Optimal Channel Pairing and Power Allocation for Multi-channel Multi-hop Relay Networks

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Abstract. We study the problem of channel pairing and power allocation in a multi-channel, multi-hop relay network to enhance the end-to-end data rate. OFDM-based relays are used as an illustrative example, and the amplify-and-forward and decode-and-forward relaying strategies are considered. Given fixed power allocation to the OFDM subcarriers, we observe that a sorted-SNR subcarrier pairing strategy is data-rate optimal, where each relay pairs its incoming and outgoing subcarriers by their SNR order. For the joint optimization of subcarrier pairing and power allocation, we show that it is optimal to separately consider the two subproblems, for both individual and total power constraints. This separation principle significantly reduces the computational complexity in finding the jointly optimal solution. We further establish the equivalence between sorting SNRs and sorting channel gains in the jointly optimal solution, which allows simple implementation of optimal subcarrier pairing at the relays. Simulation results are presented to demonstrate the performance gain of the jointly optimal solution over some suboptimal alternatives.

1 Introduction

The emerging 4G wireless systems adopt a multi-channel relaying architecture, through Orthogonal Frequency Division Multiplexing (OFDM) and the installation of wireless relay stations. As opposed to a narrow-band single-channel relay, an OFDM relay has access to multiple channels (called subcarriers). It may receive a signal from one subcarrier and transmit a processed version of the signal on a different subcarrier. This *multi-channel relaying capability* can be exploited to process the incoming signals adaptively, taking advantage of the the diverse strength of different channels. In this work, we consider the general problem of jointly optimizing, the pairing of incoming and outgoing channels in multi-channel capable relays, and power allocation to the transmitted signals on these channels, to maximize the end-to-end data rate in a *multi-hop relaying* network. Since the OFDM-based relay architecture is the best known example for multi-channel relaying, we refer to it throughout this paper for the purpose of illustration.

The concept of OFDM subcarrier pairing (SP), which maps incoming and outgoing subcarriers at the relay, was first proposed independently in [1] and [2] for a *dual-hop* (*i.e.,* single-relay) amplified-and-forward (AF) OFDM relay system. For relaying without the direct source-destination link available, [1] used integer programming to

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find the optimal pairing that maximizes the sum SNR. Whereas from a system-design perspective, [2] proposed an SP scheme which is optimal in the noise-free case, with the assumption of uniform power allocation. These works sparked interests for more research in this area. Subsequently, [3] proved that the *sorted-SNR subcarrier pairing* (sorted-SNR SP) scheme, in the absence of the source-destination link, is optimal for both AF and decode-and-forward (DF) dual-hop OFDM relaying systems. In sorted-SNR SP, the subcarriers are paired in the order of SNR seen on each subcarrier. In [4], the authors considered the direct source-destination path to achieve full diversity and presented two suboptimal SP schemes for this case.

The related problem of optimal power allocation (PA) for a *dual-hop* OFDM system was studied by many [5,6,7] for different relay strategies and power constraints. The problem of jointly optimizing SP and PA in a dual-hop OFDM system was studied for DF relaying in [8] and [9], where direct transmission links were assumed unavailable. [8] assumed a total power constraint shared between the source and the relay, while [9] considered individual power constraints imposed on the source and the relay separately. In both cases, it was shown that the joint optimization solution can be obtained by separately optimizing SP and PA. Moreover, the SP scheme which maps the subcarriers solely based on their channel gains is the optimal pairing scheme independent of the optimal PA solution.

Similar study on SP and PA in a multi-hop setting has been scarce. The authors of [10] proposed an adaptive PA algorithm to maximize the end-to-end rate under the total power constraint. For a similar network with DF relaying, [11] studied the problem of joint power and time allocation under the long-term total power constraint to maximize the end-to-end rate. Furthermore, in [10], the idea of subcarrier pairing was mentioned for further performance enhancement in addition to PA. However, no claim was provided on the optimality of sorted SP under the influence of PA. The optimal joint SP and PA solution remained unknown.

In this paper, we present a comprehensive solution for jointly optimizing SP and PA to maximize the source-destination data rate in a multi-channel (e.g., OFDM) multi-hop relay network. The main observations from our work are summarized as follows:

– Given fixed power allocation, the sorted-SNR SP scheme is optimal in multi-hop relaying. Specifically, SP can be separated into individual pairing problems at each relay, where the relay matches the incoming subcarriers to the outgoing subcarriers according to the order of SNRs seen over the subcarriers.
– The problem of joint SP and PA optimization can be decomposed into separate problems that can be solved independently. This separation principle holds for both AF and DF relaying strategies, and for either total or individual power constraints imposed on the transmitting nodes.
– With joint SP and PA optimization, the subcarriers are optimally paired according to their channel-gain order, without the need for knowledge of power allocation on each subcarrier. This allows simple relay implementation for optimal operation.

The generalization from the dual-hop case to the multi-hop case is non-trivial. Intuitively, to maximize the end-to-end rate, the choice of SP at each relay would affect the choices of SP at other relays, which also depend on the specific power allocation scheme used. Therefore, it is not apparent that the optimal SP can be decomposed into independent pairing problems at each relay, or that SP and PA can be separately considered. Besides, we will see later that the techniques used in the dual-hop case cannot be simply extended to the multi-hop case. New techniques are required.

The rest of this paper is organized as follows. In Section 2, we present the system model and problem formulation. Given a fixed PA solution, Section 3 shows that the optimal SP scheme is one based on the sorted SNR. The joint optimization problem of PA and SP is considered in Section 4, and the separation principle between PA and SP is proven. The optimal SP and PA solution is discussed in Section 5 for multi-hop relaying under both total and individual power constraints. The simulation results are provided in Section 6, and finally a summary is given in Section 7.

