilon_j}^2$  should be adjusted using experimental data.

In particular, assume that we collect  $k$  RSS measurements for a sensor-to-target distance  $d_i$  and this is repeated for  $l$  different distances,  $d_1, \dots, d_l$ . Then, the parameters  $L_0$  and  $\gamma$  are selected as the solution to the optimization problem

$$(L_{0,j}, \gamma_j) = \arg \min_{L_{0,j}, \gamma_j} \left\{ \sum_{i=1}^l \sum_{n=1}^k \left( y_{n,i} - L_{0,j} - \gamma_j 10 \log_{10} \left( \frac{d_0}{d_i} \right) \right)^2 \right\},$$

where  $y_{n,i}$  is the  $n$ -th observation measured at distance  $d_i$  in the experiments,  $k$  is the number of measurements we have at distance  $d_i$  and  $l$  is the number of distances at which we have measurements.

The variance parameter is fitted as

$$\hat{\sigma}_{\varepsilon,j}^2 = \frac{1}{l} \frac{1}{k} \sum_{i=1}^l \sum_{n=1}^k \left( y_{n,i} - L_{0,j} - \gamma_j 10 \log_{10} \left( \frac{d_0}{d_i} \right) \right)^2.$$

Note that  $L_{0,j}$ ,  $\gamma_j$  and  $\sigma_{\varepsilon,j}^2$  are adjusted for each sensor separately.

### 3 Tracking Algorithms

#### 3.1 Kalman Filter

When both the dynamic and observation models are linear and Gaussian, the density  $p(\mathbf{x}_t|\mathbf{y}_{1:t})$ ,  $t = 1, 2, \dots$  is also Gaussian and can be exactly computed using the Kalman filter [14]. Specifically if we assume

Specifically,  $p(\mathbf{x}_t|\mathbf{y}_{1:t})$  is Gaussian if we assume that

- $\mathbf{u}_t$  y  $\boldsymbol{\varepsilon}_t$  are independent and have known Gaussian distributions,
- the dynamic model is a linear function of  $\mathbf{x}_{t-1}$  and  $\mathbf{u}_t$ , and
- the observation model is a linear function of  $\mathbf{x}_t$  and  $\boldsymbol{\varepsilon}_t$ .

Obviously these requirements are satisfied by the outdoor linear model of Section 2.1.

Assume that at time  $t = 0$  the prior distribution of  $\mathbf{x}_t$  is Gaussian with mean  $\hat{\mathbf{x}}_{0|0}$  and covariance matrix  $\mathbf{P}_{0|0}$  denoted  $\mathbf{x}_0 \sim N(\mathbf{x}_0; \hat{\mathbf{x}}_{0|0}, \mathbf{P}_{0|0})$ . The Kalman filter consists on the set of recursive equations [14]

$$\left. \begin{aligned} \hat{\mathbf{x}}_{t|t-1} &= \mathbf{A}\hat{\mathbf{x}}_{t-1|t-1} \\ \mathbf{P}_{t|t-1} &= \mathbf{Q}_{u,t-1} + \mathbf{A}\mathbf{P}_{t-1|t-1}\mathbf{A}^\top \\ \hat{\mathbf{x}}_{t|t} &= \hat{\mathbf{x}}_{t|t-1} + \mathbf{K}_t(\mathbf{y}_t - \mathbf{B}\hat{\mathbf{x}}_{t|t-1}) \\ \mathbf{P}_{t|t} &= \mathbf{P}_{t|t-1} - \mathbf{K}_t\mathbf{S}_t\mathbf{K}_t^\top \end{aligned} \right\} \quad (8)$$

where  $\mathbf{Q}_{u,t} = \sigma_u^2\mathbf{Q}\mathbf{Q}^\top$  is the covariance matrix of the process noise,

$$\mathbf{S}_t = \mathbf{B}_{t-1}\mathbf{P}_{t|t-1}\mathbf{B}_{t-1}^\top + \mathbf{Q}_{v,t} \quad (9)$$

is the covariance matrix of the innovation  $\boldsymbol{\nu}_t = \mathbf{y}_t - \mathbf{B}_t\hat{\mathbf{x}}_{t|t-1}$ ,  $\mathbf{Q}_{v,t} = \boldsymbol{\Sigma}_\varepsilon$  is the covariance matrix of the observation noise and

$$\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{B}_t^\top\mathbf{S}_t^{-1} \quad (10)$$

is the Kalman gain. The recursive application of these equations gives us the mean and covariance matrix of the posterior probability distribution function  $p(\mathbf{x}_t|\mathbf{y}_{1:t})$ , namely  $p(\mathbf{x}_t|\mathbf{y}_{1:t}) = N(\mathbf{x}_t; \hat{\mathbf{x}}_{t|t}, \mathbf{P}_{t|t})$ .

Table 1 summarizes the Kalman filter algorithm for outdoor tracking using GPS observation data.

