A Strategy-Proof and Non-monetary Admission Control Mechanism for Wireless Access Networks

Xiaohan Kang, Juan José Jaramillo, and Lei Ying

Department of Electrical and Computer Engineering
Iowa State University, Ames IA 50011
{xkang,jjjarami,leiying}@iastate.edu

Abstract. We study admission control mechanisms for wireless access networks where (i) each user has a minimum service requirement, (ii) the capacity of the access network is limited, and (iii) the access point is not allowed to use monetary mechanisms to guarantee that users do not lie when disclosing their minimum service requirements. To guarantee truthfulness, we use auction theory to design a mechanism where users compete to be admitted into the network. We propose admission control mechanisms under which the access point intelligently allocates resources based on the announced minimum service requirements to ensure that users have no incentive to lie and the capacity constraint is fulfilled. We also prove the properties that any feasible mechanism should have.

Keywords: Auctions, truth-telling, admission control, resource allocation.

1 Introduction

Resource allocation has been one of the most important issues in the design of communication networks. Given a wireless access network, in which the users have various QoS requirement and the access point has limited resource, admission control mechanisms are vital to achieve stability, fairness and efficiency of the system. In this paper, we consider a wireless access network with multiple users and a single access point. We assume that the access point network is a public network, and is not allowed to charge users for accessing the network. We study the case when the QoS requirements are private, and users are allowed to selfishly disclose any value that would give them better service. Then the problem is to design an admission control mechanism such that the true QoS of the users can be collected without the use of any pricing scheme, and as many users are admitted as possible. A natural choice is to set up a game with the users, such that the selfish users are incentivized to tell the truth. Originated in economics theory, auction mechanisms have been found very useful in this kind of situation, since they are designed to entice selfish bidders tell the truth when allocating limited resources.

X. Zhang and D. Qiao (Eds.): QShine 2010, LNICST 74, pp. 172–187, 2011.

Various auction mechanisms have been well-studied. Myerson [1] has obtained the optimal auction mechanism in closed form mathematical expression. His result, however, only works for limited utility functions without constraint over the resource. The well-known VCG mechanism has been proved to guarantee truth-telling while achieving the social optimum [2]. The challenge of designing an effective auction mechanism in the proposed setting is the access point is not allowed to charge the users, so the auction mechanism has to be non-monetary. For example, using a simple example, it will be shown that the VCG mechanism cannot be easily adopted in our scenario due to the non-monetary requirement. Credit schemes that were developed to incentivize cooperation in wireless networks could also be adapted to guarantee users do not lie about their true requirements, but they would also require secure mechanisms to avoid tampering with the virtual money [7], [8], [9], [10], [11], [12], [13]. Recently, Hou and Kumar have proposed a bidding game between users and access point that maximizes the total utility, but in the iterative process the users are forced to bid specific values instead of bidding selfishly [4]. In this paper, we seek to design auction mechanisms that are truthful and do not use any money-based scheme for general utility functions.

Our contributions are therefore fourfold.

i. We model the admission control problem as an auction mechanism design with resource constraint, where the payment is related to the allocated service rate.
ii. We use a simple example to demonstrate that a direct adoption of VCG auction mechanism does not work in the non-monetary setting.
iii. Third, we come to two theorems that help us understand the essence of strategy-proof mechanisms. The first theorem (Theorem 2) shows the impossibility of a reasonable truth-telling mechanism to be based on probabilistic decisions, and the second theorem (Theorem 3) shows that the truthfulness of a mechanism is equivalent to the existence of a lowest winning bid regardless of one's own bid.
iv. We propose our mechanism and show that it has the desired properties and admits at least half of the optimal number of users with high probability in an asymptotic sense.

This paper is organized as follows. Section 2 gives the model of this problem and the set of assumptions. Section 3 illustrates the failure of VCG in our setting. Section 4 analyzes the problem and characterizes the properties for feasible auction mechanisms. Section 5 gives our proposed mechanism and shows feasibility and the performance bound. Section 6 concludes this paper.

2 Model

Consider a multiple access network with n users and a single access point (AP), where only one user can transmit at any given time. Let $\mathcal{N} = \{1, 2, \cdots, n\}$ be the set of all users. The service rate of the AP is assumed to be 1. Each user is assumed to have an arrival rate $\lambda_i = 1$ and a quality of service (QoS) q_i, which indicates the fraction of packages that has to be delivered successfully and is only known to user i. We refer to $t_i = 1 - q_i$ as the maximum allowed drop rate of user i and suppose that the range of t_i is $T_i = [a_i, b_i] \subset [0, 1]$. We let $T = T_1 \times T_2 \times \cdots \times T_n$ and $T_{-i} = T_1 \times T_2 \times \cdots \times T_{i-1} \times T_{i+1} \times \cdots \times T_n$ where $\times$ denotes the Cartesian product.