2 System Model and Problem Statement

We consider an M-hop relay network where a source node communicates with a destination node via $(M-1)$ intermediate relay nodes as illustrated in Fig. 1. For broadband communication between the nodes, the frequency bandwidth is split into multiple subbands for data transmission. A practical system with such an approach is the OFDM system where the bandwidth is divided into N equal-bandwidth subcarriers. We denote by $h_{m,n}$, for $m = 1, \cdots, M$ and $n = 1, \cdots, N$, the channel on subcarrier n over hop m. The additive noise at hop m is modeled as an i.i.d. zero mean Gaussian random variable with variance σ_m^2. We define $a_{m,n} = \frac{|h_{m,n}|^2}{\sigma_m^2}$ as the *normalized* channel gain, often shortened to channel gain, over subcarrier n of hop m. We make the common assumption that the full knowledge of global channels is available at a central controller, which determines the optimal subcarrier pairing and power allocation. We further assume that the destination is out of the transmission zone of the source, and therefore, there is no direct transmission link. For M-hop relaying, a transmission from source to destination occupies M equal time slots, one for each hop. In the mth slot, $m = 1, \cdots, M$, the mth node (the source node if $m = 1$, otherwise the $(m-1)$th relay node) transmits a data block to the $(m+1)$th node (the destination node if $m = M$, otherwise the mth relay node) on each subcarrier. Our study is constrained to half-duplex transmissions, where the relay nodes cannot send and receive at the same time on the same frequency. However, the transmission of different data blocks in different hops may occur concurrently, depending on the scheduling pattern for spatial reuse of spectrum.

2.1 Relaying Strategies

In this work, we consider two types of relaying strategies: AF and DF. In AF relaying, a relay amplifies the data received from an incoming subcarrier and directly forwards it to the next node over an outgoing subcarrier. In DF relaying, a relay attempts to decode the received data from the previous node over each incoming subcarrier and forwards a version of the decoded data on an outgoing subcarrier to the next node. We consider the simple repetition-coding based DF relaying [12,13], where the relay is required to fully decode the incoming message, re-encodes it with repetition coding, and forwards it to the intended user.

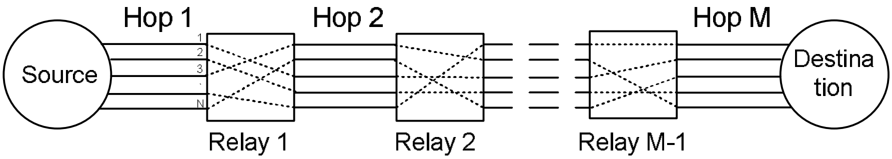

Fig. 1. Illustration of multi-hop OFDM-based network employing subcarrier pairing scheme

2.2 Subcarrier Pairing

The relay conducts SP, matching each incoming subcarrier with an outgoing subcarrier. As the channels on different subcarriers exhibit various quality, a judicious SP scheme can potentially lead to significant improvement in system spectral efficiency.

We denote path $\mathcal{P}_i = (c(1,i), \cdots, c(M,i))$, where $c(m,i)$ specifies the index of the subcarrier at hop m that belongs to path $\mathcal{P}_i$. For example, $\mathcal{P}_i = (3,4,2)$ indicates that path $\mathcal{P}_i$ consists of the third subcarrier at hop 1, the fourth subcarrier at hop 2, and the second subcarrier at hop 3. Once subcarrier pairing is determined at all the relays, the total N disjoint paths $\mathcal{P}_1, \cdots, \mathcal{P}_N$ can be identified from the source to the destination.

2.3 Power Allocation

Denote the power allocated to subcarrier n over hop m by $P_{m,n}$. The SNR obtained on this subcarrier is represented by $\gamma_{m,n} = P_{m,n} a_{m,n}$. For each path $\mathcal{P}_i$, let $\tilde{\gamma}_{m,i} \triangleq \gamma_{m,c(m,i)}$ represent the SNR seen over hop m on this path.

Let $\mathbf{P}_i = (P_{1,c(1,i)}, \cdots, P_{M,c(M,i)})$ be the PA vector for all subcarriers along path $\mathcal{P}_i$. The source-to-destination equivalent SNR of path $\mathcal{P}_i$ is denoted by $\gamma_{\text{SD}}(\mathcal{P}_i, \mathbf{P}_i)$. For AF relaying, it is given by [14],

$$\gamma_{\text{SD}}^{\text{AF}}(\mathcal{P}_i, \mathbf{P}_i) = \left(\prod_{m=1}^{M} \left(1 + \frac{1}{\tilde{\gamma}_{m,i}} \right) - 1 \right)^{-1}, \tag{1}$$

and, in Section 5, we will also use its upper bound [14],

$$\gamma_{\text{SD}}^{\text{AF}}(\mathcal{P}_i, \mathbf{P}_i) \approx \left(\sum_{m=1}^{M} \frac{1}{\tilde{\gamma}_{m,i}} \right)^{-1}, \tag{2}$$

whose approximation gap vanishes as the SNR becomes large. For DF relaying, we have

$$\gamma_{\text{SD}}^{\text{DF}}(\mathcal{P}_i, \mathbf{P}_i) = \min_{m=1,\cdots,M} \tilde{\gamma}_{m,i} \,. \tag{3}$$

We consider two types of power constraints:

Total power constraint: The power assignment $P_{m,n}$, for $m = 1 \cdots M$ and $n = 1 \cdots N$, must satisfy the following aggregated power constraint

$$\sum_{l=1}^{M} \sum_{n=1}^{N} P_{m,n} = P_t. \tag{4}$$

Individual power constraint: The power assignment $P_{m,n}$, for $n = 1, \cdots, N$, needs to satisfy the power constraint of the individual node m, *i.e.,*

$$\sum_{n=1}^{N} P_{m,n} = P_{mt}\ , \qquad m = 1, \cdots, M, \tag{5}$$

where P_{mt} denotes the maximum allowable power at node m.