#### 3.2 Particle Filter for the Indoor Nonlinear Model

**Sequential Monte Carlo approximation.** From a Bayesian point of view, the smoothing pdf

$$p(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t}|\mathbf{y}_{1:t}) = \sum_{\mathbf{m}_{0:t}} \int_{\mathbf{v}_{0:t}} p(\mathbf{x}_{J+4,0:t}|\mathbf{y}_{1:t})d\mathbf{v}_{0:t} \quad (11)$$

**Table 1.** Kalman filter for online tracking in outdoor environments using GPS data

<p>1. Initialization, at <math>t = 0</math>:</p> <ul style="list-style-type: none"> <li>– Assign a mean vector <math>\hat{\mathbf{x}}_0</math> and a covariance matrix <math>\mathbf{P}_0</math> to the first time instant <math>t = 0</math>, all taken from the prior distribution <math>p(\mathbf{x}_0; \hat{\mathbf{x}}_0, \mathbf{P}_0)</math>.</li> </ul> <p>2. Recursive step, for <math>t &gt; 0</math>:</p> <ul style="list-style-type: none"> <li>– Obtain a new GPS observation <math>\mathbf{y}_t</math>.</li> <li>– Apply the recursive formulae (8), (9) and (10) to obtain <math>p(\mathbf{x}_t   \mathbf{y}_{1:t}) = N(\mathbf{x}_t; \hat{\mathbf{x}}_{t t}, \hat{\mathbf{P}}_{t t})</math>.</li> </ul>
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contains all relevant statistical information for the estimation of  $\mathbf{r}_{0:t}$ . The elimination of  $\mathbf{v}_{0:t}$  and  $\mathbf{m}_{0:t}$  by marginalization is often termed Rao-Blackwellization and reduces the estimation variance [7,3]. Unfortunately, the density of (11) cannot be obtained analytically and we have to resort to numerical approximation techniques. Our approach, is to build a point-mass approximation of the distribution with density  $p(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t} | \mathbf{y}_{1:t})$ , consisting of  $M$  random samples in the space of  $\{\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t}\}$ , denoted  $\{\mathbf{r}_{0:t}^{(i)}, \omega_{0:t}^{(i)}, a_{0:t}^{(i)}\}_{i=1}^M$ , and associated importance weights,  $\{w_t^{(i)}\}_{i=1}^M$ . Each pair  $\left\{ \left( \mathbf{r}_{0:t}^{(i)}, \omega_{0:t}^{(i)}, a_{0:t}^{(i)} \right), w_t^{(i)} \right\}$  is called a *particle* and we can use them to build the random measure

$$p_M(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t} | \mathbf{y}_{1:t}) = \sum_{i=1}^M \delta_i(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t}) w_t^{(i)}, \tag{12}$$

where  $\delta_i$  is a unit delta measure located at  $\left( \mathbf{r}_{0:t}^{(i)}, \omega_{0:t}^{(i)}, a_{0:t}^{(i)} \right)$  and the weights are assumed normalized, i.e.,  $\sum_{i=1}^M w_t^{(i)} = 1$ . If the approximation is properly constructed, meaning that the moments of  $p_M(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t} | \mathbf{y}_{1:t})$  converge to those of  $p(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t} | \mathbf{y}_{1:t})$  in some adequate sense [4], then it is straightforward to use (12) in order to approximate any estimators of  $\mathbf{r}_{0:t}$  or  $\mathbf{r}_t$ . In particular, since

$$p_M(\mathbf{r}_t | \mathbf{y}_{1:t}) = \sum_{a_{0:t}} \int_{\omega_{0:t}} \int_{\mathbf{r}_{0:t-1}} p_M(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t} | \mathbf{y}_{1:t}) d\mathbf{r}_{0:t-1} d\omega_{0:t} = \sum_{i=1}^M \delta_i(\mathbf{r}_t) w_t^{(i)}, \tag{13}$$

where  $\delta_i$  is the delta unit measure located at  $\mathbf{r}_t^{(i)}$ , we readily calculate the (approximate) minimum mean square error (MMSE) estimate of  $\mathbf{r}_t$  as

$$\hat{\mathbf{r}}_t^{mmse} = \int \mathbf{r}_t p_M(\mathbf{r}_t | \mathbf{y}_{1:t}) d\mathbf{r}_t = \sum_{i=1}^M \mathbf{r}_t^{(i)} w_t^{(i)}. \tag{14}$$

The generation of samples and the computation of weights is carried out by means of the sequential importance sampling (SIS) principle [7]. Specifically, we can decompose the smoothing pdf using a Bayes theorem

$$\begin{aligned}
 p(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t} | \mathbf{y}_{1:t}) &\propto p(\mathbf{y}_t | \mathbf{r}_t) p(\mathbf{r}_t | \mathbf{r}_{0:t-1}, \omega_{0:t-1}) p(a_t | a_{t-1}) \\
 &\times p(\omega_t | \omega_{t-1}, a_{t-1}) p(\mathbf{r}_{0:t-1}, \omega_{0:t-1}, a_{0:t-1} | \mathbf{y}_{1:t-1}), \quad (15)
 \end{aligned}$$

and, if we draw the particles using the transition pdf's

$$\begin{aligned}
 a_t^{(i)} &\sim p(a_t | a_{t-1}^{(i)}) \\
 \omega_t^{(i)} &\sim p(\omega_t | \omega_{t-1}^{(i)}, a_{t-1}^{(i)}) \\
 \mathbf{r}_t^{(i)} &\sim p(\mathbf{r}_t | \mathbf{r}_{0:t-1}^{(i)}, \omega_{0:t-1}^{(i)}, \mathbf{y}_{1:t-1}) \quad (16)
 \end{aligned}$$

then the importance weight becomes

$$w_t^{(i)} \propto w_{t-1}^{(i)} p(\mathbf{y}_t | \mathbf{r}_t^{(i)}). \quad (17)$$

Eqs. (16) and (17) together yield a sequential IS (SIS) type of algorithm for the construction of  $p_M(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t} | \mathbf{y}_{1:t})$  [7].