In order to decide which users to serve and the QoS the AP should provide, the AP sets up an auction, in which each user, or bidder, bids a requested drop rate $s_i \in T_i$, which can be different from the true drop rate t_i, and the AP decides the set of users that get admitted and assigns drop rate x_i to user i. By Myerson's revelation principle [1], we only consider direct revelation mechanisms, i.e., the mechanisms in which users submit bids in the form of drop rate values rather than in the form of any other strategies. Then the mechanism can be described by the *outcome functions* (p, x), where p and x are the set of functions

$$p(s) = (p_\phi(s), \phi \subset \mathcal{N})$$
$$x(s) = (x_{i,\phi}(s), \phi \subset \mathcal{N})$$

with $p_\phi \colon T \to \mathbb{R}$ being the probability that only the subset of users ϕ of $\mathcal{N}$ get admitted and $x_{i,\phi} \colon T \to \mathbb{R}$ being the actually assigned drop rate for user i if the subset ϕ is admitted. For convenience we set $x_{i,\phi}(s)$ to be the assigned drop rate by mechanism (p, x) if $i \in \phi$ and $p_\phi(s) > 0$, and 1 otherwise, i.e., all packets of user i are dropped, which is equivalent to not admitting user i.

The utility function for user i, given the true drop rate t_i and the assigned drop rate x_i, is $u_i(t_i, x_i)$. It is assumed that u_i is non-increasing with x_i when $x_i \le t_i$, and equals 0 when $x_i > t_i$.

Given mechanism (p, x), the utility for user i with true drop rate t_i, bidded drop rate s_i and others bidding s_{-i} is

$$U_i(p, x, t_i, s_i, s_{-i}) = \sum_{\phi : i \in \phi} u_i(t_i, x_{i,\phi}(s)) p_\phi(s) \ . \tag{1}$$

Throughout the paper we evaluate the utility of one user by examining each of the possible bid vectors of other users.

We now present some definitions we will use throughout the paper.

Definition 1 (Incentive compatibility - IC). *A mechanism* (p, x) *is* incentive compatible *if for any utility function, any* $i \in \mathcal{N}$*, any* $t \in T$ *and any* $s_i \in T_i$*, we have*

$$U_i(p, x, t_i, t_i, t_{-i}) \ge U_i(p, x, t_i, s_i, t_{-i}) \tag{2}$$

where $U(\cdot)$ *is defined in (1). That is any possible true drop rate vector* t *is a Nash equilibrium, in which any user has no incentive to lie if all the other users bid their true drop rates.*

Definition 2 (Weak-incentive compatibility - weak-IC). *A mechanism* (p, x) *is* weakly-incentive compatible *if for any utility function, for any* $i \in \mathcal{N}$ *and any* $s_i \in T_i$*, we have*

$$U_i(p, x, t_i, t_i, t_{-i}) \geq U_i(p, x, t_i, s_i, t_{-i}), \quad \forall t \in T^* \tag{3}$$

for some $T^* \subset T$ *with* $\mathcal{L}(T \backslash T^*) = 0$*, where* $\mathcal{L}(\cdot)$ *is the Lebesgue measure.*

Definition 3 (Feasibility). *A mechanism* (p, x) *is* feasible *if it satisfies*

i. *Probability constraint (P):*
   For any $\phi \subset \mathcal{N}$ *and any* $t \in T$,

$$\sum_{\psi \subset \mathcal{N}} p_\psi(t) = 1 \quad and \quad p_\phi(t) \geq 0 \ . \tag{4}$$

ii. *Capacity constraint (CC):*
   For any $i \in \mathcal{N}$*, any* $t \in T$*, and any* $\phi \subset \mathcal{N}$ *with* $p_\phi(t) > 0$,

$$\sum_{j \in \phi} (1 - x_{j,\phi}(t)) \leq 1 \quad and \quad x_{i,\phi}(t) \geq 0 \ . \tag{5}$$

iii. *Individual rationality (IR):*
   For any $\phi \subset \mathcal{N}$ *with* $p_\phi(t) > 0$*, any* $t \in T$*, and any* $i \in \phi$,

$$x_{i,\phi}(t) \leq t_i \ . \tag{6}$$

iv. *Incentive compatibility (IC) in (2).*

Note that IC is also known as strategy-proofness or truthfulness.

Definition 4 (Weak-determinism - weak-D). *A mechanism* (p, x) *is* weakly-deterministic *if for any* $t \in T$ *and any* $i \in \mathcal{N}$,

$$(\forall j \in \mathcal{N}, j \neq i \Rightarrow t_j \neq t_i) \Rightarrow \sum_{\phi : i \in \phi} p_\phi(t) \in \{0, 1\} \ .$$

That is, for a given bid vector t*, if user* i *is the only one who bids* t_i*, then either the user always gets admitted, or the user never gets admitted.*

Definition 5 (Determinism - D). *A mechanism* (p, x) *is* deterministic *if there exists a function* $\psi : T \rightarrow \mathcal{P}(\mathcal{N})$ *such that for any* $t \in T$,

$$p_\phi(t) = \begin{cases} 1 & \textit{if } \phi = \psi(t) \\ 0 & \textit{otherwise} \end{cases}$$

where $\mathcal{P}(A)$ *is the power set of* A*. Determinism implies that the winning set is always unique. We call* $\psi(\cdot)$ *the* winning set function*.*

Note that for a deterministic mechanism (p, x) we can have the payment denoted as

$$x_i(t) = \begin{cases} x_{i,\psi(t)}(t) & \text{if } i \in \psi(t) \\ 1 & \text{otherwise} \end{cases} \ . \tag{7}$$