2.4 Objective

Our goal is to design a jointly optimal SP and PA strategy to maximize the source-destination rate under multi-hop relaying. The source-destination rate achieved through path $\mathcal{P}_i$ is given by

$$R_{\text{SD}}(\mathcal{P}_i, \mathbf{P}_i) = \frac{1}{F_s} \log_2(1 + \gamma_{\text{SD}}(\mathcal{P}_i, \mathbf{P}_i)),$$

where F_s is the spatial reuse factor. In a multi-hop relaying that allows concurrent transmissions, F_s takes value between 2 and M ($F_s \geq 2$ under the half-duplex assumption). The sum rate of all paths determines the total source-destination rate of the system, *i.e.,*

$$R_t = \sum_{i=1}^{N} R_{\text{SD}}(\mathcal{P}_i, \mathbf{P}_i). \tag{6}$$

It is a function of both $\{\mathcal{P}_i\}$ and $\{\mathbf{P}_i\}$, which should be jointly optimized:

$$\max_{\{\mathcal{P}_i\}, \{\mathbf{P}_i\}} \quad R_t \tag{7}$$

$$\text{subject to} \quad (4) \quad \text{or} \quad (5),$$

$$\mathbf{P}_i \succeq 0, \quad i = 1, \cdots, N, \tag{8}$$

where $\succeq$ signifies the element-wise inequality.

3 Optimal Multi-hop Subcarrier Pairing under Fixed Power Allocation

In this section, we first consider the case when PA is fixed and given. In this case, the optimization problem in (7) can be re-written as

$$\max_{\{\mathcal{P}_i\}} \sum_{i=1}^{N} R_{\text{SD}}(\mathcal{P}_i, \mathbf{P}_i) \tag{9}$$

and the optimal SP $\{\mathcal{P}_i^*\}$ is a function of $\{\mathbf{P}_i\}$. To simplify the notation, in this section we rewrite $R_{\text{SD}}(\mathcal{P}_i)$ and $\gamma_{SD}(\mathcal{P}_i)$ and drop their dependency on $\mathbf{P}_i$ with the understanding that $\{\mathbf{P}_i\}$ is fixed. In the following, we solve (9) to obtain the optimal SP scheme under this fixed PA. We emphasize that here the generalization from the dual-hop case

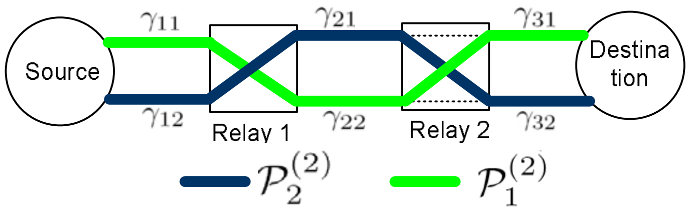

Fig. 2. Three-hop relay with two subcarriers

to the multi-hop case is non-trivial. Intuitively, there is no obvious way to decouple the sequence of pairings at all $(M-1)$ relays. Indeed, the *equivalent* incoming channel from a source to a relay and the *equivalent* outgoing channel from that relay to the destination depend on how the subcarriers are paired over multiple hops. However, we will show that the optimal SP solution over multiple hops can in fact be decomposed into $(M-1)$ independent SP problems, where the mapping of incoming and outgoing subcarriers at each relay is only based on the sorted SNR over those subcarriers, and therefore can be performed individually per hop.

In the following, we first establish the optimality of the sorted-SNR SP scheme for the case of $M=3$ and $N=2$, and then we extend the result to arbitrary M and N.

3.1 Optimal Subcarrier Pairing for Three-Hop Relaying

Two-Subcarrier Case ($N=2$). We first consider a three-hop relaying network with two subcarriers, as depicted in Fig. 2. Without loss of generality, we assume subcarrier 1 exhibits equal or larger SNR than subcarrier 2 over all the three hops, *i.e.,*

A1: $\gamma_{m,1} \geq \gamma_{m,2}$, for $m=1,2,3$.

The optimal SP scheme for this case is given in Proposition 1.

Proposition 1. *For $M=3$ and $N=2$, the solution to* (9) *is the sorted-SNR SP scheme performed on each relay, i.e.,* $\{\mathcal{P}_i^*\} = \{(1,1,1),(2,2,2)\}$ *under assumption A1.*

Proof. At relay 1, there are two ways to pair the subcarriers: (1) subcarriers 1 and 2 over hop 1 are matched with subcarriers 1 and 2 over hop 2, respectively; (2) subcarriers 1 and 2 over hop 1 are matched with subcarriers 2 and 1 over hop 2, respectively. These two ways of pairing lead to the following two sets of disjoint paths from the source to the destination: $\{\mathcal{P}_i^{(1)}\} = \{(1,1,c(3,1)),(2,2,c(3,2))\}$ and $\{\mathcal{P}_i^{(2)}\} = \{(1,2,c(3,1)),(2,1,c(3,2))\}$, where the superscript j in $\{\mathcal{P}_i^{(j)}\}$ indicates a different set of path selection.

By considering the *equivalent* subcarrier channels from the source to the second relay, using the known optimality result for dual-hop relaying, it is easy to see that $c(3,1)=1$ and $c(3,2)=2$ are optimal for $\{\mathcal{P}_i^{(1)}\}$. Furthermore, we only need to show

$$\log_2\left(1+\gamma_{\mathrm{SD}}(\mathcal{P}_1^{(1)})\right) + \log_2\left(1+\gamma_{\mathrm{SD}}\left(\mathcal{P}_2^{(1)}\right)\right) \geq$$
$$\log_2\left(1+\gamma_{\mathrm{SD}}\left(\mathcal{P}_1^{(2)}\right)\right) + \log_2\left(1+\gamma_{\mathrm{SD}}\left(\mathcal{P}_2^{(2)}\right)\right), \quad (10)$$

for the case of $c(3,1) = 1$ and $c(3,2) = 2$ for both $\{\mathcal{P}_i^{(1)}\}$ and $\{\mathcal{P}_i^{(2)}\}$, since the case of $c(3,1) = 2$ and $c(3,2) = 1$ for $\{\mathcal{P}_i^{(2)}\}$ can be similarly proven. Inequality (10) for the AF and DF relaying cases are separately proven as follows:

AF Relaying: By inserting (1) into inequality (10) we need to show

$$\left(1 + (Q_1^{(1)} - 1)^{-1}\right)\left(1 + (Q_2^{(1)} - 1)^{-1}\right) \geq \left(1 + (Q_1^{(2)} - 1)^{-1}\right)\left(1 + (Q_2^{(2)} - 1)^{-1}\right), \tag{11}$$

where

$$\begin{aligned} Q_1^{(1)} &= (1 + \frac{1}{\gamma_{1,1}})(1 + \frac{1}{\gamma_{2,1}})(1 + \frac{1}{\gamma_{3,1}}), \\ Q_2^{(1)} &= (1 + \frac{1}{\gamma_{1,2}})(1 + \frac{1}{\gamma_{2,2}})(1 + \frac{1}{\gamma_{3,2}}), \\ Q_1^{(2)} &= (1 + \frac{1}{\gamma_{1,1}})(1 + \frac{1}{\gamma_{2,2}})(1 + \frac{1}{\gamma_{3,1}}), \\ Q_2^{(2)} &= (1 + \frac{1}{\gamma_{1,2}})(1 + \frac{1}{\gamma_{2,1}})(1 + \frac{1}{\gamma_{3,2}}). \end{aligned} \tag{12}$$

The following lemma is used to prove (11).

Lemma 1. *With assumption A1, we have*

$$(Q_1^{(1)} - 1)(Q_2^{(1)} - 1) \leq (Q_1^{(2)} - 1)(Q_2^{(2)} - 1). \tag{13}$$

Proof. We omit the proof for brevity.

We proceed by considering the subtraction of the RHS from the LHS of (11),

$$\begin{aligned} &\underbrace{(Q_1^{(1)} - 1)^{-1} + (Q_2^{(1)} - 1)^{-1} + (Q_1^{(1)} - 1)^{-1}(Q_2^{(1)} - 1)^{-1}}_{A} - \\ &\left(\underbrace{(Q_1^{(2)} - 1)^{-1} + (Q_2^{(2)} - 1)^{-1} + (Q_1^{(2)} - 1)^{-1}(Q_2^{(2)} - 1)^{-1}}_{B}\right) \\ &\geq A(Q_1^{(1)} - 1)(Q_2^{(1)} - 1) - B(Q_1^{(2)} - 1)(Q_2^{(2)} - 1) && (14) \\ &= Q_2^{(1)} + Q_1^{(1)} - Q_1^{(2)} - Q_2^{(2)} \\ &= (\frac{1}{\gamma_{2,2}} - \frac{1}{\gamma_{2,1}})\left[(1 + \frac{1}{\gamma_{1,2}})(1 + \frac{1}{\gamma_{3,2}}) - (1 + \frac{1}{\gamma_{1,1}})(1 + \frac{1}{\gamma_{3,1}})\right] && (15) \\ &\geq 0, && (16) \end{aligned}$$

where the inequality in (14) holds because of (13) and the fact that $Q_i^{(j)} - 1 > 0$ for $i = 1, 2$ and $j = 1, 2, 3$.

DF Relaying: Inserting (3) into inequality (10), we need to show

$$(1+\min(\gamma_{1,1},\gamma_{2,1},\gamma_{3,1}))\,(1+\min(\gamma_{1,2},\gamma_{2,2},\gamma_{3,2})) \geq (1+\min(\gamma_{1,1},\gamma_{2,2},\gamma_{3,1}))\,(1+\min(\gamma_{1,2},\gamma_{2,1},\gamma_{3,2}))\,. \quad (17)$$

We can consider all possible relations among $\gamma_{m,n}$, for all $m = 1,2,3$ and $n = 1,2$ subject to assumption A1. For example, when $\gamma_{1,1} \leq \gamma_{2,1} \leq \gamma_{3,1}$, $\gamma_{1,2} \leq \gamma_{2,2} \leq \gamma_{3,2}$, $\gamma_{2,2} \leq \gamma_{1,1} \leq \gamma_{3,2}$, and $\gamma_{3,2} \leq \gamma_{2,1}$, (17) yields

$$(1+\gamma_{1,1})(1+\gamma_{1,2}) \geq (1+\gamma_{2,2})(1+\gamma_{1,2}), \quad (18)$$

which is clearly true. It is not hard to list all other relations, and (17) can be similarly verified. The details are omitted for brevity. ■

Multi-subcarrier case ($N \geq 2$). Here, we provide an argument to extend the result in Proposition 1 to a system with an arbitrary number of subcarriers.

Proposition 2. *For $M = 3$ and $N \geq 2$, the solution to* (9) *is the sorted-SNR SP scheme performed on each relay.*

Proof. Suppose the optimal pairing does not follow the pairing rule of sorted SNR. There is at least one relay (say, Relay 2) that has two pairs of incoming and outgoing subcarriers that are mis-matched according to their SNR. That is, there exist two subcarriers i_1 and i_2 over hop 2, and two subcarriers j_1 and j_2 over hop 3 that are respectively paired with each other while $\gamma_{2,i_1} < \gamma_{2,i_2}$ and $\gamma_{3,j_1} > \gamma_{3,j_2}$. Note that these two subcarrier pairs belong to two disjoint source-destination paths that can be regarded as a 2-subcarrier relay system. From Proposition 1, we know that pairing subcarriers i_1 with j_2 and i_2 with j_1 at relay 2 achieves a higher rate than the existing pairing over these two paths. Hence, by switching to this new pairing while keeping the other paths the same, we could increase the total rate. This contradicts our assumption on the optimality of a non-sorted SNR SP scheme. Hence, there is no better scheme than sorted-SNR SP to obtain the maximum sum rate. ■

3.2 Optimal Subcarrier Pairing for Multi-hop Relaying

We next extend the result in Section 3.1 to a relaying network with an arbitrary number of hops ($M \geq 3$) in the following proposition.