It is well known, however, that the sequential application of (16) and (17) with a finite number of samples,  $M < \infty$ , quickly leads to a degenerate set of particles [7]. Indeed, the variance of the weights increases stochastically with time and, after a few time steps, one single particle tends to accumulate all the weight and the approximation  $p_M(\mathbf{r}_{0:t}, \omega_{0:t}, a_{0:t} | \mathbf{y}_{1:t})$  becomes useless. This difficulty is commonly overcome by adding a resampling step [7,2] which, intuitively, consists in stochastically discarding the particles with low weights while the particles with higher weights are replicated. Although several resampling schemes exist (and all of them can be plugged into the tracking algorithm without any added difficulty), in this paper we adopt the conceptually simple multinomial resampling method [7,4]. A resampling step can be taken every time the approximate effective sample size [7]  $\hat{M}_{eff} = \frac{1}{\sum_{i=1}^M w_t^{(i)2}}$  falls below a user-defined threshold. Since  $\hat{M}_{eff} \leq M$ , typical threshold values are  $\lambda M$  for some  $0 < \lambda < 1$ .

**Evaluation of the weights.** In order to ensure that the weights of (17) can be computed, we must be able to draw from  $p(\mathbf{r}_t | \mathbf{r}_{0:t-1}, \omega_{0:t-1})$  and to evaluate the factors  $p(a_t | a_{t-1})$ ,  $p(\omega_t | \omega_{t-1}, a_{t-1})$  and  $p(\mathbf{y}_t | \mathbf{r}_t)$ . The transition densities  $p(a_t | a_{t-1})$  and  $p(\omega_t | \omega_{t-1}, a_{t-1})$  are part of the model, hence known by assumption. The prior density of the position at time  $t$ ,  $p(\mathbf{r}_t | \mathbf{r}_{0:t-1}, \omega_{0:t-1})$ , is Gaussian and can be obtained in closed form for each particle. Indeed, given  $\mathbf{r}_{0:t-1}^{(i)}$  and  $\omega_{0:t-1}^{(i)}$ , the system

$$\begin{bmatrix} v_{1,t} \\ v_{2,t} \\ r_{1,t}^{(i)} \\ r_{2,t}^{(i)} \end{bmatrix} = \begin{bmatrix} \cos(\omega_{t-1}^{(i)} T) & -\sin(\omega_{t-1}^{(i)} T) & 0 & 0 \\ \sin(\omega_{t-1}^{(i)} T) & \cos(\omega_{t-1}^{(i)} T) & 0 & 0 \\ \frac{\sin(\omega_{t-1}^{(i)} T)}{\omega_{t-1}^{(i)}} & -\frac{\cos(\omega_{t-1}^{(i)} T) - 1}{\omega_{t-1}^{(i)}} & 1 & 0 \\ \frac{1 - \cos(\omega_{t-1}^{(i)} T)}{\omega_{t-1}^{(i)}} & \frac{\sin(\omega_{t-1}^{(i)} T)}{\omega_{t-1}^{(i)}} & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{1,t-1} \\ v_{2,t-1} \\ r_{1,t-1}^{(i)} \\ r_{2,t-1}^{(i)} \end{bmatrix} + \begin{bmatrix} T & 0 & 0 & 0 \\ 0 & T & 0 & 0 \\ 0 & 0 & \frac{1}{2} T^2 & 0 \\ 0 & 0 & 0 & \frac{1}{2} T^2 \end{bmatrix} \begin{bmatrix} u_{3,t} \\ u_{4,t} \\ u_{1,t} \\ u_{2,t} \end{bmatrix} \quad (18)$$



is linear and Gaussian, with known parameters, and all posterior pdf's, including  $p(\mathbf{r}_t | \mathbf{r}_{0:t-1}^{(i)}, \omega_{0:t-1}^{(i)})$ , are Gaussian and can be computed exactly using a Kalman filter [3,10]. In the sequel, we denote

$$p(\mathbf{r}_t | \mathbf{r}_{0:t-1}^{(i)}, \omega_{0:t-1}^{(i)}) = N(\mathbf{r}_t; \bar{\mathbf{r}}_{t|t-1}^{(i)}, \bar{\Sigma}_{t|t-1}^{(i)}). \quad (19)$$

The pdf  $p(\mathbf{y}_t | \mathbf{r}_t)$  is usually referred to as the likelihood of  $\mathbf{r}_t$ . If we write  $p(y_{j,t} | \mathbf{r}_t)$  as a marginal of the joint density  $p(y_{j,t}, m_{j,t} | \mathbf{r}_t)$ , then it is straightforward to obtain the expression

$$p(\mathbf{y}_t | \mathbf{r}_t) = \prod_{j=1}^J p(y_{j,t} | \mathbf{r}_t) = \prod_{j=1}^J \sum_{m_{j,t}} p(y_{j,t} | \mathbf{r}_t, m_{j,t}) p(m_{j,t}), \quad (20)$$

(note that the observations are conditionally independent given the position  $\mathbf{r}_t$ ) where both

$$p(y_{j,t} | \mathbf{r}_t, m_{j,t}) = N(y_{j,t}; f_{m_{j,t}}(\mathbf{r}_t), \sigma_{m_{j,t}}^2) \quad (21)$$

and  $p(m_{j,t})$  are known from the model, for all  $j = 1, \dots, J$ .

Table 2 summarizes the proposed SIS algorithm for the system model described in Section 2.2.