Definition 6 (Anonymity). *A mechanism* (p, x) *is* anonymous *if for any* $t \in T$*, any* $\pi \in \Gamma_n$*, and any* $\phi \subset \mathcal{N}$*,*

$$p(\pi(t)) = \pi(p(t))$$
$$x_\phi(\pi(t)) = \pi(x_\phi(t))$$

where Γ_n *is the set of all permutations of* n *indices. That is, the outcome of the auction does not depend on the identity of the bidders.*

Definition 7 (Monotonicity). *A mechanism* (p, x) *is* monotonic *if for any* $i \in \mathcal{N}$*, any* $t_{-i} \in T_{-i}$*, and any* $s_i, s_i' \in T_i$ *with* $s_i < s_i'$*,*

$$\sum_{\phi: i \in \phi} p_\phi(t_{-i}, s_i) \leq \sum_{\phi: i \in \phi} p_\phi(t_{-i}, s_i') \ .$$

That is, given that others' bids are fixed, a user's chance of getting admitted should not decrease when the user bids higher.

We are interested in feasible auction mechanisms that are weakly-deterministic, anonymous and monotonic. Feasibility implies that the decision is in the capacity region, no user is forced to participate, and no one has incentive to lie about his type. Weak-determinism, anonymity and monotonicity are properties we consider desirable for a fair mechanism.

3 VCG Auctions without Money

In this section we show why a simple VCG algorithm is not suitable for our problem.

3.1 VCG Auctions

In this section we temporarily turn our attention to deterministic mechanisms (p, c), where $p\colon T \to \{0,1\}^n$ is the indicator function of users' admittance and $c\colon T \to \mathbb{R}^n$ is the payment, which can be money.

We say that a mechanism (p, c) is called a *VCG mechanism* [5] if

$$p \in \arg\max_{p'} \sum_i \theta_i(t_i) p_i'(t)$$

and for any $i \in \mathcal{N}$, there exists $h_i\colon T_{-i} \to \mathbb{R}$ such that

$$c_i(t) = h_i(t_{-i}) - \sum_{j \neq i} \theta_j(t_j) p_j(t)$$

where $\theta_i(\cdot)$ is the valuation function of user i in [5].

It has been proved that the VCG mechanism satisfies IC in (2) [5]. Usually the functions h_i's are chosen according to the *Clarke pivot rule*, that is,

$$h_i(t_{-i}) = \max_{p'} \sum_{j \neq i} \theta_j(t_j) p_j'(t)$$

and it can be shown that with Clarke pivot rule, IR is always satisfied and $h_i \geq 0$ [5]. Intuitively, user i pays an amount equal to the total damage that the user causes to the other users. Then the utility for user i given bid vector t is

$$u_i(p, c, t) = \theta_i(t_i)p_i(t) - c_i(t) \ .$$

Notice that the utility function is quasi-linear with payment function c [6].

3.2 Adaptation of VCG Auctions to Money-Free Scenario

In this subsection we apply a simple VCG auction mechanism in this money-free, capacity-constrained scenario and show why it is not suitable.

Let us consider the deterministic mechanism above. Let $\theta_i(t_i) = t_i$ and use the Clarke pivot payment

$$c_i(t) = \max_{p'} \sum_{j \neq i} t_j p'_j(t) - \sum_{j \neq i} t_j p_j(t)$$

then we have $0 \leq c_i(t) \leq t_i$. We consider the drop rate as the payment for those admitted users, that is,

$$x_i(t) = \begin{cases} c_i(t) & \text{if } p_i(t) = 1 \\ 1 & \text{otherwise} \end{cases}$$

then this drop rate is well defined and is no larger than t_i. The utility function for user i would be

$$u_i(p, x, t) = \begin{cases} t_i - x_i & \text{if } p_i(t) = 1 \\ 0 & \text{otherwise} \end{cases}$$

and is quasi-linear with x.

However, one may wonder if CC in (5) is still satisfied, since $x_i(t) \leq t_i$ for $p_i(t) = 1$. The answer is no. To see this, we consider the following simple example.

Example 1. Let $n = 3$ and $t = (0.6, 0.6, 0.2)$, then the optimal decision should be $\phi = \{1, 2\}$, that is, the first two users are admitted. If either of the first two users is removed, the optimal decision will be admitting the other of the two only, so the harm is 0. If user 3 is removed, nothing changes, so the harm is still 0, then we have the payment

$$c(t) = \{0, 0, 0\}$$

and the drop rate

$$x(t) = \{0, 0, 1\}$$

which violates CC because the QoS's of the first two users are then both 1, which cannot be supported by the AP simultaneously.

In summary, even though the simple VCG auction satisfies P, IR, IC, and maximizes the number of admitted users, it does not fulfill the CC constraint. Therefore the simple VCG algorithm is not a feasible mechanism for our scenario.

3.3 Further Analysis of the Failure of VCG

The failure comes from the interaction between the allocation results and the payment. As we mentioned above, to make a VCG algorithm work, the utility function has to be quasi-linear, where the payment is the linear part which can be freely chosen after allocation decision is made. This kind of behavior resembles money. Without money, however, the choice of drop rate has a feedback effect on the allocation decision, which leads to a violation on the capacity constraint.

Hence the lack of monetary payments makes VCG auction mechanisms not feasible. Thus, we must come up with better solutions in our setting.

