Time and Power Scheduling in a Wireless Network with Network Coding and Bidirectional Relaying

Maryam Mohseni and Dongmei Zhao

Department of Electrical and Computer Engineering
McMaster University
Hamilton, Ontario, L8S 4K1
`dzhao@mcmaster.ca`

Abstract. Using network coding in a wireless network can potentially improve the network throughput. On the other hand, it increases the complexity of resource allocations as the quality of one transmission is often affected by the link conditions of the transmitter node to multiple destination nodes. In this paper we consider a wireless network using network coding, where one relay node is available for each bidirectional link. Both digital and analog network coding strategies, referred to as DNC and ANC, respectively, are considered. All transmissions share the same frequency channel, and therefore simultaneous transmissions cause interference to each other. For ANC, transmission power of the nodes at different time slots can be co-related. We study link transmission scheduling, and in specific, the transmission time and power for each node so that the overall network throughput is maximized. Our results indicate that the two types of network coding strategies can outperform each other depending on the network conditions.

Keywords: Digital network coding, analog network coding, scheduling, power distribution.

1 Introduction

The notion of network coding (NC) was first introduced in [1] in the context of wired multicast networks. It is a technique for intermediate nodes to combine the received packets from multiple links and forward to subsequent nodes. Using NC can reduce the amount of transmitted data and potentially improve the network throughput. Significant efforts have been put to design different strategies for applying network coding in wireless networks [2]-[5]. Based on whether messages are decoded at the relay nodes, wireless network coding (WNC) can be implemented in two different ways. In the conventional WNC, multiple signals received at a relay node should be first decoded and then mixed together before being forwarded to subsequent nodes. Alternatively, the relay node can simply amplify and forward the analog signals, and this type of physical layer network coding (PNC) is also referred to as analog network coding (ANC) [6][7].

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In contrast, the conventional WNC that requires the relay node to decode the received signals is referred to as digital network coding (DNC) in this paper.

By exploiting the broadcast nature of the wireless channel, WNC may not only improve the network throughput [8], but also combat transmission errors, save transmission power, and improve routing efficiency. While WNC has a great potential to improve the performance of wireless networks, applying WNC increases the complexity in managing the radio resources. In particular, transmission power allocations become different in a network using WNC from that in a traditional wireless network, as one message often needs to reach multiple destinations. In order for a mixed message to reach all the destinations, the transmission power of the relay station has to consider the worst link condition. This can negatively affect the network performance. The relationship between transmission power and symbol error rate is studied in [9]. The advantage of using network coding, such as throughput improvement, can be reduced significantly in a fading channel [10]. The problem of power management using WNC becomes particularly challenging in interference-limited networks, such as UWB and CDMA-based networks. However, little work has been done along this direction.

Network coding is closely related to packet transmission scheduling, which is important in order to coordinate the resource allocations and optimize the network performance. Different opportunistic scheduling schemes have been proposed for wireless networks using NC by taking advantage of the random channel conditions so that packets that are mixed in the same transmission have similar channel conditions to their respective destinations. In [11] the scheduling problem is studied jointly with channel and power allocations for broadcast traffic in an OFDMA-based WiMAX network using random network coding. In [12] joint scheduling and automatic modulation and coding is studied for a network having one transmitter and multiple destination nodes. Joint coding, routing and scheduling is studied in [13] for a wireless mesh network with orthogonal channels. Although determining the transmission time and power is one of the main aspects for scheduling, not very much work has been done on this topic for interference-limited networks with network coding.

In this paper we study time and power allocations for an interference-limited network with WNC. Both DNC and ANC are considered. All the transmissions in the network share the same frequency channel, and simultaneous transmissions can interfere with each other. Furthermore, for a network using ANC, transmissions at different time slots are co-related. An optimization problem is first formulated for the network using DNC and ANC, respectively. The objective is to maximize the system throughput, subject the transmission power and time constraints. Heuristic scheduling schemes are then proposed. The remainder of the paper is organized as follows. In Section 2 we describe the system that this work is based on. The scheduling problems for networks with DNC and ANC, respectively, are formulated and heuristic schemes are proposed in Sections 3 and 4. Numerical results are demonstrated in Section 5 to show the performance of the scheduling schemes. Section 6 concludes the paper.

2 System Description

We consider a network with M bidirectional links, indexed by $m = 1, 2, \ldots, M$. For each link, there is a relay node (R-node) that forwards data packets between the two end nodes, referred to as source node (or S-node) and destination node (or D-node), respectively. For preliminary study, we consider that the R-node for each pair of the S-node and D-node is fixed. Both the S-node and the D-node always have packets to transmit.