**Table 2.** SIS for indoor tracking with RSS data

1. Initialization, at  $t = 0$ :
  - For  $i = 1, \dots, M$ , sample  $\mathbf{r}_0$ ,  $\mathbf{v}_0$ ,  $\omega_0$  and  $a_0$  from the priors  $p(\mathbf{r}_0)$ ,  $p(\mathbf{v}_0)$ ,  $p(\omega_0)$  and  $p(a_0)$ . Initialize weights to  $w_0^{(i)} = \frac{1}{M}$ .
2. Recursive step, for  $t > 0$ :
  - For  $i = 1, \dots, M$ , sample  $a_t^{(i)} \sim p(a_t | a_{t-1}^{(i)})$ ,  $\omega_t^{(i)} \sim p(\omega_t | \omega_{t-1}^{(i)}, a_{t-1}^{(i)})$  and obtain  $\mathbf{r}_t^{(i)}$  from the distribution  $p(\mathbf{r}_t | \mathbf{r}_{0:t-1}^{(i)}, a_{0:t-1}^{(i)}, \omega_{0:t-1}^{(i)}) = N(\mathbf{r}_t; \bar{\mathbf{r}}_{t|t-1}^{(i)}, \bar{\Sigma}_{t|t-1}^{(i)})$  obtained with the Kalman filter.
  - For  $i = 1, \dots, M$ , update weights,  $w_t^{(i)} \propto w_{t-1}^{(i)} p(\mathbf{y}_t | \mathbf{r}_t^{(i)})$
  - Compute the particle effective size  $\hat{M}_{eff} = 1 / \sum_{k=1}^M w_t^{(k)2}$ . If  $\hat{M}_{eff} < \lambda M$  perform resampling and set  $w_t^{(i)} = \frac{1}{M} \forall i$ .

### 3.3 Particle Filter for the Outdoor Nonlinear Model

The algorithm that we are going to use for outdoor tracking with GPS and RSS data is a particle filter similar to the one used for indoor tracking. The main difference is the use of a different proposal pdf for the generation of particles and the subsequent change in the computation of the weights. Recall that the state vector for our nonlinear outdoor model is defined as  $\mathbf{x}_{4,t} = [\mathbf{r}_t^\top \mathbf{v}_t^\top]^\top$  and, as a consequence, the probability density function of interest is

$$p(\mathbf{r}_{0:t} | \mathbf{y}_{1:t}) = \int_{\mathbf{v}_{0:t}} p(\mathbf{x}_{0:t} | \mathbf{y}_{1:t}) d\mathbf{v}_{0:t},$$

where the velocity is integrated to reduce the variance in the estimation.

In order to obtain an efficient proposal pdf, we note that even though the complete vector of observations is not linear, the part of the vector that corresponds to the GPS observations is linear, therefore if we take only this part of the vector we can construct a posterior Gaussian pdf of  $\mathbf{r}_t$  given partial data. To be specific, if we define the vector of GPS observations as

$$\mathbf{y}_{S,t} = \mathbf{r}_t + \boldsymbol{\varepsilon}_{S,t},$$

then we can derive an analytic expression for the posterior density  $p(\mathbf{r}_t | \mathbf{r}_{0:t-1}, \mathbf{y}_{S,t})$ , namely

$$\begin{aligned} p(\mathbf{r}_t | \mathbf{r}_{0:t-1}, \mathbf{y}_{S,t}) &\propto p(\mathbf{y}_{S,t} | \mathbf{r}_t) p(\mathbf{r}_t | \mathbf{r}_{0:t-1}) = \\ &= \frac{1}{2\pi |\boldsymbol{\Sigma}_{t|t-1}|^{\frac{1}{2}} |\boldsymbol{\Sigma}_{\varepsilon}|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2} [(\mathbf{y}_{S,t} - \mathbf{r}_t)^\top \boldsymbol{\Sigma}_{\varepsilon}^{-1} (\mathbf{y}_{S,t} - \mathbf{r}_t) + \right. \\ &\quad \left. (\mathbf{r}_t - \mathbf{r}_{t|t-1})^\top \boldsymbol{\Sigma}_{t|t-1}^{-1} (\mathbf{r}_t - \mathbf{r}_{t|t-1})\right\}, \end{aligned} \quad (22)$$

where  $\boldsymbol{\Sigma}_{\varepsilon}$  is the covariance matrix of  $\mathbf{y}_{S,t}$  and  $p(\mathbf{r}_t | \mathbf{r}_{0:t-1}) = N(\mathbf{r}_t | \mathbf{r}_{t|t-1}, \boldsymbol{\Sigma}_{t|t-1})$ .

Equations 22 can be written in a more compact form if we use the equality

$$\begin{aligned} (\mathbf{y} - \mathbf{B}\mathbf{r})^\top \mathbf{V}(\mathbf{y} - \mathbf{B}\mathbf{r}) + (\mathbf{r} - \mathbf{r}_a)^\top \mathbf{U}(\mathbf{r} - \mathbf{r}_a) &= (\mathbf{r} - \mathbf{r}_p)^\top \mathbf{C}(\mathbf{r} - \mathbf{r}_p) + \\ + (\mathbf{y}^\top \mathbf{V}\mathbf{y} + \mathbf{r}_a^\top \mathbf{U}\mathbf{r}_a) - (\mathbf{B}^\top \mathbf{V}\mathbf{y} + \mathbf{U}\mathbf{r}_a)^\top \mathbf{C}^{-1}(\mathbf{B}^\top \mathbf{V}\mathbf{y} + \mathbf{U}\mathbf{r}_a) \end{aligned} \quad (23)$$

