Cross-Layer Control for Utility Maximization in Multihop Cognitive Radio Networks

Long Bao Le

INRS-EMT, University of Quebec
Montreal, Quebec, Canada
long.le@emt.inrs.ca

Abstract. We investigate the cross-layer control problem for utility maximization in a multihop cognitive radio network. Specifically, we consider a scenario where wireless links of a secondary multihop wireless network opportunistically exploit a frequency band for data delivery when their neighboring primary users do not access it. We assume that the busy/idle status of the underlying channel in view of any particular secondary wireless link follows a two-state Markov chain and this information is only available at each secondary wireless link within one time slot delay. We develop a joint flow control, routing and scheduling algorithm that can achieve the total network utility arbitrarily close to optimality. In addition, we show that the proposed algorithm can maintain stability of all network queues while keeping collision probabilities with primary users below predetermined desirable values. To the best of our knowledge, this paper is the first attempt to design optimal control algorithms for multihop cognitive radio networks.

Keywords: Network control, routing, scheduling, utility and backlog tradeoff, stability/throughput region, cognitive radio, multihop wireless networks.

1 Introduction

Recent measurements have shown that spectrum utilization on many frequency bands is very low [1]. This has motivated a great deal of research interests from FCC, wireless industry as well as academia [2], [3], [4], [5], [6]. These research interests are mostly driven by growing bandwidth demands of emerging broadband wireless applications. In general, wireless technologies that aim at improving spectrum utilization through efficient spectrum sharing/allocation techniques are commonly referred to as *cognitive radio technologies*. In fact, the first standard that specifies physical, MAC and air interface for spectrum sharing in the TV broadcast band has been under active development [4]. There have been growing research activities on information theoretic, protocol and system engineering issues of cognitive radio systems.

Although the cognitive radio can be very broad and abstract in concept [3], research activities in this area mainly focus on developing efficient hierarchical spectrum sharing techniques between primary and secondary users/networks

X. Zhang and D. Qiao (Eds.): QShine 2010, LNICST 74, pp. 28–40, 2011.

[5], [16]. In particular, primary users usually have strictly higher priority than secondary users in accessing one or several frequency bands. Here, there are two important spectrum sharing paradigms between primary and secondary users, namely spectrum underlay and spectrum overlay [5]. In the spectrum underlay paradigm, secondary users are allowed to transmit simultaneously with primary users on the same frequency band. However, transmission powers of secondary users should be carefully controlled so that the total interference they create to primary users must be smaller some allowable limit. The spectrum underlay can be realized by CDMA or Ultra-wide bandwidth (UWB) radio access technologies [7], [8]. In the spectrum overlay paradigm, secondary users can only access the channels that are not being used by primary users [9]. Here, secondary users have to detect or sense the presence of primary users by employing some form of spectrum sensing [10], [11].

Several spectrum sharing protocols based on the spectrum overlay paradigm have been proposed for both single-hop and multihop cognitive radio networks. In particular, significant efforts have been made to develop efficient contention-based medium access control (MAC) protocols for opportunistic spectrum access [18]-[23]. These MAC protocols aim at incorporating spectrum sensing with spectrum sharing functionalities in an intelligent manner. There have been also some recent works that proposed scheduling-based spectrum sharing solutions for multihop cognitive radio networks [17]. However, these spectrum sharing solutions consider static network settings, which, therefore, do not consider the network stability issue.

In this paper, we investigate the network control problem for a secondary multihop cognitive radio network using spectrum overlay paradigm within the stability framework proposed by Tassiulas and Ephremides [12]. There are several research challenges in designing network control protocols for this research problem. First, a secondary user can only access a channel if no primary users in its neighborhood is using the channel. Therefore, new cognitive interference constraints need to be defined, which must capture the conflict relationship among secondary users and between primary and secondary users. Second, secondary users have to periodically sense the channel to detect the presence of primary users, which will introduce several forms of imperfection, namely delayed and/or erroneous spectrum sensing.

We will consider the scenario where spectrum sensing outcomes are available to secondary users within one time slot delay. Due to this delayed spectrum sensing, spectrum access of secondary users should be controlled to keep collision rates with primary users to be within a tolerable limit. We investigate the utility optimization problem for the secondary network and propose a cross-layer control algorithm, which is proved to achieve optimal network utility. The considered model captures conflict relationships between primary and secondary users as well as imperfect aspects of spectrum sensing. The most similar work to ours was published in [14]. However, this paper considered a special network setting where primary and secondary users communicate with their access points. In addition, delayed spectrum sensing was not explicitly considered in their paper.

The remaining of this paper is organized as follows. System model is described in section II. We propose optimal control algorithm in section III and analyze its performance in section IV. Then, we conclude the paper in section V.