Proposition 3. *The solution to* (9) *is the sorted-SNR SP scheme individually performed at each relay.*

Proof. (By induction.) It is shown in Proposition 2 that the sorted-SNR SP is optimal for $M = 3$. Suppose the claim holds for $M \leq L$. Now consider $M = L + 1$ as shown in Fig. 3(a). Let $\gamma_{eq,n}$ be the nth equivalent channel SNR from the source to Relay $L-1$, corresponding to the nth incoming subcarrier of that relay. Then, the $L+1$-hop network can be converted to a 3-hop network, with an equivalent relay whose incoming subcarriers have SNR $\{\gamma_{eq,n}\}$ and outgoing subcarriers remain the same as those of

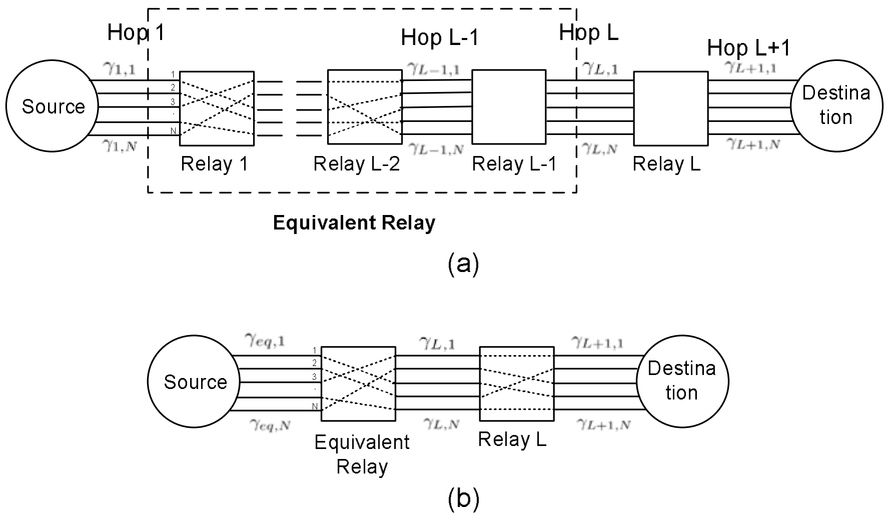

Fig. 3. Converting a $(L+1)$-hop network to a 3-hop network via an equivalent relay

Relay $L-1$, as shown in Fig. 3(b). Hence, from Propositon 2, the optimal SP is one where $\{\gamma_{eq,n}\}$ and $\{\gamma_{L,n}\}$ are sorted and paired at the equivalent relay, and $\{\gamma_{L,n}\}$ and $\{\gamma_{L+1,n}\}$ are sorted and paired at Relay L. Note that the sorted-SNR pairing at Relay L is independent of how the subcarriers are paired at the other relays.

Next, ignore Relay L and replace it by equivalent subcarriers from Relay $L-1$ to the destination. We now have a L-hop network. From the induction hypothesis, the sorted-SNR SP is optimal. In particular, the subcarriers at each of Relays $1, 2, \ldots, L-2$ are sorted by their SNR and paired. Since the equivalent SNR $\{\gamma_{eq,n}\}$ at the equivalent relay is computed by applying (1) or (3) over these sorted and paired subcarriers, it is easy to see that $\{\gamma_{L-1,n}\}$ and $\{\gamma_{eq,n}\}$ are ordered in the same way. Therefore, sorting and pairing $\{\gamma_{eq,n}\}$ and $\{\gamma_{L,n}\}$ at the equivalent relay is the same as sorting and pairing $\{\gamma_{L-1,n}\}$ and $\{\gamma_{L,n}\}$ at Relay $L-1$. ■

4 Jointly Optimal Subcarrier Pairing and Power Allocation: A Separation Principle

So far, given a fixed PA scheme, we have found that the optimal SP scheme for (9) is SNR based, which depends on the transmission power allocated to each subcarrier. We next present the solution for (7) by jointly optimizing SP and PA.

The apparent coupling of SP and PA makes a direct exhaustive search for the jointly optimal solution prohibitively complex. Instead, we will show that the joint optimization problem can be decoupled into two separate PA and SP subproblems. Specifically, we prove that the jointly optimal solution is obtained by pairing subcarriers based on the order of their *channel gains*, followed by optimal PA over the paired subcarriers.

This separation principle holds for a variety of scenarios, including AF and DF relaying under either total or individual power constraints. The solution obtained through such separate optimization bears drastically lower computational complexity in comparison with the exhaustive-search alternative.

Our argument for the separation principle is briefly summarized as follows. We first show that, at a global optimum, between two subcarriers, the subcarrier with a higher channel gain exhibits a larger SNR. This relation implies that the SNR-based ordering of subcarriers is the same as one based on channel gain. Hence, we conclude that the sorted SP scheme based on channel gain is optimal when PA is also optimized. In the following, the above argument is first verified for a multi-hop system consisting of two subcarriers ($N = 2$). We then explain how it can be extended to a system with an arbitrary number of subcarriers.

4.1 Two-Subcarrier Case ($N = 2$)

For both types of power constraint, the following proposition holds:

Proposition 4. *For $N = 2$, in the optimal PA and SP solution for (7), at each hop, the subcarrier with better channel gain also provides a higher received SNR, i.e., if $a_{m,i} \geq a_{m,j}$, then $\gamma^*_{m,i} \geq \gamma^*_{m,j}$, for $m = 1, \cdots, M$; $i, j \in \{1, 2\}$; and $i \neq j$.*

Proof. We prove the proposition by contradiction. Let $\mathcal{P}_1$ and $\mathcal{P}_2$ represent the two disjoint source-destination paths corresponding to the optimal SP scheme. Consider any hop m along these paths. Without loss of generality, let subcarrier 1 belong to $\mathcal{P}_2$, subcarrier 2 belong to $\mathcal{P}_1$, and $a_{m,1} \geq a_{m,2}$. Suppose at optimality $\gamma^*_{m,i} < \gamma^*_{m,j}$, i.e., $P^*_{m,2}a_{m,2} > P^*_{m,1}a_{m,1}$, where $P^*_{m,1}$ and $P^*_{m,2}$ are the power allocated to subcarriers 1 and 2, respectively. Let $P_{mt} = P^*_{m,1} + P^*_{m,2}$.