where all vectors have compatible dimensions,  $\mathbf{C} = \mathbf{B}^\top \mathbf{V}\mathbf{B} + \mathbf{U}$  and  $\mathbf{r}_p = \mathbf{C}^{-1}(\mathbf{B}^\top \mathbf{V}\mathbf{y} + \mathbf{U}\mathbf{r}_a)$ . if we identify  $\mathbf{y} = \mathbf{y}_{S,t}$ ,  $\mathbf{V} = \boldsymbol{\Sigma}_{\varepsilon}^{-1}$ ,  $\mathbf{B} = \mathbf{I}$ ,  $\mathbf{r} = \mathbf{r}_t$ ,  $\mathbf{U} = \boldsymbol{\Sigma}_{t|t-1}^{-1}$ ,  $\mathbf{r}_a = \mathbf{r}_{t|t-1}$ ,  $\tilde{\boldsymbol{\Sigma}}_t = \mathbf{C}^{-1}$  and  $\tilde{\mathbf{r}}_t = \mathbf{r}_p$ , then we can see that substitution of (23) into (22) gives a Gaussian density

$$p(\mathbf{r}_t | \mathbf{r}_{0:t-1}, \mathbf{y}_t) \propto \frac{1}{2\pi |\tilde{\boldsymbol{\Sigma}}^{-1}|^{1/2}} \exp\left\{-\frac{1}{2} [(\mathbf{r}_t - \tilde{\mathbf{r}}_t)^\top \tilde{\boldsymbol{\Sigma}}_t^{-1} (\mathbf{r}_t - \tilde{\mathbf{r}}_t)]\right\}.$$

Therefore, the proposed pdf that incorporates GPS observations can be characterized as a Gaussian density,  $N(\mathbf{r}_t; \tilde{\mathbf{r}}_t, \tilde{\boldsymbol{\Sigma}}_t)$ , where the vector of means and the covariance matrix are computed as

$$\tilde{\boldsymbol{\Sigma}}_t^{-1} = \boldsymbol{\Sigma}_{\varepsilon}^{-1} + \boldsymbol{\Sigma}_{t|t-1}^{-1} \quad \text{and} \quad \tilde{\mathbf{r}}_t = \tilde{\boldsymbol{\Sigma}}_t (\boldsymbol{\Sigma}_{\varepsilon}^{-1} \mathbf{y}_{S,t} + \boldsymbol{\Sigma}_{t|t-1}^{-1} \mathbf{r}_{t|t-1}), \quad (24)$$

respectively. The weight update equation becomes

$$w_t^{(i)} \propto w_{t-1}^{(i)} \frac{p(\mathbf{y}_t | \mathbf{r}_t^{(i)}) p(\mathbf{r}_t^{(i)} | \mathbf{r}_{0:t-1}^{(i)})}{N(\mathbf{r}_t; \tilde{\mathbf{r}}_t^{(i)}, \tilde{\boldsymbol{\Sigma}}_t^{(i)})}. \quad (25)$$

Note that the pdf  $p(\mathbf{y}_t | \mathbf{r}_t)$  incorporates both GPS and RSS data as

$$p(\mathbf{y}_t | \mathbf{r}_t) = \prod_{n=1}^{J+S} p(\mathbf{y}_{n,t} | \mathbf{r}_t) = \prod_{s=1}^S p(\mathbf{y}_{s,t} | \mathbf{r}_t) \prod_{j=1}^J p(y_{j,t} | \mathbf{r}_t), \quad (26)$$

where both  $p(\mathbf{y}_{s,t} | \mathbf{r}_t) = N(\mathbf{y}_{s,t}; \mathbf{r}_t, \sigma_v^2 \mathbf{I}_2)$  and  $p(y_{j,t} | \mathbf{r}_t) = N(y_{j,t}; f_j(\mathbf{r}_t), \sigma_{\varepsilon_j}^2)$  are known, for  $j = 1, \dots, J$   $s = 1, \dots, S$ .

Table 3 shows a summary of the SIS algorithm for the nonlinear outdoor model.

**Table 3.** SIS algorithm with a more efficient proposal function for tracking in an outdoor environment with GPS and RSS data

<p>1. Initialization, at <math>t = 0</math>:</p> <ul style="list-style-type: none"> <li>- For <math>i = 1, \dots, M</math>, sample <math>\mathbf{r}_0</math> and <math>\mathbf{v}_0</math> from the prior functions <math>p(\mathbf{r}_0)</math> and <math>p(\mathbf{v}_0)</math>. Initialize weights <math>w_0^{(i)} = \frac{1}{M}</math>.</li> </ul> <p>2. Recursive step, for <math>t &gt; 0</math>:</p> <ul style="list-style-type: none"> <li>- For <math>i = 1, \dots, M</math>, compute the vector of means <math>\mathbf{r}_{t t-1}^{(i)}</math> and the covariance matrices <math>\Sigma_{t t-1}^{(i)}</math> with the Kalman filter.</li> <li>- With the current GPS observation, <math>\mathbf{y}_{S,t}</math>, compute the vector of means <math>\tilde{\mathbf{r}}_t^{(i)}</math> and the covariance matrices <math>\tilde{\Sigma}_t^{(i)}</math> of the proposal pdf as defined in (24).</li> <li>- For <math>i = 1, \dots, M</math>, draw <math>\mathbf{r}_t^{(i)}</math> from the distribution <math>p(\mathbf{r}_t   \mathbf{r}_{0:t-1}^{(i)}, \mathbf{y}_{S,t}) = N(\mathbf{r}_t; \tilde{\mathbf{r}}_t^{(i)}, \tilde{\Sigma}_t^{(i)})</math>.</li> <li>- For <math>i = 1, \dots, M</math>, update the weights, <math>w_t^{(i)} \propto w_{t-1}^{(i)} \frac{p(\mathbf{y}_t   \mathbf{r}_t^{(i)}) p(\mathbf{r}_t^{(i)}   \mathbf{r}_{0:t-1}^{(i)})}{N(\mathbf{r}_t; \tilde{\mathbf{r}}_t^{(i)}, \tilde{\Sigma}_t^{(i)})}</math> with the complete vector of observations of the current time instant <math>\mathbf{y}_t</math>.</li> <li>- Compute the effective sample size <math>\hat{M}_{eff} = 1 / \sum_{k=1}^M w_t^{(k)2}</math>. If <math>\hat{M}_{eff} &lt; \lambda M</math> resample and set <math>w_t^{(i)} = \frac{1}{M} \forall i</math>.</li> </ul>
---