4 Analysis of the Problem

In this section we analyze feasible strategy-proof mechanisms in non-monetary scenarios. We start with a mechanism (p^*, x^*), which is inspired by the uniform-price auction [2]. We then show the flaws of (p^*, x^*) and prove the general properties of feasible strategy-proof non-monetary auction mechanisms.

4.1 The First Mechanism

We now propose the mechanism (p^*, x^*), and prove that this mechanism satisfies the capacity constraint and weak-IC in (3), and admits at least half of the maximal possible number of users. After that we point out the flaws of the weak-IC concept.

Given bid vector $t \in T$, the mechanism (p^*, x^*) is described as follows.

Step i. Let α be a rearrangement of the indices such that $t_{\alpha(1)} \geq t_{\alpha(2)} \geq \cdots \geq t_{\alpha(n)}$. If several users bid the same, just arrange them randomly. Introduce a pseudo-bidder with $t_{n+1} = 0$ and $\alpha(n+1) = n+1$.

Step ii. Find the largest index m^* with $1 \leq m^* \leq n$ such that $m^* \cdot (1 - t_{\alpha(m^*+1)}) \leq 1$. The winning set is $\psi^*(t) = \{\alpha(1), \alpha(2), \cdots, \alpha(m^*)\}$.

Step iii. Set the payment $x_i^*(t) = x^*(t) = t_{\alpha(m^*+1)}$ if $i \in \psi^*(t)$.

The idea of the mechanism is basically that we start from the higher bidders and try to admit as many users as possible, with payment equal to the highest losing bid.

Notice that the rearrangement α and the winning set ψ^* here might not be unique if several users bid the same value, in which case both of them will be probabilistic functions instead of deterministic functions. However, when there are no two users bidding the same value, α and ψ^* are deterministic functions.

Given the bids $t \in T$ and a corresponding rearrangement α, we let

$$\sigma_i(t) = t_{\alpha(i)}, \quad 1 \leq i \leq n+1$$

and

$$\sigma(t) = (\sigma_1(t), \sigma_2(t), \cdots, \sigma_{n+1}(t))$$

where $\sigma(t)$ is the unique sorted vector of t with $\sigma_1(t) \geq \sigma_2(t) \geq \cdots \geq \sigma_{n+1}(t)$, regardless of the possible different choices of α. Then the index chosen in step ii is a function of the bids given by

$$m^*(t) = \max\{m \in \mathcal{N} | m \cdot (1 - \sigma_{m+1}(t)) \leq 1\}$$

where $m^*(t)$ is always well-defined because

$$1 \cdot (1 - t_{\alpha(2)}) \leq 1 \ .$$

Similarly, the payment for those admitted users is also determined by

$$x^*(t) = t_{\alpha(m^*(t)+1)} \ .$$

Example 2. Let the bid vector be $t = (t_1, t_2, t_3, t_4) = \{0.5, 0.6, 0.7, 0.6\}$.

- Step i. The rearrangement could be $\alpha = (\alpha(1), \alpha(2), \alpha(3), \alpha(4), \alpha(5)) = (3, 2, 4, 1, 5)$. ($\alpha$ could also be $(3, 4, 2, 1, 5)$) So $t_{\alpha(1)} \geq t_{\alpha(2)} \geq t_{\alpha(3)} \geq t_{\alpha(4)} \geq t_{\alpha(5)} = 0$ and the sorted vector of t is $\sigma(t) = (0.7, 0.6, 0.6, 0.5)$.
- Step ii. $2 \cdot (1 - 0.6) = 0.8 \leq 1$ and $3 \cdot (1 - 0.5) = 1.5 > 1$, so $m^*(t) = 2$ and the winning set is $\psi^*(t) = \{2, 3\}$ since $\alpha = (3, 2, 4, 1, 5)$. ($\psi^*(t)$ would be $\{3, 4\}$ if $\alpha = (3, 4, 2, 1, 5)$.)
- Step iii. The payment for either of the two winners is $x^*(t) = t_{\alpha(3)} = t_4 = 0.6$. (Note that if $\alpha = (3, 4, 2, 1, 5)$, then $x^*(t)$ would still be 0.6.)

Lemma 1. *The mechanism (p^*, x^*) satisfies the P, CC, IR and weak-IC constraints.*

The proof of Lemma 1 is deferred to Appendix A.1. Note that (p^*, x^*) does not satisfy feasibility because it is not IC. To see this, just consider two users bidding the same drop rate. The chance of getting admitted is half for either user. However the chance of either user increases to 1 when he raises his bid by a small amount and the other user keeps the original bid.

We now show that (p^*, x^*) admits at least half of the maximal possible number of users.

Theorem 1 (Scalability of (p^*, x^*)). *For any true value of drop rate $t \in T$, if there exists some feasible mechanism that admits M users, then (p^*, x^*) can admit at least $\lfloor \frac{M}{2} \rfloor$ users.*

The proof of Theorem 1 is deferred to Appendix A.2.

The problem about (p^*, x^*) is that it is only weakly-IC but not IC. This means that if equal bids happen, although with low probability, users might have incentive to lie. For example, in a two-user case, if both users bid the same value, each of them would have half chance of getting admitted. But if one of them increases his bid by a small amount, he would win with the same payment and probability 1. Thus indistinguishable bids make (p^*, x^*) fail for IC.