We use $x = s, d, r$ to represent the type of the transmitting node with s for the S-node, d for the D-node, and r for the R-node, and use $t = 1, 2, \ldots, T$ to represent the time slots. Define a set of binary variables $A_{x_m,t}$'s with $A_{x_m,t} = 1$ denoting that node x of link m transmits at time slot t and $A_{x_m,t} = 0$ denoting that the node does not transmit at the time slot. Let $P_{x_m,t}$ represent the transmission power of node x of link m, and $G_{x_m,y_n,t}$ represent the link gain between node x of link m and node y of link n at time slot t.

For DNC, we consider a simple XOR operation for the R-node to combine the packets from the S-node and D-node. It takes three time slots for the S-node and D-node of a given link to exchange one pair of packets, i.e., one packet is transmitted in each direction. In the first time slot, the S-node transmits packet k_s to the R-node; and in the second time slot, the D-node transmits packet k_d to the R-node. The two packets are decoded at the R-node, which transmits $k_d \oplus k_s$ to both the S-node and D-node in the third time slot. Upon receiving the XORed packet, the S-node recovers k_d and the D-node recovers k_s.

When using ANC, it takes two time slots for the S-node and D-node of a given link to exchange one packet in each direction. In the first time slot, the S-node transmits packet k_s and the D-node transmits packet k_d simultaneously to the R-node. The R-node does not try to decode the packets, but amplifies the mixed analog signals and forwards to both the S-node and D-node in the second time slot. Upon receiving the forwarded signal, the S-node recovers k_d and the D-node recovers k_s.

We consider that a single frequency channel is shared by all the links, and therefore simultaneous transmissions interfere with each other. The number of transmitting nodes at any given time should be limited, and the transmission power of each node should be carefully distributed, so that data can be correctly recovered at the destinations. When decoding is required, the signal-to-interference-plus-noise ratio (SINR) of the desired signal at the receiver should be above a certain threshold, γ. For ANC, transmission power at the current time slot is dependent on the transmission power and link conditions at previous time slots.

3 Scheduling for a Network Using DNC

Consider a typical bidirectional link m. For DNC, the SINR at a given time slot only depends on the transmission power and link conditions at the current time slot. If the S-node transmits at time slot t, its transmission power should satisfy the following condition in order for the R-node to correctly decode its transmitted packet k_s:

$$\frac{P_{s_m,t}G_{s_m,r_m,t}}{I_{r_m,t}+P_n} \geq \gamma, \tag{1}$$

where P_n is the power of the background noise, and $I_{r_m,t}$ is the co-channel interference that the R-node experiences from all other transmissions at time t and is given by

$$I_{r_m,t} = \textstyle\sum_{\text{all } z_n \neq s_m} P_{z_n,t}G_{z_n,r_m,t}. \tag{2}$$

Similarly, if the D-node of link m transmits at time slot t, its transmission power should satisfy the following condition in order for the R-node to correctly decode its transmitted packet k_d.

$$\frac{P_{d_m,t}G_{d_m,r_m,t}}{I_{r_m,t}+P_n} \geq \gamma, \tag{3}$$

where

$$I_{r_m,t} = \textstyle\sum_{\text{all } z_n \neq d_m} P_{z_n,t}G_{z_n,r_m,t}. \tag{4}$$

For the R-node, its transmission power should satisfy the following two conditions in order for the S-node to correctly decode packet k_d and for the D-node to correctly decode packet k_s:

$$\frac{P_{r_m,t}G_{r_m,s_m,t}}{I_{s_m,t}+P_n} \geq \gamma, \tag{5}$$

$$\frac{P_{r_m,t}G_{r_m,d_m,t}}{I_{d_m,t}+P_n} \geq \gamma, \tag{6}$$

where $I_{s_m,t}$ and $I_{d_m,t}$, respectively, represent the co-channel interference that the S-node and D-node experience from all other transmissions at time t, and their expressions can be obtained similarly to (4) as

$$I_{s_m,t} = \textstyle\sum_{\text{all } z_n \neq r_m} P_{z_n,t}G_{z_n,s_m,t}, \tag{7}$$

$$I_{d_m,t} = \textstyle\sum_{\text{all } z_n \neq r_m} P_{z_n,t}G_{z_n,d_m,t}. \tag{8}$$

Combining (1), (3), (5) and (6), and using x_m to represent the transmitter node and y_n the receiver node, we have

$$\frac{P_{x_m,t}G_{x_m,y_m,t}}{I_{y_m,t}+P_n} \geq \gamma, \tag{9}$$

where

$$I_{y_m,t} = \textstyle\sum_{\text{all } z_n \neq x_m} P_{z_n,t}G_{z_n,y_m,t}. \tag{10}$$

In (9), when $x_m = s_m$ or $x_m = d_m$, $y_m = r_m$; and when $x_m = r_m$, node $y_m = s_m$ or $y_m = d_m$.