### 3.4 Switching between Algorithms

As the targets move from an indoor environment to an outdoor environment, or vice versa, we have to switch between different tracking algorithms. There are essentially three cases. If the target moves from indoors to outdoors, or outdoors to indoors, but RSS data are available in *both* environments, then the tracking algorithms are particle filters and it is straightforward to go from one to another.

In order to switch from an outdoor environment with GPS data alone to an indoor environment with RSS data, we have to generate a collection of particles from the Gaussian distribution computed by the Kalman filter immediately before the transition, which plays the role of a prior pdf for the particle filter. Specifically if  $p(\mathbf{x}_{t-1} | \mathbf{y}_{1:t-1}) = N(\mathbf{x}_{t-1}; \hat{\mathbf{x}}_{t-1|t-1}, \mathbf{P}_{t-1|t-1})$  then we can draw  $M$  samples with equal weights,

$$\mathbf{x}_{t-1}^{(i)} \sim N(\mathbf{x}_t; \mathbf{x}_{t-1|t-1}, \mathbf{P}_{t-1|t-1}), \quad w_{t-1}^{(i)} = \frac{1}{M} \quad i = 1, \dots, M, \quad (27)$$

from which the positions of the particles,  $\mathbf{r}_{t-1}^{(i)}, i = 1, \dots, M$ , are extracted.

The prior  $p(\mathbf{v}_{t-1}|\mathbf{y}_{1:t-1})$  is a Gaussian marginal of  $N(\mathbf{x}_{t-1}; \hat{\mathbf{x}}_{t-1|t-1}, \mathbf{P}_{t-1|t-1})$  that is straightforward to compute.

$$\begin{aligned}\hat{\mathbf{x}}_{t-1|t-1} &= \sum_{i=1}^M w_{t-1}^{(i)} \mathbf{x}_{t-1}^{(i)} \\ \mathbf{P}_{t-1|t-1} &= \sum_{i=1}^M w_{t-1}^{(i)} (\mathbf{x}_{t-1}^{(i)} - \hat{\mathbf{x}}_{t-1|t-1})(\mathbf{x}_{t-1}^{(i)} - \hat{\mathbf{x}}_{t-1|t-1})^\top.\end{aligned}\quad (28)$$

where  $\mathbf{x}_t^{(i)} = [\mathbf{r}_t^{(i)\top} \mathbf{v}_t^{(i)\top}]^\top$  and  $\mathbf{v}_t^{(i)}$  is the mean of  $p(\mathbf{v}_t|\mathbf{r}_{0:t}^{(i)})$  which is obtained from a Kalman filter for the  $i$ -th particle (this is the *same* Kalman filter that is used to compute  $p(\mathbf{r}_t|\mathbf{r}_{0:t-1}^{(i)}, \omega_{0:t-1}^{(i)})$ ).

The algorithms require of a variable that will indicate them the available technology at each instant  $t$ . To do so, we introduce a new variable  $K_t \in \{0, 1, 2\}$  that can take 3 values:  $K_t = 0$  indicates that we only have ZigBee observations and that we must use particle filters for indoor tracking,  $K_t = 1$  indicates us that we only have GPS observations available and that we must use the Kalman filter, and lastly  $K_t = 2$  indicates that we have GPS and ZigBee observations and we therefore use the data fusion particle filter for outdoor.

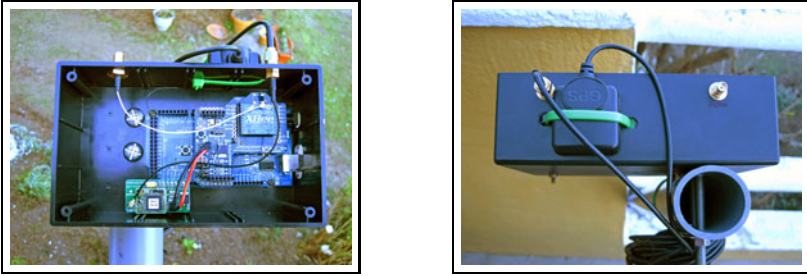
## 4 Experimental Setup and Results

### 4.1 Experimental Setup

We have developed two kind of hardware nodes using standard parts. For the collection of RSS data, we have set up  $J=8$  anchors, acting as ZigBee transmitters, built using Arduino boards and XBee (series 1) modules (IEEE 802.15.4 compliant). The mobile device, that acts as a target, uses an Arduino Mega board, a XBee (series 1) module, and adds an small OEM GPS receiver. The antennas for 2.4 GHz band were all equal, 2 dBi monopole omnidirectional antennas. For the GPS receiver we used a RHCP external antenna. Each node was mounted inside a plastic enclosure and attached to a 2 m long non-metal pole. This was done to minimize the interferences caused by the person who carries the device. Figure 1 shows the setup of the mobile device. The anchors look really similar, with a smaller Arduino inside and without the GPS receiver (and its antenna).