4.2 Impossibility for Probabilistic Decisions of Equal Bids

We now show that to fulfill strict IC with some assumptions mentioned above, a weak-deterministic mechanism has to be deterministic. That is, if several users bid exactly the same value, then the only choice for guaranteeing truth-telling is to admit either all or none of them.

Theorem 2 (Impossibility). *For a mechanism (p, x) that satisfies P, IR, IC, anonymity and monotonicity, (p, x) is weakly-deterministic if and only if (p, x) is deterministic. That is, (p, x) admits either all or none of the equal bids.*

The proof of Theorem 2 is omitted due to the page limit and can be found in [16].

We should note that (p^*, x^*) is weakly-deterministic because users might be randomly admitted when bidding the same. Then by Theorem 2, to achieve IC we need to design deterministic mechanisms, that is, mechanisms with only deterministic outcomes.

4.3 Lowest Winning Bid Theorem

We further show that any deterministic mechanism with IC must be illustrated by a *lowest winning bid function.*

Definition 8 (Lowest winning bid mechanism). *A deterministic mechanism (p, x) is a* lowest winning bid mechanism *if there exists some function $z \colon T_{-i} \to T_i$ such that for any $t_{-i} \in T_{-i}$,*

$$\begin{cases} \text{if } s_i \geq z(t_{-i}) & \text{then } i \in \psi(t_{-i}, s_i) \text{ and } x_i(t_{-i}, s_i) = z(t_{-i}) \\ \text{if } s_i < z(t_{-i}) & \text{then } i \notin \psi(t_{-i}, s_i) \text{ and } x_i(t_{-i}, s_i) = 1 \end{cases}$$

where $\psi(\cdot)$ is the winning set function of (p, x) defined in the definition of determinism (Definition 5).

The function $z(\cdot)$ is called the lowest winning bid function *of (p, x).*

Theorem 3 (Lowest winning bid). *A deterministic mechanism satisfies IC if and only if it is a lowest winning bid mechanism.*

The proof of Theorem 3 is omitted due to the page limit and can be found in [16].

The lowest winning bid theorem shows us what a deterministic truth-telling mechanism should look like. Notice that no other assumptions are needed for this theorem, so it remains valid in a general setting. More importantly, this theorem gives us an efficient approach to design deterministic truth-telling mechanisms.

Although Theorem 3 does not work for (p^*, x^*) due to weak-determinism, we do have the following similar result.

Lemma 2 (Infimum winning bid function for (p^*, x^*)). *Under mechanism (p^*, x^*), knowing others' bid t_{-i}, the infimum of user i's winning bids is given by*

$$\bar{z}^*(t_{-i}) = \min\{\sigma_j(t_{-i}) | j(1 - \sigma_j(t_{-i})) \leq 1\} . \quad (8)$$

The proof of Lemma 2 is deferred to Appendix A.3.

Note that $\bar{z}^*(t_{-i})$ is not the lowest winning bid for user i because bidding this value does not guarantee winning.

5 Our Proposed Mechanism

Now we introduce the mechanism (p^{**}, x^{**}), which is a truth-telling mechanism based on the previous (p^*, x^*). We first construct (p^{**}, x^{**}) by the so-called *lifting trick*. After that we show that (p^{**}, x^{**}) is feasible, and has very close performance to (p^*, x^*) in an asymptotic sense.

5.1 Lifting Trick

Let

$$z^{**}(t_{-i}) = \begin{cases} \bar{z}^*(t_{-i}) + a(1 - \max\limits_{\substack{j \neq i \\ t_j \neq 1}} t_j) & \text{if } t_k \neq 1 \text{ for some } k \in \mathcal{N} \\ 1 & \text{otherwise} \end{cases} \tag{9}$$

where $0 < a < 1$ is a constant. Recall that $\bar{z}^*(\cdot)$ here is the infimum winning bid function of (p^*, x^*) in (8). We note that (p^{**}, x^{**}) is a deterministic mechanism based on the lowest winning bid function z^{**} in (9) and by Theorem 3 we know that (p^{**}, x^{**}) satisfies IC. Pictorially, we lift one corner of the infimum winning bid function $\bar{z}^*$, so that bidding the new function guarantees winning.

Example 3. Take $a = 0.1$ in (9). Again, let the bid vector be $t = (t_1, t_2, t_3, t_4) = \{0.5, 0.6, 0.7, 0.6\}$. By (8) we can calculate the infimum winning bid under (p^*, x^*) for each user:

$$\begin{aligned} \bar{z}^*(t_{-1}) &= \bar{z}^*(0.6, 0.7, 0.6) = 0.6 \ , \\ \bar{z}^*(t_{-2}) &= \bar{z}^*(0.5, 0.7, 0.6) = 0.6 \ , \\ \bar{z}^*(t_{-3}) &= \bar{z}^*(0.5, 0.6, 0.6) = 0.6 \ , \\ \bar{z}^*(t_{-4}) &= \bar{z}^*(0.5, 0.6, 0.7) = 0.6 \ . \end{aligned}$$

Then the lowest winning bid under (p^{**}, x^{**}) for each user is

$$\begin{aligned} z^{**}(t_{-1}) &= \bar{z}^*(t_{-1}) + a(1 - \max_{j \neq 1} t_j) = 0.63 \ , \\ z^{**}(t_{-2}) &= \bar{z}^*(t_{-2}) + a(1 - \max_{j \neq 2} t_j) = 0.63 \ , \\ z^{**}(t_{-3}) &= \bar{z}^*(t_{-3}) + a(1 - \max_{j \neq 3} t_j) = 0.64 \ , \\ z^{**}(t_{-4}) &= \bar{z}^*(t_{-4}) + a(1 - \max_{j \neq 4} t_j) = 0.63 \ . \end{aligned}$$

Since only user 3's bid is higher than or equal to his lowest winning bid, we have the winning set $\psi^{**}(t) = \{3\}$ and $x_3^{**}(t) = 0.64$.