2 System Model

Consider a scenario where secondary users form a multihop wireless network, in which they share a single frequency band with primary users for multihop communications. We assume a hierarchical spectrum sharing model where a secondary user can only transmit to its intended receiver when the channel is not used by any primary users in its local neighborhood. In essence, this local neighborhood determines the conflict relationship among primary and secondary users. We model the secondary multihop wireless network as a network graph $G = (V, E)$ where V is the set of secondary nodes and E is the set of secondary links. Assume that the cardinality of E is N. Also, suppose there are M primary links in the network, which will be referred to as primary users in the following.

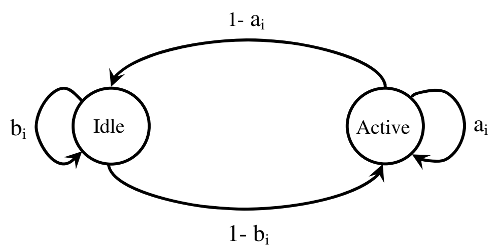

Fig. 1. Two-state Markov chain to capture idle/active state of primary user

Consider a time slotted system where a secondary user can transmit one packet/slot to its intended neighbor if there is no conflict transmissions from other secondary links and primary users. We model the status of a particular primary user i as two-state Markov chain, which is illustrated in Fig. 1. This model has been justified by several recent practical measurements [24]. We assume that the statuses of different primary users are independent from one another. For the secondary network, we assume that there are a set of traffic flows, each of which corresponds to a source and destination node. We will allocate different buffers at each secondary node to queue secondary traffic flows with the corresponding destination (i.e., per destination queueing). Traffic with the same destination is said to belong to the same commodity. It is assumed that arrival traffic waits outside the network in overflow buffers for being admitted into the network by flow controllers.

Now, we describe the conflict relationship of different transmissions in the network. Let Ψ_i be the set of secondary links that can cause collision with primary user i if any of these secondary links and primary user i transmit at the same time. Let Π_{mn} be the set of primary users, which are impacted by the transmission of link (m, n), i.e., if $i \in \Pi_{mn}$ then $(m, n) \in \Psi_i$. In addition, there is a finite number of feasible activation sets of secondary links, where secondary links in each feasible activation set can be activated simultaneously. We denote the set of feasible activation sets as Δ. In general, the set Δ is determined by some underlying interference model. Examples of interference models are k-hop interference model [26] and SINR interference model [25]. We denote a schedule as an N-dimensional vector $\boldsymbol{I}$ whose element is equal to one if the corresponding link is activated and equal to zero, otherwise.

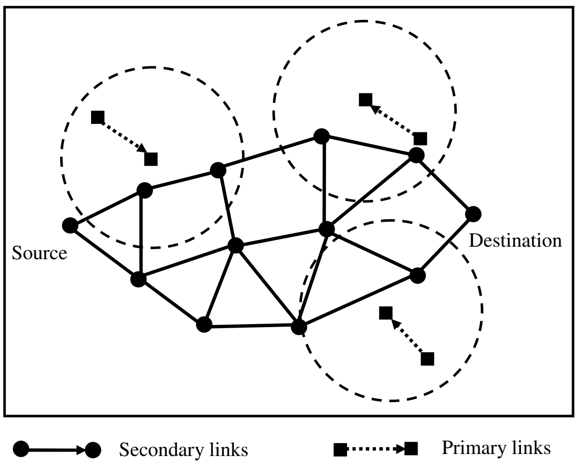

Fig. 2. Multihop cognitive radio network

2.1 Sensing Model and System Constraints

We assume that each secondary link (m, n) will sense the active/idle status of primary users in set Π_{mn} in each time slot. Ideally, link (m, n) is only allowed to transmit if all primary users in Π_{mn} are idle in that time slot. We assume that each secondary user performs sensing continuously without errors. However, secondary users can only report the sensing results to the network controller within one time slot delay, based on which control actions are determined. Due to delayed sensing, collisions between primary and secondary users can occur. Specifically, this collision occurs when secondary links mis-detect the presence of a conflict primary user, i.e., secondary links transmit when conflict primary users are in active state. Let $X_i(t)$ represent a collision variable, which captures the collision event between primary user i and some secondary users in Ψ_i. Specifically, we have

$$X_i(t) = \begin{cases} 1 \text{ if there is a collision with primary} \\ \quad \text{user } i \text{ in slot } t \\ 0 \text{ otherwise.} \end{cases} \tag{1}$$

Let $\overline{X}_i$ be the time average collision rate experienced by primary user i. We have

$$\overline{X}_i = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} E\left\{X_i(\tau)\right\}. \tag{2}$$

To protect primary users, we require that the time average collision rate of primary user i be smaller than some desired value δ_i, i.e.,

$$\overline{X}_i \leq \delta_i, \ \forall i = 1, 2, \cdots, M \tag{3}$$

where δ_i is small number chosen in advance.