In addition to transmission power, transmission time of the nodes for the same link should also be constrained. For each link, at most one node can transmit at any given time. That is,

$$A_{s_m,t} + A_{r_m,t} + A_{d_m,t} \leq 1. \tag{11}$$

The R-node should transmit later than the S-node and D-node of the same link. That is,

$$\textstyle\sum_{\tau=1}^{t} A_{r_m,\tau} \le \sum_{\tau=1}^{t} A_{x_m,\tau} \tag{12}$$

should be true for all $t > 0$, where $x = s$ or d.

Let C_m represent the total number of packets successfully transmitted in each direction of the bidirectional link m until time T. We have

$$C_m = \textstyle\sum_{t=1}^{T} A_{r_m,t}. \tag{13}$$

Define C_m/T as the average throughput in number of packets per time slot in one direction of link m, and our objective is to maximize $\sum_m C_m/T$.

By putting together all the above constraints, we can formulate the optimization problem as follows:

$$\begin{align}
&\text{P1: } \max \textstyle\sum_m C_m/T \tag{14}\\
&\text{s.t. } \frac{A_{x_m,t}P_{x_m,t}G_{x_m,y_m,t}}{I_{y_m,t}+P_n} \ge \gamma A_{x_m,t}, \text{for all } m,\ x, \text{ and } t \tag{15}\\
&\quad I_{y_m,t} = \textstyle\sum_{\text{all } z_n \ne x_m} A_{z_n,t}P_{z_n,t}G_{z_n,y_m,t} \tag{16}\\
&\quad C_m = \textstyle\sum_{t=1}^{T} A_{r_m,t}, \text{ for all } m \tag{17}\\
&\quad A_{s_m,t} + A_{r_m,t} + A_{d_m,t} \le 1, \text{ for all } m \text{ and } t \tag{18}\\
&\quad \textstyle\sum_{\tau=1}^{t} A_{r_m,\tau} \le \sum_{\tau=1}^{t} A_{s_m,\tau}, \text{ for all } m \text{ and } t \tag{19}\\
&\quad \textstyle\sum_{\tau=1}^{t} A_{r_m,\tau} \le \sum_{\tau=1}^{t} A_{d_m,\tau}, \text{ for all } m \text{ and } t \tag{20}\\
&\quad A_{x_m,t} \in \{0,1\}, \text{ for all } m,\ x, \text{ and } t \tag{21}\\
&\quad P_{x_m,t} \le P_{\max}, \text{ for all } m\ ,\ x, \text{ and } t. \tag{22}
\end{align}$$

There are two sets of unknown variables in the above problem, $A_{x_m,t}$'s and $P_{x_m,t}$'s. The problem is a non-linear mixed-integer problem. Finding solutions to this optimization problem is not efficient. Moreover, implementing the solutions is impossible since the scheduling decisions at all the time slots are jointly optimized and it requires future information in order to make the current scheduling. Below we design a heuristic scheme to schedule the node transmissions.

We define S_m, D_m and R_m, respectively, as the number of time slots that the S-, D-, and R-nodes of link m have successfully transmitted till the current time slot. The scheduling process is given in Algorithm 1, where $\mathcal{A}_t$ is a set of the nodes that are allowed to transmit at time slot t. At the beginning of each time slot, the values of S_m, D_m and R_m are used to decide which nodes of each link can possibly be scheduled to transmit. Exchanging one pair of the packets in each bidirectional link requires one transmission from each of the S-, D-, and R-nodes. According to the time constraints, all the three nodes of the same link transmit at different time slots. For link m, if both $S_m = D_m$ and $S_m > R_m$, then the R-node of the link should be scheduled to transmit next. This is shown in Lines 6-7 of Algorithm 1. If $S_m = D_m = R_m$, either the S- or the D-node can be scheduled to transmit next, and we arbitrarily choose the S-node to transmit. This is shown in Lines 9-10. If $S_m > D_m$, then the D-node should be scheduled to transmit next. This is shown in Lines 12-13. Note that Lines 6, 9 and 12 list all possible relations among S_m, D_m and R_m. The R-node always transmits later than the S- and D-nodes of the same link, and therefore $R_m \leq S_m$ and $R_m \leq D_m$. Although there is no order between transmissions of the S- and D-nodes for the same link, we choose to have the S-node transmit earlier than the D-node in the scheduling, and therefore $D_m \leq S_m$.