The ZigBee and GPS measurements were taken in an outdoor scenario, in the middle of a clear area (without walls or trees) of about  $600 m^2$ . We deployed an 8 ZigBee node network (anchors) covering a  $6 \times 10$  m area and, on the other hand, another mobile node, with ZigBee and GPS receivers. This mobile node acted as a gateway, receiving both the ZigBee packets (from the 8 anchors) and the GPS signal every 1 s. The measurement were tagged by this node and sent to a small laptop using the USB port, where they were stored for future off line process.

In order to test the performance of the proposed algorithms and to be able to build realistic observation models we collected a large number of RSS and GPS observations in the mentioned area. The GPS observations are modelled as the position of the target with an added Gaussian noise. Therefore once known the



**Fig. 1.** Mobile device setup, with ZigBee and GPS receivers

true position where the GPS data are being taken,  $\mathbf{x}_{n,real}$ , we may translate the GPS global coordinates onto ENU coordinates (east north up) [14] and we may compute the covariance matrix applying a maximum likelihood criteria

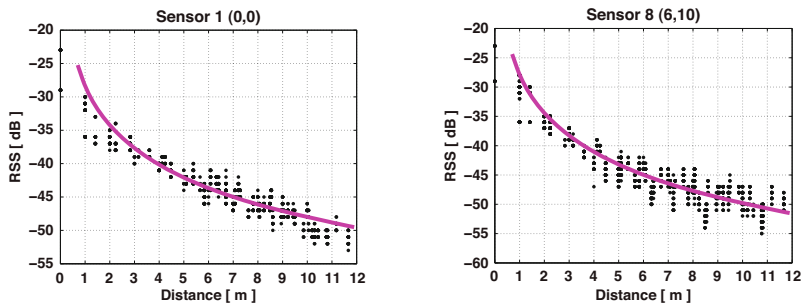
$$\Sigma_{\varepsilon} = \frac{1}{N} \sum_{n=1}^N (\mathbf{x}_n - \mathbf{x}_{n,real})(\mathbf{x}_n - \mathbf{x}_{n,real})^{\top}, \quad (29)$$

where  $N$  is the total number of GPS observations taken in the experiment,  $n$  is the index of a unique observation and  $\mathbf{x}_{n,real}$  is the true position where the  $n$ -th observation has been taken. The resulting covariance matrix is

$$\Sigma_{\varepsilon} = \begin{bmatrix} 7.5 & -0.58 \\ -0.58 & 11.3 \end{bmatrix}. \quad (30)$$

For the modeling of the ZigBee observation functions we have followed the criteria described in 2.2 to fit the real data taken in the experiments. Figure 2 shows experimental data taken from sensors 1 and 8 and the log-distance path loss models adjusted to them. Note we have fitted one model for each sensor.

The details of the hardware and experiments performed for the indoor model construction can be found in [1].

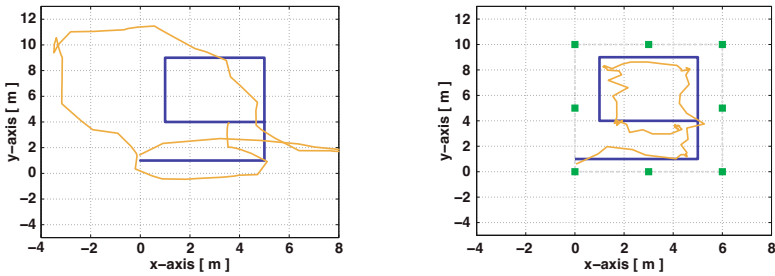


**Fig. 2.** Real ZigBee outdoor measurements taken in an outdoor environment for Sensors 1 and 8 and the observation functions adjusted to them

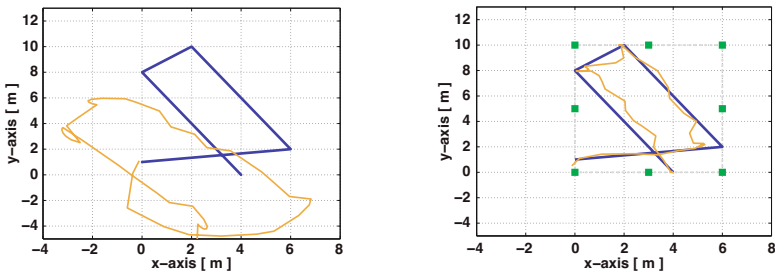
### 4.2 Results

In order to illustrate the performance of the outdoor algorithms we have taken experimental data of 8 ZigBee sensors and experimental GPS data from a moving target following three specific trajectories (see Figures 3, 4 and 5).

Figures 3, 4 and 5 show a tracking example of the two outdoor algorithms: the Kalman filter with GPS data alone and SIS algorithm. The figures in the left show the estimation of the target trajectory with the Kalman filter and the GPS observations and the figures in the right show the estimated trajectories with the SIS filter that uses GPS and RSS data. The true trajectory is drawn with a dark colored line, the estimated trajectory is drawn in a light colored line and the ZigBee nodes are depicted with squares.