2.2 Queueing Dynamics and Problem Formulation

Let $S_i(t)$ denote the status of primary user i in time slot t where $S_i(t) = 1$ if primary user i is active in time slot t, and $S_i(t) = 0$, otherwise. Let us define the following quantity

$$\omega_{mn}(t) = \prod_{i \in \Pi_{mn}} (1 - S_i(t)). \tag{4}$$

Then, to avoid collisions with primary users, a particular secondary link (m, n) should be silent in time slot t if $\omega_{mn}(t) = 0$. Let $Q_n^{(c)}(t)$ be the backlog at secondary node n for commodity c in time slot t. Also, let $R_n^{(c)}(t)$ be the number packets admitted into the network at secondary node n for commodity c in time slot t. We assume the constantly backlogged scenario where there are always enough packets to admit into the secondary network at all times. Let $\mu_l^{(c)}(t)$ be the number of commodity c packets transmitted over secondary link l in time slot t. For brevity, we sometimes use a single letter l to denote a wireless link. The queue evolution can be written as

$$Q_n^{(c)}(t+1) = Q_n^{(c)}(t) - \sum_{l \in \Omega_n^{\mathsf{out}}} \mu_l^{(c)}(t)\omega_l(t) + \sum_{l \in \Omega_n^{\mathsf{in}}} \mu_l^{(c)}(t)\omega_l(t) + R_n^{(c)}(t) \tag{5}$$

where Ω_n^{out} and Ω_n^{in} denote the set of outgoing and incoming links at node n, respectively. The equation (5) can be interpreted as follows. The backlog for commodity c at node n decreases by a value, which is equal to the number of packets successfully transmitted over all outgoing links and increases by a value, which is equal to the total number of admitted packets and successfully transmitted packets over incoming links. In particular, transmission over link l is only successful if $\omega_l(t) = 1$ (i.e., no collision with active primary users occurs).

Let $\overline{R}_n^{(c)}(t)$ be the time average rate of admitted traffic for commodity c at node n up to time t, that is

$$\overline{R}_n^{(c)}(t) \triangleq \frac{1}{t} \sum_{\tau=0}^{t-1} E\left\{R_n^{(c)}(\tau)\right\}. \tag{6}$$

The long-term time-average admitted rate for commodity c at node n is defined as

$$\overline{R}_n^{(c)} \triangleq \lim_{t\to\infty} \overline{R}_n^{(c)}(t). \tag{7}$$

Now, we recall the definitions of network stability and the maximum throughput region (or throughput region for brevity) [12], which will be used in our analysis. A queue for a particular commodity c at node n is called strongly stable if

$$\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} E\left\{Q_n^{(c)}(\tau)\right\} < \infty. \tag{8}$$

In addition, the network is called strongly stable (or stable for simplicity) if all individual queues in the network are stable. The maximum throughput region Λ contains the union of all traffic arrival/admitted rate vectors such that there exists a network control algorithm to stabilize all individual queues in the network.

Let $U_n^{(c)}\left(\overline{R}_n^{(c)}\right)$ be the utility achieved by admitting an average rate $\overline{R}_n^{(c)}$ for commodity c at node n. We assume that utilities functions $U_n^{(c)}(.)$ are concave, increasing, and differentiable. We seek to optimize the total network utility subject to constraints on network throughput and collisions with primary users due to delayed sensing performed by secondary links. Specifically, we are interested in solving the following optimization problem

$$\text{maximize} \quad \sum_{n,c} U_n^{(c)}\left(\overline{R}_n^{(c)}\right) \tag{9}$$

$$\text{subject to} \quad \left(\overline{R}_{n_c}^{(c)}\right) \in \Lambda \tag{10}$$

$$\overline{X}_i \leq \delta_i,\ \forall i = 1, 2, \cdots, M \tag{11}$$

where $\overline{R}_n^{(c)}$ is the time average admitted rate for commodity c at node n, and δ_i are the desired collision rates. Constraints (11) ensure that long-term collision rates with primary users are below desired levels.

2.3 Discussion of Formulated Problem

The optimization problem (9)-(11) is the network utility maximization (NUM) problem, which seeks to achieve a fair resource sharing for different traffic flows. Here, the desired fairness for radio resource sharing can be achieved by choosing appropriate utility functions. One popular class of utility functions is the α-fair utilities for which different fairness criteria can be achieved by changing a parameter α [27]. Consideration of NUM under the stability framework of Tassiulas and Ephremides [12] has been done in [13], [15]. Investigation of this problem in the cognitive radio setting has been recently performed in [14]. However, this paper considers a simple setting with single-hop traffic flows. In the current paper, we extend this problem to the multihop setting, where we need to design a joint flow control, routing, and scheduling algorithm. In addition, we consider the realistic scenario where only delayed spectrum sensing information is available at the network controller.