5.2 Properties of (p^{**}, x^{**})

Lemma 3. *(p^{**}, x^{**}) is feasible, and $0 \leq z^{**}(t_{-i}) \leq 1$ for any $t_{-i} \in T_{-i}$.*

The proof of Lemma 3 is deferred to Appendix A.4.

Now we show that (p^{**}, x^{**}) has very close performance to (p^*, x^*) in an asymptotic sense. Assume that the drop rate vector t is drawn from a distribution with joint probability density function $f: T \rightarrow \mathbb{R}^+$. Then, we have the following theorem.

Theorem 4. *If the distribution function f is bounded by K, the probability that (p^{**}, x^{**}) behaves differently from (p^*, x^*) is at most aK.*

The proof of Theorem 4 is deferred to Appendix A.5.

We notice from Theorem 4 that the probability of different behaviors between the two mechanisms is of order $O(a)$.

6 Conclusion

In this paper, we studied the problem of designing a strategy-proof non-monetary auction mechanism for wireless networks. The motivation is to let the users tell the truth when bidding their resource requirements, and to admit as many users as possible. We gave a general model for this problem and showed that due to the lack of monetary payments in this scenario, a simple adoption of VCG mechanism would violate the capacity constraint. We analyzed the problem and found some properties that any strategy-proof auction mechanism should satisfy. Finally we proposed a feasible mechanism which is truthful even with equal bids, and showed that it could admit at least half of the maximal number of users with high probability in an asymptotic sense.

As possible topic for future work, discrete pricing models might be considered rather than continuous pricing models. Also, the assumption of weak-determinism could be weakened, and more specific utility functions could be considered for better performance. Furthermore, the lower bound of the number of admitted user might be improved.

Acknowledgement. Research supported by NSF Grant CNS-0953165, and DTRA Grants HDTRA1-08-1-0016 and HDTRA1-09-1-0055.

References

1. Myerson, R.B.: Optimal Auction Design. Mathematics of Operations Research 6(1), 58–73 (1981)
2. Krishna, V.: Auction Theory. Academic Press, San Diego (2002)
3. Varian, H.R.: Position Auctions. International Journal of Industrial Organization 25, 1163–1178 (2007)
4. Hou, I.H., Kumar, P.R.: Utility Maximization for Delay Constrained QoS in Wireless. In: Proceedings of IEEE INFOCOM, pp. 1–9 (2010)

5. Nisan, N., Roughgarden, T., Tardos, E., Vazirani, V.V.: Algorithmic Game Theory. Cambridge University Press, New York (2007)
6. Maillé, P., Tuffin, B.: Why VCG Auctions Can Hardly Be Applied to the Pricing of Inter-domain and Ad Hoc Networks. In: 3rd EuroNGI Conference on Next Generation Internet Networks, pp. 36–39 (2007)
7. Anderegg, L., Eidenbenz, S.: Ad Hoc-VCG: a Truthful and Cost-Efficient Routing Protocol for Mobile Ad Hoc Networks with Selfish Agents. In: Proc. ACM 9th Annual International Conference on Mobile Computing and Networking (MobiCom 2003), San Diego, pp. 245–259 (2003)
8. Buttyán, L., Hubaux, J.P.: Enforcing Service Availability in Mobile Ad-Hoc WANs. In: Proc. International Symposium on Mobile Ad Hoc Networking & Computing (MobiHoc 2000), Boston, pp. 87–96 (2000)
9. Buttyán, L., Hubaux, J.P.: Stimulating Cooperation in Self-organizing Mobile Ad Hoc Networks. ACM/Kluwer Mobile Networks and Applications 8(5), 579–592 (2003)
10. Crowcroft, J., Gibbens, R., Kelly, F., Östring, S.: Modelling Incentives for Collaboration in Mobile Ad Hoc Networks. In: Proc. WiOpt 2003, France (2003)
11. Srinivasan, V., Nuggehalli, P., Chiasserini, C.F., Rao, R.R.: Energy Efficiency of Ad Hoc Wireless Networks with Selfish Users. In: Proc. European Wireless Conference, Italy (2002)
12. Srinivasan, V., Nuggehalli, P., Chiasserini, C.F., Rao, R.R.: Cooperation in Wireless Ad Hoc Networks. In: Proc. IEEE INFOCOM 2003, San Francisco, vol. 2, pp. 808–817 (2003)
13. Zhong, S., Chen, J., Yang, Y.R.: Sprite: a Simple, Cheat-Proof, Credit-Based System for Mobile Ad-Hoc Networks. In: Proc. IEEE INFOCOM 2003, San Francisco, vol. 3, pp. 1987–1997 (2003)
14. Goldberg, A.V., Hartline, J.D., Karlin, A.R., Wright, A., Saks, M.: Competitive Auctions. Games and Economic Behavior 55(2), 242–269 (2003)
15. Mehta, A., Vazirani, V.V.: Randomized Truthful Auctions of Digital Goods are Randomizations Over Truthful Auctions. In: Proceeding of the 5th ACM Conference on Electronic Commerce 2004, New York (2004)
16. Kang, X., Jaramillo, J.J., Ying, L.: A Strategy-Proof and Non-monetary Admission Control Mechanism for Wireless Access Networks (2010), `http://arxiv.org/abs/1010.2713`