Whether the nodes with $A_{x_m,t} = 1$ can transmit depends on the required SINR, which is achieved by transmission power distribution. Given $A_{x_m,t}$'s at time slot t, the minimum required transmission power of all the nodes can be found from the following optimization problem:

$$\text{P2:} \qquad \min \max\nolimits_{x_m \in \mathcal{A}_t} P_{x_m,t} \tag{23}$$

$$\text{s.t.} \ \frac{P_{x_m,t} G_{x_m,y_m,t}}{\sum_{z_n \in \mathcal{A}_t,\, z_n \neq x_m} P_{z_n,t} G_{z_n,y_m} + P_n} \geq \gamma, \ \text{all } x_m \in \mathcal{A}_t. \tag{24}$$

If the power level for any node exceeds $P_{\max}$, the node is not allowed to transmit at the current time, i.e., the corresponding $A_{x_m,t}$ is reset to zero. This is shown in Lines 19-21. Alternatively, we can remove only the node with the highest power level from $\mathcal{A}_t$, and solve problem P2 again. The process is repeated until all the remaining nodes can have their transmission power below $P_{\max}$. However, this is not considered in this paper due to the high complexity.

Algorithm 1. scheduling when using DNC

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 1: Initialize $S_m = D_m = R_m = 0$ for all $m$.
 2: Initialize $C_m = 0$ for all $m$.
 3: Initialize $A_{x_m,t} = 0$ for all $m$, $x$ and $t$ and $\mathcal{A}_t = \emptyset$.
 4: for $t = 1 : T$ do
 5:   for $m = 1 : M$ do
 6:     if $S_m = D_m$ and $S_m > R_m$ then
 7:       Set $A_{r_m,t} = 1$ and $\mathcal{A}_t = \mathcal{A}_t \cup \{r_m\}$.
 8:     else
 9:       if $S_m = D_m$ and $S_m = R_m$ then
10:         Set $A_{s_m,t} = 1$ and $\mathcal{A}_t = \mathcal{A}_t \cup \{s_m\}$.
11:       else
12:         if $S_m > D_m$ then
13:           $A_{d_m,t} = 1$ and $\mathcal{A}_t = \mathcal{A}_t \cup \{d_m\}$
14:         end if
15:       end if
16:     end if
17:   end for
18:   Find $P_{x_m,t}$'s by solving problem P2.
19:   while There exist $m$ and $x$ so that $P_{x_m,t} > P_{\max}$ do
20:     Reset $A_{x_m,t} = 0$ and $\mathcal{A}_t = \mathcal{A}_t \setminus \{x_m\}$
21:   end while
22:   Find $P_{x_m,t}$'s by solving problem P2.
23:   Update $S_m$, $D_m$ and $R_m$.
24:   for $m = 1 : M$ do
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        $C_m = C_m + A_{r_m,t}$.
    end for
end for
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4 Scheduling for a Network Using ANC

As in Section 3, we consider a bidirectional link m. When using ANC, transmissions at different time slots are dependent on each other. This is because the R-node amplifies signals and interference received from previous time slots. At time slot τ, packet k_s transmitted by the S-node reaches the R-node with the received power $P_{s_m,\tau}G_{s_m,r_m,\tau}$, and packet k_d transmitted by the D-node reaches the R-node with the received power $P_{d_m,\tau}G_{d_m,r_m,\tau}$. At the same time, the interference level that other simultaneous transmissions cause at the R-node is given by

$$I_{r_m,\tau} = \sum\nolimits_{\text{all } z_n \neq s_m, z_n \neq d_m} A_{z_n,\tau}P_{z_n,\tau}G_{z_n,r_m,\tau} \tag{25}$$

At time slot t, $t > \tau$, the mixed signals, which includes the desired signals, the interference received from time slot τ, as well as noise, are amplified with amplification factor $\beta_{m,t}$ by the R-node. The amplified desired signal includes $\beta_{m,t}P_{s_m,\tau}G_{s_m,r_m,\tau}$ for packet k_s, $\beta_{m,t}P_{d_m,\tau}G_{d_m,r_m,\tau}$ for packet k_d, and the amplified interference and noise component is $\beta_{m,t}(I_{r_m,\tau} + P_n)$.