3 Optimal Control Algorithm

To solve the optimization problem (9)-(11), we employ the Lyapunov optimization technique developed in [15]. In particular, to capture the collisions experienced by primary user i, we define a virtual queue for each primary user i with the following evolution

$$Z_i(t+1) = \max\left[Z_i(t) - \delta_i, 0\right] + X_i(t). \tag{12}$$

We call these queues as virtual queues because their values can be maintained in software counters (i.e., no physical buffers are needed for implementation). In addition, $Z_i(t)$ captures the "backlog" in the virtual queue with "arrival processs" $X_i(t)$ and constant service rate δ_i. Therefore, it can be shown that if all virtual queues $Z_i(t)$ are stable then all collision requirements in (3) are satisfied. This is because the average arrival rate should be smaller than the service rate for a stable queue.

Specifically, an optimal control algorithm can be developed by minimizing the Lyapuniv drift minus total utility for an appropriate Lyapunov function. Now, let $\boldsymbol{\Theta}(t) = (\boldsymbol{Q}(t), \boldsymbol{Z}(t), \boldsymbol{S}(t-1))$ denote the system states. We describe the optimal cognitive control algorithm in the following, whose performance is analyzed in the next section.

Cross-Layer Cognitive Control Algorithm

- *Flow Control:* Each node n injects an amount of traffic of commodity c into the network equal to $r_n^{(c)}$, which is the optimal solution of the following optimization problem

$$\begin{array}{l} \text{maximize } V\sum_{n,c} U_n^{(c)}(r_n^{(c)}) - 2\sum_{n,c} Q_n^{(c)}(t) r_n^{(c)} \\ \text{subject to } r_n^{(c)} \leq R_n^{\mathsf{max}} \end{array} \tag{13}$$

  where $R_n^{\mathsf{max}} > 0$ is a positive number, which controls the burstiness of the admitted traffic and V is a control parameter.
- *Routing and Scheduling:* For each link (m,n), find the commodity satisfying the following

$$c^* = \underset{c}{\operatorname{argmax}}\, E\left\{\omega_{mn}(t)|\boldsymbol{\Theta}(t)\right\}\left[Q_m^{(c)}(t) - Q_n^{(c)}(t)\right]. \tag{14}$$

  The weight for link (m,n) is defined as

$$W_{mn}(t) \triangleq E\left\{\omega_{mn}(t)|\boldsymbol{\Theta}(t)\right\}\left[Q_m^{(c^*)}(t) - Q_n^{(c^*)}(t)\right] - \sum_{i \in \Pi_{mn}} Z_i(t) E\left\{S_i(t)|\boldsymbol{\Theta}(t)\right\}. \tag{15}$$

Using these weights, we find a feasible schedule $\boldsymbol{I}^*(t)$ as follows:

$$\boldsymbol{I}^*(t) = \underset{\boldsymbol{I}\in\Delta}{\operatorname{argmax}} \sum_{(m,n)} I_{mn} W_{mn}(t) \tag{16}$$

where recall that Δ denotes the set of all feasible schedules. For each scheduled link (m,n) in $\boldsymbol{I}^*$, we transmit one packet of flow c^* that satisfies (14). After the scheduled transmissions occur, based on the feedbacks of the "collisions outcomes" $X_i(t)$ from the primary users, the control queues $Z_i(t)$ are updated according to (12).

3.1 Discussion of Proposed Control Algorithm

The scheduling policy described in (16) has the max-weight structure similar to that proposed by Tassiulas and Ephremides in [12]. However, the weight in (15) is different from that in [12] to capture two important aspects of the current model, i.e., the delayed channel sensing information and the potential collisions with primary users. The modified differential backlog measure in (14) is the scaled version of the traditional measure of [12]. Here, the scaling factors are $E\{\omega_{mn}(t)|\boldsymbol{\Theta}(t)\}$, which capture the expected number of packets that can be transmitted over link (m,n) given queue length and delay sensing information.

In addition, the link weight in (15) has the term $\sum_{i\in\Pi_{mn}} Z_i(t)E\{S_i(t)|\boldsymbol{\Theta}(t)\}$, which captures the collision measure with primary users due to secondary link (m,n). In particular, secondary links with large $\sum_{i\in\Pi_{mn}} Z_i(t)E\{S_i(t)|\boldsymbol{\Theta}(t)\}$ will achieve small weights, which are therefore less likely to be scheduled. This weight structure helps avoid excessive collisions with active primary users.