A Appendix

A.1 Proof of Lemma 1

i. Probability constraint (P)
The probability constraint is obviously satisfied. For those t such that the winning set $\psi^*(t)$ is determined, $\sum_{\phi \subset \mathcal{N}} p_\phi(t) = p_{\psi^*(t)}(t) = 1$. For those t such that there are M possible winning sets, the probability for each of them would be $\frac{1}{M}$ and $\sum_{\phi \subset \mathcal{N}} p_\phi(t) = M \cdot \frac{1}{M} = 1$.

ii. Capacity constraint (CC)
For any $t \in T$, for any $\phi \subset \mathcal{N}$ with $p_\phi(t) > 0$,

$$\sum_{i \in \phi}(1 - x^*_{i,\phi}(t)) = m^*(t) \cdot (1 - x^*(t))$$

$$= m^*(t) \cdot \left(1 - t_{\alpha(m^*(t)+1)}\right)$$
$$\leq m^*(t) \cdot \frac{1}{m^*(t)}$$
$$= 1 \ .$$

Also $x^*_{i,\phi}(t) \geq 0$. Thus, (p^*, x^*) satisfies CC.

iii. Individual rationality (IR)
For any $t \in T$, any $\phi \subset \mathcal{N}$ with $p_\phi(t) > 0$, and any $i \in \phi$,

$$x^*_{i,\phi}(t) = x^*(t) = t_{\alpha(m^*(t)+1)} \leq t_i \ .$$

iv. Weak-incentive compatibility (weak-IC)
We only consider the set of *distinguishable* bid vectors

$$T_D = \{t \in T | t_i \neq t_j \forall i \neq j\}$$

that is, the set of bid vectors with no equal bids from any two users. For $t \in T_D$ and $i \in \mathcal{N}$, the result of user i bidding s_i would be

$$p^*_i(t_{-i}, s_i) = \begin{cases} 1 & \text{if } s_i > x^*(t) \\ \frac{1}{2} & \text{if } s_i = x^*(t) \\ 0 & \text{if } s_i < x^*(t) \end{cases}$$

with payment $x^*_i(t_{-i}, s_i) = x^*(t)$ if admitted. Note that $p^*_i(t_{-i}, s_i) = \frac{1}{2}$ when $s_i = x^*(t)$ since there is only one other user who bids $x^*(t)$.

We first consider the case of $i \in \psi^*(t)$. We then have $t_i > x^*(t)$. If $s_i > x^*(t)$, then user i still gets admitted with the same payment. If $s_i < x^*(t)$, then user i gets rejected. If $s_i = x^*(t)$, then user i either gets admitted with the same payment, or get rejected, both of which have probability 1/2. So user i cannot get better utility in the first case.

We then consider the case of $i \notin \psi^*(t)$. Now we have $t_i < x^*(t)$. If $s_i > x^*(t)$, then user i gets admitted with payment $x^*(t)$ higher than true value t_i. If $s_i < x^*(t)$, the user i still does not get admitted. If $s_i = x^*(t)$, then user i either gets admitted with payment too high to accept, or does not get admitted at all, both of which have probability 1/2. So user i cannot get better utility in the second case.

Thus, for any $t \in T_D$, no user has incentive to lie. As $T \backslash T_D$ has measure zero, we have weak-IC.

□

A.2 Proof of Theorem 1

Suppose (p^*, x^*) admits only m users, that is,

$$m^*(t) = m$$

and $(\bar{p}, \bar{x})$ has a chance of admitting at least $2m + 2$ users given bid vector t, that is,

$$\exists \phi \subset \mathcal{N}, |\phi| \geq 2m + 2, \bar{p}_\phi(t) > 0 \ .$$

Then we have

$$\begin{aligned}
\sum_{i \in \phi}(1 - \bar{x}_{i,\phi}(t)) &\geq \sum_{i \in \phi}(1 - t_i) &(10)\\
&\geq \sum_{i=1}^{2m+2}(1 - \sigma_i(t)) \\
&\geq \sum_{i=m+2}^{2m+2}(1 - \sigma_i(t)) \\
&\geq (m+1)(1 - \sigma_{m+2}(t)) \\
&> 1 &(11)
\end{aligned}$$

where the inequality (10) comes from IR and (11) comes from the definition of the mechanism.