The above amplified mixed signal reaches both the S-node and the D-node of link m. At the S-node, the received power becomes $\beta_{m,t}P_{s_m,\tau}G_{s_m,r_m,\tau}G_{r_m,s_m,t}$ for packet k_s, $\beta_{m,t}P_{d_m,\tau}G_{d_m,r_m,\tau}G_{r_m,s_m,t}$ for packet k_d, $\beta_{m,t}(I_{r_m,\tau}+P_n)G_{r_m,s_m,t}$ for the interference and noise. Meanwhile, other nodes that transmit at time t cause interference at the S-node, and the interference level is given by

$$I_{s_m,t} = \sum\nolimits_{\text{all } z_n \neq r_m} A_{z_n,t}P_{z_n,t}G_{z_n,s_m,t}. \tag{26}$$

Assume $\beta_{m,t}$ is known to all the nodes, $G_{s_m,r_m,t} = G_{r_m,s_m,t}$ and $G_{s_m,r_m,\tau} = G_{r_m,s_m,\tau}$ are known to the S-node, the S-node can remove the component of k_s from the received signal. It should recover packet k_d from the remaining signal with the desired signal power $\beta_{m,t}P_{d_m,\tau}G_{d_m,r_m,\tau}G_{r_m,s_m,t}$ and all other components as interference. In order for the S-node to correctly decode packet k_d, the following condition should be satisfied:

$$\frac{A_{r_m,t}\beta_{m,t}P_{d_m,\tau}G_{d_m,r_m,\tau}G_{r_m,s_m,t}}{\beta_{m,t}G_{r_m,s_m,t}(P_n + I_{r_m,\tau}) + I_{s_m,t} + P_n} \geq \gamma A_{r_m,t} \tag{27}$$

where P_n in the last term in the denominator is the local background noise power at the S-node.

Similarly, the SINR condition for the D-node to decode packet k_s is given by

$$\frac{A_{r_m,t}\beta_{m,t}P_{s_m,\tau}G_{s_m,r_m,\tau}G_{r_m,d_m,t}}{\beta_{m,t}G_{r_m,d_m,t}(I_{r_m,\tau} + P_n) + I_{d_m,t} + P_n} \geq \gamma A_{r_m,t}, \tag{28}$$

where $I_{d_m,t}$ is given by

$$I_{d_m,t} = \sum\nolimits_{\text{all } z_n \neq r_m} A_{z_n,t}P_{z_n,t}G_{z_n,d_m,t}. \tag{29}$$

Define $A_{x_m,t}$ in the same way as in the previous section, the optimization problem P3 can be formulated, where (36) gives the total transmission power of the R-node at time t, and (37)-(41) together specify the time constraints. Among the time constraints, (37) specifies the most recent time slot when the S-node and D-node transmit, (38) specifies that the S-node and D-node of the same link should transmit at the same time, (39) specifies that the S-node and R-node cannot transmit at the same time, and (40) indicates that the R-node transmits after the S-node for the same link. In problem P3, the unknown variables are $A_{x_m,t}$'s, $P_{x_m,t}$'s, and $\beta_{m,t}$'s. The problem is non-linear and non-convex, and cannot be solved efficiently. Below we design a heuristic scheduling scheme.

$$
\begin{align}
\text{P3: } \max \sum_m C_m/T \tag{30}\\
\text{s.t. } \frac{A_{r_m,t}\beta_{m,t}P_{d_m,\tau}G_{d_m,r_m,\tau}G_{r_m,s_m,t}}{I_{s_m,t}+P_n+\beta_{m,t}G_{r_m,s_m,t}(P_n+I_{r_m,\tau})} \ge \gamma A_{r_m,t} \tag{31}\\
\frac{A_{r_m,t}\beta_{m,t}P_{s_m,\tau}G_{s_m,r_m,\tau}G_{r_m,d_m,t}}{I_{d_m,t}+P_n+\beta_{m,t}G_{r_m,d_m,t}(P_n+I_{r_m,\tau})} \ge \gamma A_{r_m,t} \tag{32}\\
I_{r_m,\tau} = \textstyle\sum_{\text{all } z_n \neq s_m, z_n \neq d_m} A_{z_n,\tau}P_{z_n,\tau}G_{z_n,r_m,\tau} \tag{33}\\
I_{s_m,t} = \textstyle\sum_{\text{all } z_n \neq r_m} A_{z_n,t}P_{z_n,t}G_{z_n,s_m,t} \tag{34}\\
I_{d_m,t} = \textstyle\sum_{\text{all } z_n \neq r_m} A_{z_n,t}P_{z_n,t}G_{z_n,d_m,t} \tag{35}\\
P_{r_m,t} = \beta_{m,t}(P_{s_m,\tau}G_{s_m,r_m,\tau} + P_{d_m,\tau}G_{d_m,r_m,\tau} + P_n + I_{r_m,\tau}) \tag{36}\\
\tau = \max\{t_1 < t : A_{d_m,t_1} = 1\} \tag{37}\\
A_{s_m,t} = A_{d_m,t}, \text{ for all } m \text{ and } t \tag{38}\\
A_{s_m,t} + A_{r_m,t} \le 1, \text{ for all } m \text{ and } t \tag{39}\\
\textstyle\sum_{\tau=1}^{t} A_{r_m,\tau} \le \sum_{\tau=1}^{t} A_{s_m,\tau}, \text{ for all } m \tag{40}\\
A_{x_m,t} \in \{0,1\}, \text{ for all } m, x, \text{ and } t \tag{41}\\
P_{x_m,t} \le P_{\max}, \text{ for all } m \text{ , } x, \text{ and } t \tag{42}\\
C_m = \textstyle\sum_{t=1}^{T} A_{r_m,t}, \text{ for all } m \tag{43}
\end{align}
$$