Note that given $\boldsymbol{S}(t-1)$, we can easily calculate $E\{\omega_{mn}(t)|\boldsymbol{\Theta}(t)\}$ and $E\{S_i(t)|\boldsymbol{\Theta}(t)\}$ using the transition probabilities of the corresponding Markov chains. Specifically, we have

$$\begin{aligned}
E\{\omega_{mn}(t)|\boldsymbol{\Theta}(t)\} &= \prod_{i\in\Pi_{mn}} \left(1 - \Pr\{S_i(t)=1|S_i(t-1)\}\right) \\
&= \prod_{i\in\Pi_{mn}} \Pr\{S_i(t)=0|S_i(t-1)\} \\
E\{S_i(t)|\boldsymbol{\Theta}(t)\} &= \Pr\{S_i(t)=1|S_i(t-1)\}.
\end{aligned}$$

This is because $S_i(t)$ is independent of queue length $\boldsymbol{Q}(t)$ and $\boldsymbol{X}(t)$. The performance of the proposed cross-layer control algorithm is stated in the following theorem.

Theorem: Let $\boldsymbol{R}^*$ be the optimal solution of the considered optimization problem. The proposed control algorithm achieves the following performance bounds:

$$\liminf_{M\to\infty} \sum_{n,c} U_n^{(c)}\left(\overline{R}_n^{(c)}(M)\right) \geq \sum_{n,c} U_n^{(c)}\left(R_n^{(c)*}\right) - \frac{B}{V} \tag{17}$$

$$\limsup_{M\to\infty} \frac{1}{M} \sum_{\tau=0}^{M-1} \sum_{n,c} E\left\{Q_n^{(c)}(\tau)\right\} \leq \frac{B+VG_{\mathsf{max}}}{2\lambda_{\mathsf{max}}} \quad (18)$$

where B is a finite number, V is a control parameter of the control algorithm, G_{max} is maximum achievable utility, and λ_{max} is the largest value such that $\boldsymbol{\lambda}_{\mathsf{max}} \in \Lambda$.

It can be observed that the control parameter V can be used to control the utility and backlog tradeoff. Specifically, larger V leads to better achievable utility with the penalty on the average backlog bound in the network.

3.2 Further Discussion and Possible Extension

The flow controller of the proposed control algorithm can be implemented in a distributed manner by each source node. This is because to make the flow control decision in each time slot, flow controllers only need to know the backlog information of its own buffer for the corresponding commodity. In addition, the link weight in (15) can also be calculated by each link using the updated "virtual queue value" $Z_i(t)$ upon collecting collision outcomes $X_i(t)$ from the conflict primary users. However, the scheduling scheme in (16) requires centralized implementation in general. This is because we need to find one feasible schedule that achieves the maximum total weight among all possible feasible schedules in each time slot.

Fortunately, there are several techniques available in the literature, which enable us to decentralize the scheduling operation [28], [29], [30]. In particular, there is a tradeoff between performance and complexity in implementing these decentralization techniques. Specifically, the PICK-COMPARE based scheduling schemes [29], [30] can achieve very close to optimum throughput but requires high communication complexity. Other greedy scheduling schemes such as the one proposed in [28] have low complexity but only achieve a fraction of the throughput region. Overall, it is feasible to apply any of these available scheduling techniques instead of the max-weight scheme to our proposed cross-layer control algorithm.

4 Performance Analysis

We analyze the performance of the proposed control algorithm and prove the main theorem of the paper in this section. As mentioned above, the analysis is based on the Lyapunov optimization technique proposed in [13], [15]. Now, consider the following Lyapunov function

$$L(\boldsymbol{Q}) \triangleq \sum_{n,c} \left(Q_n^{(c)}(t)\right)^2 + \sum_i (Z_i(t))^2. \quad (19)$$

Consider the Lyapunov drift defined as follows:

$$\Delta(t) \triangleq E\left\{L(\boldsymbol{Q})(t+1) - L(\boldsymbol{Q})(t) | \boldsymbol{\Theta}(t)\right\} \quad (20)$$

where recall that $\boldsymbol{\Theta}(t) = (\boldsymbol{Q}(t), \boldsymbol{Z}(t), \boldsymbol{S}(t-1))$ denotes the system states. Using the queue evolution equations in (5) and (12), we have