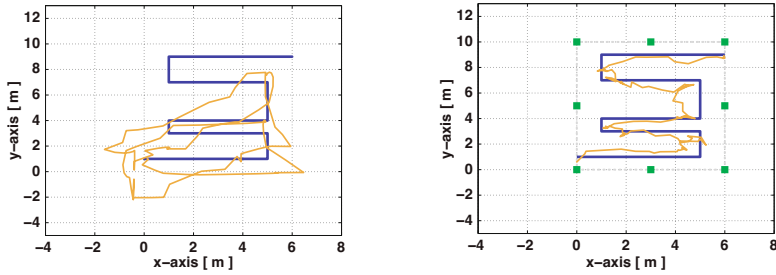


**Fig. 3.** Outdoor tracking example using ZigBee and GPS real data. The figure in the left uses GPS only for the estimation (via de Kalman filter) and the figure in the right uses both GPS and ZigBee data (vis de SIS algorithm).



**Fig. 4.** Outdoor tracking example using ZigBee and GPS real data. The figure in the left uses GPS only for the estimation (via de Kalman filter) and the figure in the right uses both GPS and ZigBee data (vis de SIS algorithm).

As we can observe the trajectories with GPS data have a greater error and the improvement we obtain by incorporating RSS data is high. This is as expected, because the precision that the civil GPS system can obtain is from 5 to 10 meters and depends on open sky conditions [16,8]. The area in which we perform the



**Fig. 5.** Outdoor tracking example using ZigBee and GPS real data. The figure in the left uses GPS only for the estimation and the figure in the right uses both GPS and ZigBee real data.

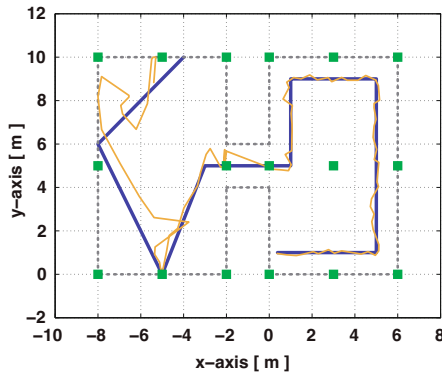
tracking is, therefore, too small for the precision that the technology provides. The ZigBee technology, on the other hand, has a much lower range but achieves a greater precision.

In order to illustrate the performance of the tracking scheme we have generated synthetic trajectories that switch between environments (from indoor to outdoor) and we have synthetically generated observations that switch between technologies (from indoor ZigBee to outdoor ZigBee with GPS, to outdoor with GPS only).

We have simulated a scenario consisting of an indoor room of dimension  $6 \times 10$  meters, linked to a 2 meter long and 2 meters wide corridor that leads on to an outdoor environment, which is also an area of  $6 \times 10$  meters. This is shown in Figure 6. In the  $6 \times 10$  meters area on the right and in the corridor we assume we are in an indoor environment. In the  $6 \times 10$  meter area of the left we assume we are in an outdoor environment.

A random trajectory that switches between environments, generated according to the corresponding dynamic models, has been randomly generated for exemplification. At each time instant  $t$ , we check the position of the target in the previous time instant,  $\mathbf{r}_{t-1}$ , and the new position,  $\mathbf{r}_t$ , is generated according to the environment-specific dynamic model. In order to generate the observations that switch between technologies we also check where the target is at every time instant, and we generate the observations according to the environment-specific model. On the other hand, as we have two observation models for the outdoor environment (GPS only and GPS and ZigBee) we have introduced a new random variable which in outdoor environments chooses the available technology. When  $K_t = 1$  we assume we only have GPS technology available and when  $K_t = 2$  we assume we have the two technologies and we, therefore, simulate both types of observations. We have also associated a transition probability for the technology selection variable so that we do not change technologies too fast, that is,  $p(K_t = i | K_{t-1} = i) = 0.95$ ,  $i = 1, 2$ .

Figure 6 shows a tracking example in this mixed scenario. The dark line is the true trajectory, the light line is the estimated trajectory and the squares are the ZigBee nodes. The beginning of the simulation is situated in the right *indoor* room, and the simulation lasts 300 seconds. The observation period is  $T = 0.4$  seconds. In the first 200 seconds the target moves in an indoor environment and we have used the indoor specific algorithm. From the 200-th second, the target has moved on to the corridor and then on to the outdoor environment. In the corridor we assume we have both GPS and outdoor ZigBee measurements available. Then, in the left outdoor environment we have allowed variable  $K_t$  to choose which technology is available in each time instant.



**Fig. 6.** Example of tracking a simulated trajectory using different technologies in different time instants and commuting between environment and technology specific algorithms. The beginning of the trajectory is situated in the right *indoor* room and the trajectory end in an outdoor environment (in the left area). For the indoor environment we have used ZigBee measurements to perform the tracking and for the outdoor environment we have used for some instant GPS measurements only and others GPS and ZigBee outdoor measurements.

### 4.3 Conclusions

We have introduced a target tracking algorithm for mixed indoor and outdoor scenarios based on the particle filtering methodology and a class of generalized interacting multiple-model state-space systems. The resulting method enables the fusion of GPS and RSS data in such a way that a target moving between indoor and outdoor environments, and therefore switching between different available data (GPS alone, RSS alone or both), can be seamlessly tracked. Within the particle filter, we use Kalman filtering techniques to reduce the variance of Monte Carlo estimates and to design efficient importance sampling schemes when GPS data are available. The performance improvement, compared to outdoor trackers that use only GPS data, is demonstrated using experimental data. Computer simulation results are presented to illustrate the application of the method in mixed indoor/outdoor scenarios.



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