Define S_m and R_m as the number of time slots that the S-node and R-node of link m have transmitted till the current time slot. Algorithm 2 gives the scheduling scheme for a network using ANC. The time constraints for the scheduling is that for each link, the S-node and D-node should transmit at the same time slot, and they should transmit at an earlier time slot than the R-node. Therefore, $S_m \ge R_m$. If $S_m = R_m$, the S-node and D-node should transmit next; if $S_m > R_m$, the R-node should transmit next. This is shown in Lines 6-12.

The scheme then finds transmission power for all the nodes in $\mathcal{A}_t$. The transmission power of any S-nodes and D-nodes at the current time slot is dependent on the transmission power of the R-node at a future time slot. Without any future information available, setting the transmission power for the S- and D-nodes involves two contradictory effects. For a given link, allowing the S- and D-nodes to transmit at high power can potentially reduce the required transmission power of the R-node at a future time slot. On the other hand, this increases the interference to other nodes at the current time slot. Given the transmission power of the S-nodes and D-nodes, the scheme finds the minimum required transmission power for all the R-nodes in $\mathcal{A}_t$ by solving problem P4 in (44). In the solution, if the required power for any of the R-nodes exceeds $P_{\max}$, either some nodes (can be S-, D-, or R-nodes) are removed from the current time slot, or the transmission power of some S-nodes and D-nodes is reduced. Therefore, different criteria can be used depending the objective. As a preliminary study, we fix the transmission power of the S- and D-nodes to $P_{\max}$ and reset $A_{r_m,t} = 0$ for all R-nodes with $P_{r_m,t} > P_{\max}$. Results of using other criteria will be reported in future work.

Algorithm 2. scheduling when using ANC

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Initialize $S_m = R_m = 0$ for all $m$.
Initialize $C_m = 0$ for all $m$.
$A_{x_m,t} = 0$ for all $m$, $x$ and $t$ and $\mathcal{A}_t = \emptyset$.
for $t = 1 : T$ do
    for $m = 1 : M$ do
        if $S_m = R_m$ then
            $A_{s_m,t} = 1$, $A_{d_m,t} = 1$, and $\mathcal{A}_t = \mathcal{A}_t \cup \{s_m, d_m\}$
        else
            if $S_m > R_m$ then
                $A_{r_m,t} = 1$ and $\mathcal{A}_t = \mathcal{A}_t \cup \{r_m\}$
            end if
        end if
    end for
    if There exists $r$ so that $A_{r_m,t} = 1$ then
        Find $P_{r_m,t}$'s by solving problem P4.
        while There exists $m$ so that $P_{r_m,t} > P_{\max}$ do
            $A_{r_m,t} = 0$ and $\mathcal{A}_t = \mathcal{A}_t \setminus \{r_m\}$
        end while
    end if
    if There exists $r$ so that $A_{r_m,t} = 1$ then
        Find $P_{r_m,t}$'s by solving problem P4.
    end if
    Update $S_m$ and $R_m$ for all $m$.
    for $m = 1 : M$ do
        $C_m = C_m + A_{r_m,t}$.
    end for
end for
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$$\text{P4: } \min \max_{r_m \in \mathcal{A}_t} P_{r_m,t} \tag{44}$$

$$\text{s.t. } \frac{\beta_{m,t} P_{d_m,\tau} G_{d_m,r_m,\tau} G_{r_m,s_m,t}}{I_{s_m,t} + P_n + \beta_{m,t} G_{r_m,s_m,t}(P_n + I_{r_m,\tau})} \geq \gamma \tag{45}$$

$$\frac{\beta_{m,t} P_{s_m,\tau} G_{s_m,r_m,\tau} G_{r_m,d_m,t}}{I_{d_m,t} + P_n + \beta_{m,t} G_{r_m,d_m,t}(P_n + I_{r_m,\tau})} \geq \gamma \tag{46}$$