Consider the following alternate allocation of power between subcarriers 1 and 2 over hop m

$$\hat{P}_{m,1} = \frac{a_{m,2}}{a_{m,1}} P^*_{m,2}, \qquad \hat{P}_{m,2} = \frac{a_{m,1}}{a_{m,2}} P^*_{m,1}. \tag{19}$$

We further swap the two subcarriers so that subcarrier 1 belongs to path $\mathcal{P}_1$ and subcarrier 2 belongs to path $\mathcal{P}_2$. Since $\hat{P}_{m,1}a_{m,1} = P^*_{m,2}a_{m,2}$ and $\hat{P}_{m,2}a_{m,2} = P^*_{m,1}a_{m,1}$, the above procedure of power re-allocation and subcarrier swapping does not change the end-to-end rate. However, the sum power after the procedure is reduced:

$$\begin{aligned}\hat{P}_{m,1} + \hat{P}_{m,2} &= \frac{a_{m,2}}{a_{m,1}}(P_{mt} - P^*_{m,1}) + \frac{a_{m,1}}{a_{m,2}} P^*_{m,1} \\ &= \frac{a_{m,2}}{a_{m,1}} P_{mt} + \frac{(a_{m,1})^2 - (a_{m,2})^2}{a_{m,1}a_{m,2}} P^*_{m,1} \\ &< \frac{a_{m,2}}{a_{m,1}} P_{mt} + \frac{(a_{m,1})^2 - (a_{m,2})^2}{a_{m,1}a_{m,2}} \frac{a_{m,2}}{a_{m,1} + a_{m,2}} P_{mt} \\ &= P_{mt},\end{aligned} \tag{20}$$

where inequality (20) is obtained from our initial assumption that $P^*_{m,1}a_{m,1} < P^*_{m,2}a_{m,2}$, which can be rewritten as $P^*_{m,1} < \frac{a_{m,2}}{a_{m,1}+a_{m,2}} P_{mt}$, and that $a_{m,1} \geq a_{m,2}$. This contradicts our initial assumption that the original PA is globally optimal. ■

4.2 Multi-subcarrier Case ($N > 2$)

The result of Proposition 4 can be generalized to $N \geq 2$. We have the following separation principle for jointly optimal SP and PA.

Proposition 5. *The joint optimization of PA and SP in* (7) *is equivalent to the following separate optimization problem:*

$$\max_{\{\mathbf{P}_i\}} \left\{ \max_{\{\mathcal{P}_i\}} \sum_{i=1}^{N} R_{SD}(\mathcal{P}_i, \mathbf{P}_i) \right\}.$$

Furthermore, the optimal SP is independent of $\{\mathbf{P}^*\}$ *and is performed individually at each relay based on sorted channel gain.*

Proof. A similar proof by contradiction as it was used in Section 3.1 for Proposition 2 can be applied. We omit the details here.

5 Optimal Power Allocation for Multi-hop Relaying

So far we have obtained the optimal SP at all relays. We next find the optimal PA solution for a given SP scheme.

5.1 Individual Power Constraint

Without loss of generality, we assume $a_{m,1} \geq a_{m,2} \geq \cdots \geq a_{m,N}$ for all hops $m = 1, \cdots, M$. From Proposition 5, the subcarriers with the same index are paired, and a path with the optimal SP consists of all the same subcarrier index, *i.e.,* $\mathcal{P}_i^* = (i, \cdots, i)$.

Hence, for DF relaying, the source-destination sum rate in (6) reduces to

$$R_t^{\mathrm{DF}} = \frac{1}{F_s} \sum_{n=1}^{N} \min_{m=1,\cdots,M} \log_2(1 + P_{m,n} a_{m,n}). \tag{21}$$

Maximizing (21) over $\{P_{m,n}\}$ under individual power constraints in (5) can be cast into the following optimization problem using a set of auxiliary variables $\mathbf{r} = [r_1, \cdots, r_N]^T$:

$$\begin{aligned} &\max_{\mathbf{r},\mathbf{P}} \frac{1}{F_s} \sum_{n=1}^{N} r_n && (22) \\ &s.t. \quad i) \quad r_n \leq \log_2(1 + P_{m,n} a_{m,n}), \quad m = 1, \cdots, M, \quad n = 1, \cdots, N \\ &\qquad ii) \quad \sum_{n=1}^{N} P_{m,n} = P_{mt}, \quad m = 1, \cdots, M \\ &\qquad iii) \quad P_{m,n} \geq 0 \quad m = 1, \cdots, M, \quad n = 1, \cdots, N \end{aligned}$$

where $\mathbf{P} \triangleq [P_{m,n}]_{M \times N}$. Since the objective function is linear, and all the constraints in problem (22) are convex, the optimization problem in (22) is convex. We can obtain the optimal solution by employing the standard convex optimization tools, and therefore we omit the details.

Unlike DF relaying, the achievable sum rate for the case of AF relaying is not generally concave in $P_{m,n}$. Therefore, we have a non-convex optimization problem formulated as

$$\max_{\mathbf{P}} \frac{1}{F_s} \sum_{n=1}^{N} \log_2 \left(1 + \left(\prod_{m=1}^{M} \left(1 + \frac{1}{P_{m,n} a_{m,n}} \right) - 1 \right)^{-1} \right) \tag{23}$$

$$s.t. \quad i) \quad \sum_{n=1}^{N} P_{m,n} = P_{mt}, \quad m = 1, \cdots, M$$

$$ii) \quad P_{m,n} \geq 0.$$

A non-convex optimization solver may be applied to produce the optimal solution, or we may resort to a suboptimal solution as follows. Instead of the exact rate formula in (23), based on (2), an upper-bound approximation can be used, which is given by

$$R_t^{\text{up}} = \frac{1}{F_s} \sum_{n=1}^{N} \log_2 \left(1 + \left(\sum_{m=1}^{M} \frac{1}{P_{m,n} a_{m,n}} \right)^{-1} \right). \tag{24}$$

Proposition 6. *R_t^{up} in* (24) *is concave with respect to* $\{P_{m,n}\}$.