$$\begin{aligned}\Delta(t) \leq B + 2E\left\{\sum_i Z_i(t)\left(X_i(t) - \delta_i\right)|\boldsymbol{\Theta}(t)\right\} + 2E\left\{\sum_{n,c} Q_n^{(c)}(t)R_n^{(c)}(t)|\boldsymbol{\Theta}(t)\right\} \\ +2\sum_{n,c} Q_n^{(c)}(t)E\left\{-\sum_{l\in\Omega_n^{\mathsf{out}}} \mu_l^{(c)}(t)\omega_l(t) + \sum_{l\in\Omega_n^{\mathsf{in}}} \mu_l^{(c)}(t)\omega_l(t)|\boldsymbol{\Theta}(t)\right\} \qquad (21)\end{aligned}$$

where B is a finite number. Now, we can bound the collision variable $X_i(t)$ as follows:

$$X_i(t) \leq \sum_{c,(m,n)\in\Psi_i} \mu_{m,n}^{(c)}(t)S_i(t). \qquad (22)$$

This is because collision with primary user i occurs if some conflict secondary links in Ψ_i transmit while primary user i is in active state. Substitute this relationship into (21), we have

$$\begin{aligned}\Delta(t) - V\sum_{n,c} E\left\{U_n^{(c)}(R_n^{(c)}(t))|\boldsymbol{\Theta}(t)\right\} \leq B - 2\sum_i \delta_i Z_i(t) \\ +E\left\{2\sum_{n,c} Q_n^{(c)}(t)R_n^{(c)}(t) - V\sum_{n,c} U_n^{(c)}(R_n^{(c)}(t))|\boldsymbol{\Theta}(t)\right\} \\ -2\sum_{(m,n),c} E\left\{\mu_{mn}^{(c)}(t)\omega_{mn}(t)|\boldsymbol{\Theta}(t)\right\}\left[Q_m^{(c)}(t) - Q_n^{(c)}(t)\right] \\ +2\sum_i Z_i(t)E\left\{S_i(t)\sum_{(mn)\in\Psi_i,c} \mu_{mn}^{(c)}(t)|\boldsymbol{\Theta}(t)\right\} \\ = B - 2\sum_i \delta_i Z_i(t) - E\left\{V\sum_{n,c} U_n^{(c)}(R_n^{(c)}(t)) - 2\sum_{n,c} Q_n^{(c)}(t)R_n^{(c)}(t)|\boldsymbol{\Theta}(t)\right\} \qquad (23) \\ -2\left[\sum_{(m,n),c} E\left\{\mu_{m,n}^{(c)}(t)\omega_{mn}(t)|\boldsymbol{\Theta}(t)\right\}\left[Q_m^{(c)}(t) - Q_n^{(c)}(t)\right] \right. \qquad (24) \\ \left. - \sum_{(m,n),c} E\left\{\mu_{m,n}^{(c)}(t)\sum_{i\in\Pi_{mn}} Z_i(t)S_i(t)|\boldsymbol{\Theta}(t)\right\}\right]. \qquad (25)\end{aligned}$$

It can be observed that the proposed control algorithm minimizes the RHS of the above inequality. Specifically, the flow controller minimizes the third term of (23) and the routing/scheduling algorithm minimizes (24) and (25) in the RHS of the above inequality. Now, to quantify the performance of the proposed control algorithm, we need some more definitions. First, let us define the ϵ-stripped throughput region as follows:

$$\Lambda_\epsilon \triangleq \left\{\left(r_n^{(c)}\right) \mid \left(r_n^{(c)} + \epsilon\right) \in \Lambda\right\} \tag{26}$$

where $\left(r_n^{(c)}\right)$ denotes the vector of admitted rates for all commodities. Also, let $(R_n^{*(c)}(\epsilon))$ be the optimal solution of the following optimization problem

$$\text{maximize} \qquad \textstyle\sum_c U_n^{(c)}\left(R_n^{(c)}\right) \tag{27}$$

$$\text{subject to} \qquad \left(R_n^{(c)}\right) \in \Lambda_\epsilon \tag{28}$$

$$\overline{X}_i \leq \delta_i,\ \forall i = 1, 2, \cdots, M. \tag{29}$$

We will quantify the performance of the considered control algorithms in terms of $(R_n^{*(c)}(\epsilon))$. Note that $(R_n^{*(c)}(\epsilon))$ tends to the optimal solution $(R_n^{*(c)})$ as $\epsilon \to 0$ where $(R_n^{*(c)})$ is the optimal solution of the optimization problem (27)-(29) where Λ_ϵ is replaced by Λ (i.e., the original throughput region). Because $R_n^{(c)*}(\epsilon)$ is inside the ϵ-stripped throughput region, there exists randomized stationary scheduling and routing scheme to support this rate, i.e., we have

$$V\sum_{n,c} U_n^{(c)}(R_n^{(c)}(t)) - 2\sum_{n,c} Q_n^{(c)}(t) R_n^{(c)}(t) \geq V\sum_{n,c} U_n^{(c)}(R_n^{(c)*}) - 2\sum_{n,c} Q_n^{(c)}(t) R_n^{(c)*} \tag{30}$$

$$E\left\{X_i(t)|\boldsymbol{\Theta}(t)\right\} \leq \delta_i \tag{31}$$

$$\sum_{n,c} Q_n^{(c)}(t)\left[\sum_{l\in\Omega_n^{\mathsf{out}}} \mu_l^{(c)}(t)\omega_l(t) - \sum_{l\in\Omega_n^{\mathsf{in}}} \mu_l^{(c)}(t)\omega_l(t)\right] \geq \sum_{n,c} Q_n^{(c)}(t)\left[R_n^{(c)*} + \epsilon\right]. \tag{32}$$