$$I_{s_m,t} = \sum\nolimits_{\text{all } z_n \in \mathcal{A}_t, z_n \neq r_m} P_{z_n,t} G_{z_n,s_m,t} \tag{47}$$

$$I_{d_m,t} = \sum\nolimits_{\text{all } z_n \in \mathcal{A}_t, z_n \neq r_m} P_{z_n,t} G_{z_n,d_m,t} \tag{48}$$

$$P_{r_m,t} = \beta_{m,t}(P_{s_m,\tau} G_{s_m,r_m,t} + P_{d_m,\tau} G_{d_m,r_m,t} + P_n + I_{r_m,\tau}) \tag{49}$$

$$\tau = \max\{t_1 < t : A_{d_m,t_1} = 1\} \tag{50}$$

5 Numerical Results

We consider a network with M bidirectional links, where $3M$ nodes are located in a $3 \times M$ grid as shown in Fig. 1. For each link, the R-node is located at the middle point between the S-node and the D-node. The distance between the R-node and the S-node of the same link is denoted as w_1, and the distance between the S-nodes of two neighboring links is denoted as w_2. The link gain between any two nodes x_m and y_n includes both distance-based path loss and log-normally distributed shadowing and is given by $G_{x_m,y_n,t} = d_{x_m,y_n}^{-\alpha} \times 10^{-X}$, where X is Gaussian distributed with zero mean and standard deviation of 1. The default values are $M = 3$, $w_1 = 100$ m, $w_2 = 500$ m, $P_n = 10^{(-6)}$W, $\alpha = 2$, and $P_{\max} = 1$W. The throughput results for $T = 350$ time slots are plotted in Figs. 2 and 3 for the network using DNC, and in Fig. 4 for the network using ANC. Analytically, it takes three time slots to exchange one pair of packets between the S-node and the D-node of a given bidirectional link when using DNC. Therefore, the maximum throughput in one direction for each link is 1/3 packets per time slot. With M bidirectional links, the maximum unidirectional throughput is $M/3$ packets per time slot, or 1 packet per time slot when $M = 3$. For ANC, it takes two time slots to exchange one pair of packets in each bidirectional link. Therefore, the maximum unidirectional throughput is 1/2 packets per time slot, or 1.5 packets per time slot when $M = 3$.