Proof. The proof follows from the concavity of (2) with respect to $\{P_{m,n}\}$, which can be shown by considering its Hessian matrix. The details are omitted. ■

Given Proposition 6, the optimization of $\{P_{m,n}\}$ to maximize R_t^{up} is a convex optimization problem, and we again may resort to the standard convex optimization tools to obtain the solution.

5.2 Total Power Constraint

With the same assumption as in the previous section, we have $a_{m,1} \geq a_{m,2} \geq \cdots, a_{m,N}$ for all hops $m = 1, \cdots, M$, and $\mathcal{P}_i^* = (i, \cdots, i)$. Let P_i be the total power allocated along path $\mathcal{P}_i^*$. We can define an equivalent channel gain, $a_{eq,i}$, corresponding to the maximum achievable end-to-end SNR by optimally allocating P_i among the subcarriers on this path:

$$a_{eq,i} = \frac{\max_{\mathbf{P}_i} \gamma_{\text{SD}}(\mathcal{P}_i, \mathbf{P}_i)}{P_i}. \tag{25}$$

For DF relaying, the equivalent channel gain corresponding to path $\mathcal{P}_i^*$ is given by [10]

$$a_{eq,i}^{DF} = \left(\sum_{m=1}^{M} \frac{1}{a_{m,i}} \right)^{-1}, \quad i = 1, \cdots, N, \tag{26}$$

where the two following facts have been applied: 1) the end-to-end rate of a path $\mathcal{P}_i^*$ is equal to the minimum rate over a subcarrier of that path; 2) to maximize the end-to-end rate on one path, the total power allocated to the path must be shared among the subcarriers on this path such that all subcarriers exhibit the same SNR. For AF relaying, the equivalent channel gain for path $\mathcal{P}_i^*$ can be expressed as

$$a_{eq,i}^{AF} = \left(\sum_{m=1}^{N} \frac{1}{\sqrt{a_{m,i}}} \right)^{-2}, \quad i = 1, \cdots, N, \tag{27}$$

where the approximation of end-to-end SNR in (2) over each path is used.

Then, power allocation among the subcarriers over all hops essentially reduces to the problem of power allocation among the paths. This problem then can be formulated as

$$\max_{P_1, \cdots, P_N} \quad \frac{1}{F_s} \sum_{i=1}^{N} \log_2 (1 + P_i a_{eq,i}) \tag{28}$$
$$s.t. \quad \sum_{i=1}^{N} P_i = P_t, \quad P_i \geq 0 \quad i = 1, \cdots, N \, .$$

This problem has the classical water-filling solution

$$P_i^* = \left[\frac{1}{\lambda F_s \ln 2} - \frac{1}{a_{eq,i}} \right]^+ , \tag{29}$$

where the Lagrange multiplier λ is chosen such that the power constraint in (28) is met.

6 Simulation Results

In this section, through Monte-Carlo simulations, we first examine the performance of the optimal SP scheme when a power allocation solution is given. We then compare the performance of the joint optimal SP and PA with other suboptimal SP and PA schemes.

For both DF and AF relaying strategies, we consider a multi-hop OFDM-based cooperative network with $N = 64$ and $M = 4$, and the direct transmission between the source and destination is not available. The source, the three relays, and the destination are placed on a straight line with equal distance from each other. The distance between every two nodes is denoted by d_r. The spatial reuse factor is set to $F_s = 3$. A frequency-selective fading channel is simulated, using the L-tap filter model [15] with $L = 11$. Moreover, the pathloss exponent of 3 and the noise variance of 10^{-4} are assumed throughout the simulation.

Uniform power allocation with sum power 100W at each node ($i.e.$, $\sum_{n=1}^{N} P_{m,n} = 100, m = 1, \cdots, 4$) is assumed for both Figs. 4 and 5. Fig. 4 depicts the performance of the achievable end-to-end sum rates vs. d_r for AF relaying under the sorted-SNR SP scheme and direct pairing scheme (*i.e.,* the indices for the incoming and outgoing subcarriers are the same). We observe that a gain of more than 10% can be achieved by conducting optimal pairing compared with direct pairing. Similarly, for DF relaying,

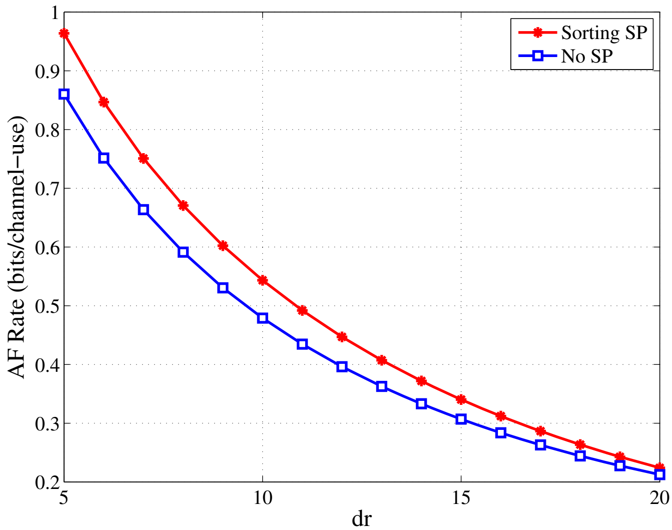

Fig. 4. Rate vs. inter-nodes distance under optimal and suboptimal SP schemes for a multi-hop AF OFDM network with $M = 4$ and $N = 64$