Using the results of (30), (31), and (32) in (23)-(25), we can obtain the following

$$\Delta(t) - V\sum_{n,c} E\left\{U_n^{(c)}(R_n^{(c)}(t))|\boldsymbol{\Theta}(t)\right\} \leq B - 2\epsilon\sum_{n,c} Q_n^{(c)}(t) - V\sum_{n,c} U_n^{(c)}(R_n^{(c)*}). \tag{33}$$

Taking the expectations over the distribution of $\boldsymbol{\Theta}(t)$ and summing over $t \in \{1, 2, \cdots, M\}$, we have

$$E\left\{L(\boldsymbol{Q}(M))\right\} - E\left\{L(\boldsymbol{Q}(0))\right\} - V\sum_{\tau=0}^{M-1}\sum_{n,c} E\left\{U_n^{(c)}\left(R_n^{(c)}(\tau)\right)\right\} \leq MB - VM\sum_{n,c} U_n^{(c)}\left(R_n^{(c)*}(\epsilon)\right) - 2\epsilon\sum_{\tau=0}^{M-1}\sum_{c} E\left\{Q_n^{(c)}(\tau)\right\}. \tag{34}$$

To prove the backlog bound, we arrange the inequality (34) appropriately and divide both sides by M, we have

$$\frac{1}{M}\sum_{\tau=0}^{M-1}\sum_{n,c} E\left\{Q_n^{(c)}(\tau)\right\} - \frac{E\left\{L(\boldsymbol{Q}(0))\right\}}{M} \leq \frac{B + VG_{\mathsf{max}}}{2\epsilon}. \tag{35}$$

Note that the above inequality holds for any $0 < \epsilon \leq \lambda_{\mathsf{max}}$. Hence, by choosing $\epsilon = \lambda_{\mathsf{max}}$ and taking the limit for $M \to \infty$ in (35), we can obtain the backlog bound. By arranging the terms of (34) appropriately and dividing both sides by VM, we have

$$\frac{1}{M}\sum_{\tau=0}^{M-1}\sum_{n,c} E\left\{U_n^{(c)}\left(R_n^{(c)}(\tau)\right)\right\} \geq \sum_{n,c} U_n^{(c)}\left(R_n^{(c)*}(\epsilon)\right) - \frac{B + E\left\{L(\boldsymbol{Q}(0))\right\}/M}{V} + \frac{2\epsilon}{VM}\sum_{\tau=0}^{M-1}\sum_{n,c} E\left\{Q_n^{(c)}(\tau)\right\} \tag{36}$$

where we have used the fact that $L(\boldsymbol{Q}(M)) \geq 0$ to obtain (36). Using the Jensen's inequality and taking the limit $M \to \infty$ in (36), we have

$$\liminf_{M\to\infty} \sum_{c} U_n^{(c)}\left(\overline{R}_n^{(c)}(M)\right) \geq \sum_{n,c} U_n^{(c)}\left(R_n^{(c)*}(\epsilon)\right) - \frac{B}{V}. \tag{37}$$

Hence, we can obtain the utility bound by letting $\epsilon \to 0$. Therefore, we have completed the proof for the main theorem of the paper.

5 Conclusion

We investigate the optimal control problem for utility maximization in multihop cognitive radio networks in this paper. Specifically, we seek to maximize the total utility achieved by different traffic flows of the secondary network subject to network stability and collision constraints with primary users. We propose a cross-layer control algorithm that is proved to achieve utility arbitrarily close to optimality and derive the corresponding utility-backlog tradeoff.