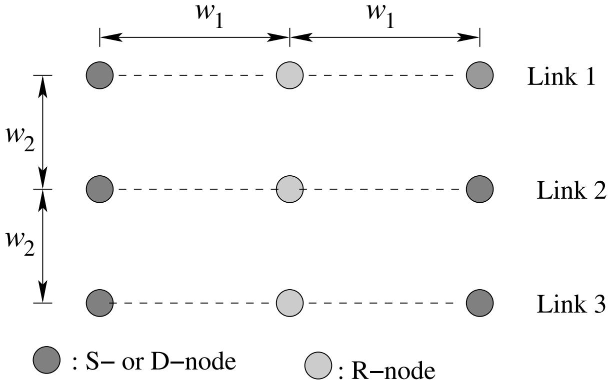

Fig. 1. Grid topology with $M = 3$

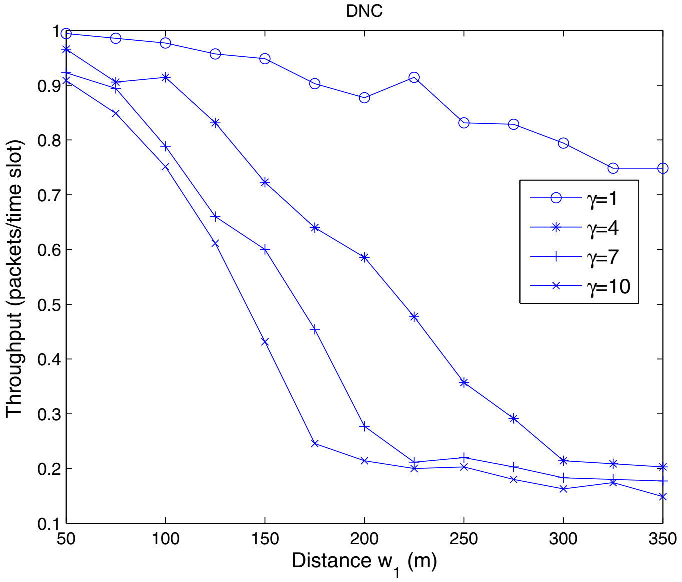

Fig. 2. Throughput versus w_1 for DNC

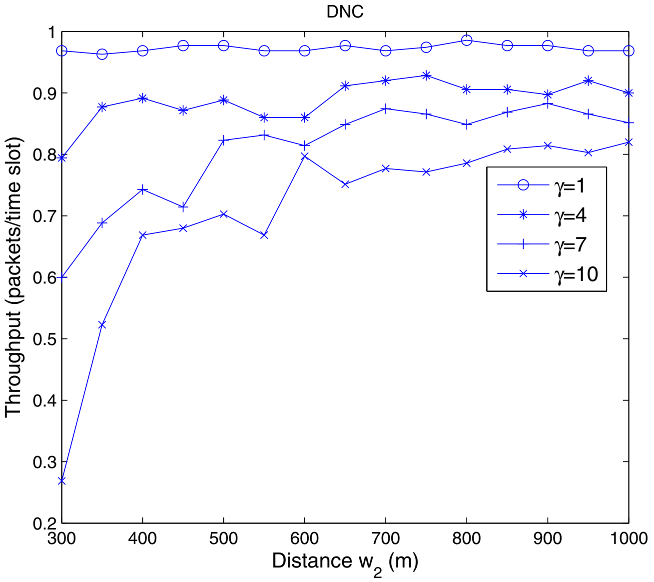

Fig. 3. Throughput versus w_2 for DNC

Fig. 2 shows that the transmission throughput decreases with w_1. This is due to the increased path loss, which increases the required transmission power. When w_1 is relatively small, the throughput is close to the upper limit, which is one packet per time slot. This indicates that the proposed scheduling scheme is efficient. On the other hand, as w_1 increases, the transmission power of the nodes increases, which increases the mutual interference and makes it more difficult to satisfy the SINR requirements. Therefore, throughput decreases as w_1 increases, and the decrease is much faster for larger γ values.

As w_2 increases, the distance between nodes of different links increases, which reduces the mutual interference between the links. Therefore, throughput increases with w_2 as shown in Fig. 3. When w_2 is relatively small, increasing w_2 can significantly increase the overall throughput. After w_2 is larger than a certain value, the effect of mutual interference on throughput performance becomes minor and further increasing w_2 does not very much increase the throughput, which is not limited by w_1. Fig. 3 also shows that the throughput is very close to the upper bound when the required SINR is very low.

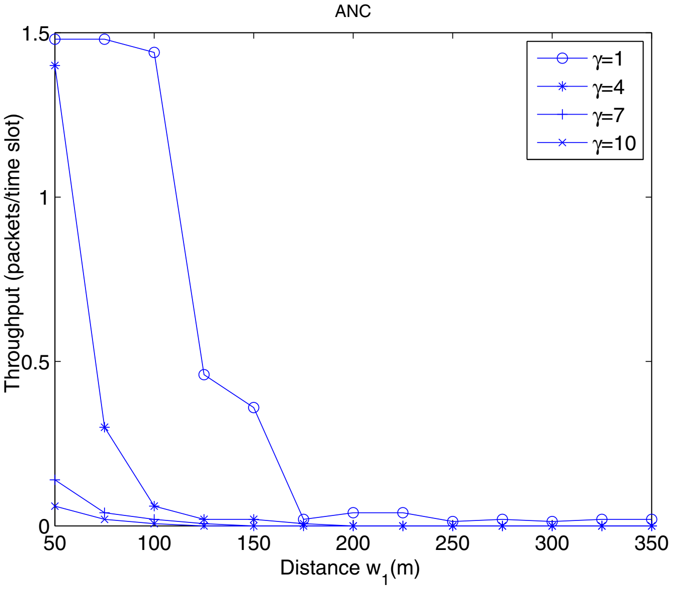

Fig. 4. Throughput versus w_1 for ANC

For ANC, the maximum throughput is achieved when both w_1 and γ are small as shown in Fig. 4. When w_1 or γ increases, the throughput can decrease very abruptly. Comparing Figs. 4 and 2 we can find that when both the required SINR and w_1 are small, using ANC can achieve much higher throughput than using DNC. On the other hand, when the required SINR is relatively high or w_1 is relatively large, using DNC is a better choice. This is basically because that when using ANC, interference and noise is also amplified by the R-node and this deteriorates the transmission quality and makes it much more difficult to satisfy relatively high SINR requirement in poor channel conditions.

6 Conclusions

We have studied transmission time and power scheduling for a network with bidirectional links and network coding. Heuristic schemes have been proposed for the network using digital network coding and analog network coding. In general, the scheduling scheme for a system using ANC is more complicated than that using DNC because transmissions at different time slots can be dependent on each other. When the SINR requirement is relatively low and channel conditions are good, using ANC can achieve higher throughput than using DNC. In other conditions, using DNC can achieve higher throughput than using ANC.

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