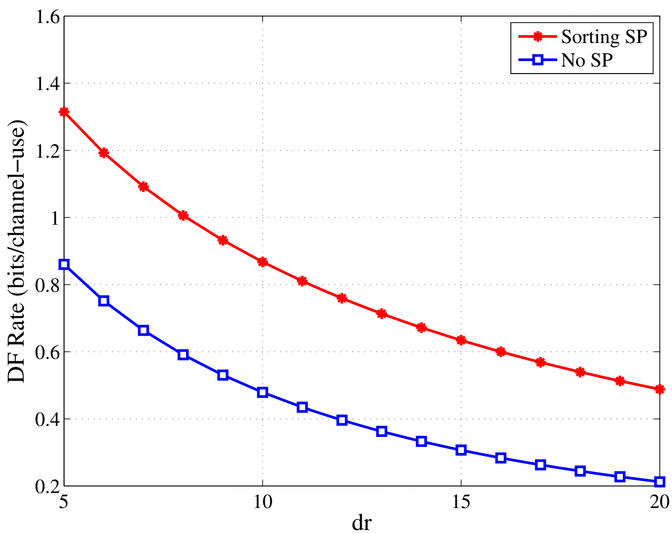

Fig. 5. Rate vs. inter-nodes distance under optimal and suboptimal SP schemes for a multi-hop DF OFDM network with $M = 4$ and $N = 64$

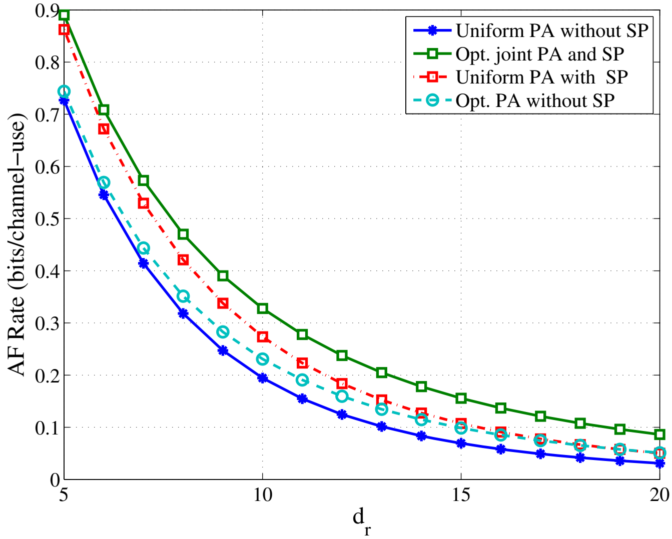

Fig. 6. Rate vs. inter-nodes distance under different SP and PA schemes with total power constraint $P_t = 100W$ for a multi-hop OFDM AF network with $M = 4$ and $N = 64$

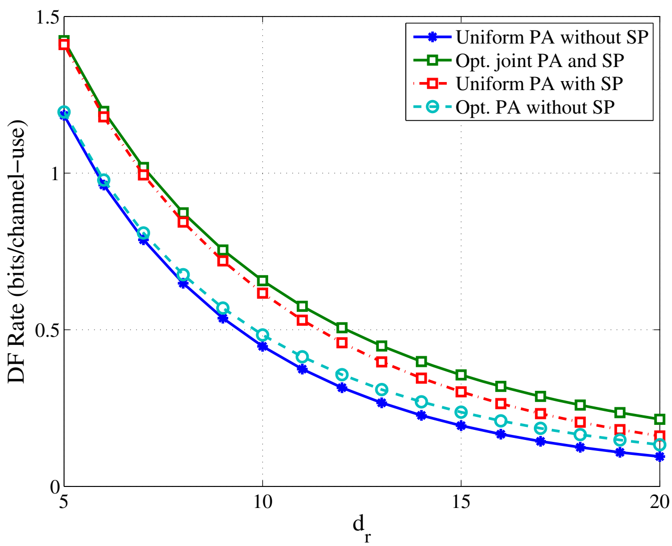

Fig. 7. Rate vs. inter-nodes distance under different SP and PA schemes with total power constraint $P_t = 100W$ for a multi-hop OFDM DF network with $M = 4$ and $N = 64$

Fig. 5 plots the achievable rates vs. d_r. We observe that the gain is even more significant, from 25% when d_r is small to over 150% when d_r is large.

Optimal power allocation leads to further performance improvement when combined with subcarrier pairing. Figs. 6 and 7 illustrate the total achievable rates vs. d_r for AF and DF relaying, respectively, for a total power constraint of 100W ($i.e.$, $\sum_{m=1}^{M} \sum_{n=1}^{N} P_{m,n} = 100$). We compare the end-to-end rate of jointly optimized SP and PA with that of suboptimal alternatives including 1) *Opt. PA without SP*, where only water-filling power allocation is applied; 2) *Uniform PA without SP*, where the total power is uniformly distributed among the N paths obtained from direct subcarrier pairing; 3) *Uniform PA with SP*, where the total power is uniformly distributed among the N paths obtained from the optimal sorted SP. We observe that, through joint optimization of PA and SP, a significant performance improvement can be obtained for both the DF and AF multi-hop relaying cases, especially when the SNR is low, e.g., when d_r is large. Similar observations are made when we increase the number of relays between the source and the destination. Those results are omitted to avoid redundancy.

7 Conclusion

In this paper, we have studied the problem of jointly optimizing spectrum and power allocation to maximize the source-to-destination data rate for a multi-channel M-hop relaying network. OFDM relays are used as example. For a fixed power allocation, we have shown that the general SP problem over multiple hops can be decomposed into $(M-1)$ independent SP problems at each relay, where the sorted-SNR scheme is optimal. We then proved that a jointly optimal solution for the SP and PA problems can be achieved by decomposing the original problem into two separate PA and SP problems solved independently. The solution obtained through the separate optimization bears considerably lower computational complexity compared with exhaustive-method alternatives. The separation principle was shown to hold for a variety of scenarios including AF and DF relaying strategies under either total or individual power constraints. For all these scenarios, the optimal SP scheme maps the subcarriers according to their channel gain order, independent of the optimal PA solution. From simulation, we observed that significant gains in data rate can be achieved from employing jointly optimal SP and PA in multi-channel multi-hop relaying, especially when the networking environment is challenging.

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