References

1. FCC. Spectrum policy task force report, FCC 02-155 (November 2002)
2. FCC. Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies, notice of proposed rule making and order, FCC 03-322 (December 2003)
3. Haykin, S.: Cognitive radio: Brain-empowered wireless communications. IEEE J. Sel. Areas Commun. 23(2), 201–220 (2005)
4. Cordeiro, C., Challapali, K., Birru, D., Shankar, N.S.: IEEE 802.22: The first worldwide wireless standard based on cognitive radios. In: DySpan 2005 (November 2005)
5. Zhao, Q., Sadler, B.M.: A survey of dynamic spectrum access: Signal processing, networking, and regulatory policy. IEEE Signal Processing Mag., 79–89 (May 2007)
6. Etkin, R., Parekh, A., Tse, D.: Spectrum sharing for unlicensed bands. IEEE J. Sel. Areas Commun. 25(3), 517–528 (2007)
7. Le, L.B., Hossain, E.: Resource allocation for spectrum underlay in cognitive radio networks. IEEE Trans. Wireless Commun. 7(12), 5306–5315 (2008)

8. Kim, D.I., Le, L.B., Hossain, E.: Joint rate and power allocation for cognitive radios in dynamic spectrum access environment. IEEE Trans. Wireless Commun. 7(12), 5517–5527 (2008)
9. Chou, C.T., Shankar, N.S., Kim, H., Shin, K.G.: What and how much to gain by spectrum agile? IEEE J. Sel. Areas Commun. 25(3), 576–588 (2007)
10. Gandetto, M., Regazzoni, C.: Spectrum sensing: A distributed approach for cognitive terminals. IEEE J. Sel. Areas Commun. 25(3), 546–557 (2007)
11. Unnikrishnan, J., Veeravalli, V.V.: Cooperative sensing for primary detection in cognitive radio. IEEE J. Sel. Signal Processing 2(1), 18–27 (2008)
12. Tassiulas, L., Ephremides, A.: Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks. IEEE Trans. Automatic Control 37(12), 1936–1948 (1992)
13. Georgiadis, L., Neely, M.J., Tassiulas, L.: Resource allocation and cross-layer control in wireless networks. Foundations and Trends in Networking 1(1), 1–144 (2006)
14. Urgaonkar, R., Neely, M.J.: Opportunistic scheduling with reliability guarantees in cognitive radio networks. In: IEEE INFOCOM (2008)
15. Neely, M.J.: Energy optimal control for time varying wireless networks. IEEE Trans. Information Theory 52(7), 2915–2934 (2006)
16. Devroye, N., Mitran, P., Tarokh, V.: Achievable rates in cognitive radio channels. IEEE Trans. Inf. Theory 52(5), 1813–1827 (2006)
17. Hou, Y.T., Shi, Y., Sherali, H.D.: Optimal spectrum sharing for multi-hop software defined radio networks. In: IEEE INFOCOM 2007 (May 2007)
18. Cao, L., Zheng, H.: Stable and efficient spectrum access in next generation dynamic spectrum networks. In: IEEE INFOCOM 2008 (2008)
19. Zhao, Q., Tong, L., Swami, A., Chen, Y.: Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework. IEEE J. Sel. Areas Commun. 25(3), 589–600 (2007)
20. Chang, N.B., Liu, M.: Optimal channel probing and transmission scheduling for opportunistic spectrum access. In: ACM MOBICOM 2007 (2007)
21. Huang, S., Liu, X., Ding, Z.: Opportunistic spectrum access in cognitive radio networks. In: IEEE INFOCOM 2008 (2008)
22. Le, L., Hossain, E.: OSA-MAC: A MAC protocol for opportunistic spectrum access in cognitive radio networks. In: IEEE WCNC 2008 (March 2008)
23. Su, H., Zhang, X.: Cross-layer based opportunistic MAC protocols for QoS provisionings over cognitive radio mobile wireless networks. IEEE J. Sel. Areas Commun. 26(1), 118–129 (2008)
24. Geirhofer, S., Tong, L., Sadler, B.M.: Dynamic spectrum access in the time domain: Modeling and exploiting white space. IEEE Commun. Mag. 45(5), 66–72 (2007)
25. Le, L.B., Modiano, E., Joo, C., Shroff, N.B.: Longest-queue-first scheduling under SINR interference model. In: ACM MobiHoc 2010 (2010)
26. Sharma, G., Mazumdar, R., Shroff, N.B.: On the complexity of scheduling in multihop wireless systems. In: ACM MOBICOM 2006 (2006)
27. Chiang, M., Low, S.H., Calderbank, A.R., Doyle, J.C.: Layering as optimization decomposition: A mathematical theory of network architectures. Proc. of the IEEE 95(1), 255–312 (2007)
28. Joo, C., Shroff, N.B.: Performance of random access scheduling schemes in multihop wireless networks. In: IEEE INFOCOM 2007 (2007)
29. Modiano, E., Shah, D., Zussman, G.: Maximizing throughput in wireless networks via gossiping. In: ACM SIGMETRICS (2006)
30. Sanghavi, S., Bui, L., Srikant, R.: Distributed link scheduling with constant overhead. In: ACM SIGMETRICS 2007 (June 2007)