Thus $(\bar{p}, \bar{x})$ admits at most $2m + 1$ users. This is equivalent to the statement that if some feasible mechanism admits M users, (p^*, x^*) can at least admit $\lfloor \frac{M}{2} \rfloor$ users. □

A.3 Proof of Lemma 2

This can be checked by directly going through the process of (p^*, x^*). If $t_i > \bar{z}^*(t_{-i})$, we can see that $x^*(t) = \bar{z}^*(t_{-i}) < t_i$, so user i wins. If $t_i < \bar{z}^*(t_{-i})$, we have $x^*(t) \geq t_i$, so user i loses. If $t_i = \bar{z}^*(t_{-i})$, user i wins with some probability between 0 and 1, which depends on the number of users bidding $\bar{z}^*(t_{-i})$. □

A.4 Proof of Lemma 3

P comes from determinism. IR comes from the definition of lowest winning bid mechanisms. IC comes from Theorem 3. So only the proof of CC requires some more effort.

By the construction of $z^{**}(\cdot)$ in (9), we have for any $t_{-i} \in T_{-i}$ with $t_k \neq 1$ for some $k \in \mathcal{N}\backslash\{i\}$, $z^{**}(t_{-i}) > \bar{z}^*(t_{-i})$. Then by Lemma 2, we note that (p^{**}, x^{**}) is stricter than (p^*, x^*), that is, for any $t \in T$, if user i gets admitted in (p^{**}, x^{**}), then user i also gets admitted in (p^*, x^*). Let $\psi^{**}(t)$ be the winning set function of (p^{**}, x^{**}). Fix $t \in T$, for any ψ with $p^*_\psi(t) > 0$, we have

$$\sum_{i\in\psi^{**}(t)} (1 - x_i^{**}(t)) = \sum_{i\in\psi^{**}(t)} (1 - z^{**}(t_{-i})) \tag{12}$$
$$\leq \sum_{i\in\psi^{**}(t)} (1 - \bar{z}^*(t_{-i})) \tag{13}$$
$$\leq \sum_{i\in\psi} (1 - \bar{z}^*(t_{-i})) \tag{14}$$
$$= \sum_{i\in\psi} (1 - x^*_{i,\psi}(t)) \tag{15}$$
$$\leq 1 \tag{16}$$

where (12) comes from the definition of (p^{**}, x^{**}), (13) and (14) comes from the fact that (p^{**}, x^{**}) is stricter than (p^*, x^*), (15) comes from the definition of (p^*, x^*), (16) is because (p^*, x^*) satisfies CC.

If all bids in t_{-i} are 1, then $z^{**}(t_{-i}) = \bar{z}^*(t_{-i}) = 1$. Otherwise, for any $t_{-i} \in T_{-i}$ with some entry not equal to 1,

$$\begin{aligned} z^{**}(t_{-i}) &= \bar{z}^*(t_{-i}) + a(1 - \max_{\substack{j\neq i\\ t_j\neq 1}} t_j) \\ &\leq \max_{\substack{j\neq i\\ t_j\neq 1}} t_j + a(1 - \max_{\substack{j\neq i\\ t_j\neq 1}} t_j) \\ &\leq \max_{\substack{j\neq i\\ t_j\neq 1}} t_j + (1 - \max_{\substack{j\neq i\\ t_j\neq 1}} t_j) \\ &\leq 1\ . \end{aligned}$$

Since $z^{**}(t_{-i}) \geq \bar{z}^*(t_{-i}) \geq 0$, we get $0 \leq z^{**}(t_{-i}) \leq 1$. □

A.5 Proof of Theorem 4

Let $\mathbf{T}$ be the vector of random variables with values taken in T, and as usual let $\mathbf{T}_i$ and $\mathbf{T}_{-i}$ be the corresponding components. Note that $T \subset [0,1]^n$. Then the probability that (p^{**}, x^{**}) and (p^*, x^*) have different results is

$$\begin{aligned} D &= \Pr\left(\bar{z}^*(\mathbf{T}_{-i}) < \mathbf{T}_i < z^{**}(\mathbf{T}_{-i}) \text{ for some } i\right) \\ &\leq \sum_{i=1}^{n} \Pr\left(\bar{z}^*(\mathbf{T}_{-i}) < \mathbf{T}_i < z^{**}(\mathbf{T}_{-i})\right) \end{aligned}$$

$$= \sum_{i=1}^{n} \int_{B_i} f(t)\mathrm{d}t$$
$$\leq \sum_{i=1}^{n} \mathcal{L}(B_i) K$$

where

$$B_i = \{t \in T | \bar{z}^*(t_{-i}) < t_i < z^{**}(t_{-i})\}$$

and $\mathcal{L}(\cdot)$ denotes the Lebesgue measure. Then

$$\begin{aligned}
\mathcal{L}(B_i) &= \int_{B_i} 1\mathrm{d}t \\
&= \int_{T_{-i}} \int_{\bar{z}^*(t_{-i})}^{z^{**}(t_{-i})} 1\mathrm{d}t_i \mathrm{d}t_{-i} \\
&\leq \int_{[0,1]^{n-1}} (z^{**}(t_{-i}) - \bar{z}^*(t_{-i}))\mathrm{d}t_{-i} \\
&= \int_{[0,1]^{n-1}} a(1 - \max_{j \neq i} t_j)\mathrm{d}t_{-i} \\
&= a - a \int_{[0,1]^{n-1}} \max_{j \neq i} t_j \mathrm{d}t_{-i} \\
&= a - a\frac{n-1}{n} \\
&= \frac{a}{n} \ .
\end{aligned}$$

Thus

$$D \leq aK \ .$$

□