Research on the Interferometer Direction-Finding and Positioning Improved Algorithm under the Influence of Ground-to-Air Channel

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Abstract. Based on analyses multipath fading of ground-to-air channel under different elevation angle circumstances, according to the characteristic and change range of ground-to-air channel Rice factor determining Weighted value, the direction-finding and positioning algorithm which airborne interferometer and spaceborne interferometer usually adopting is improved under the influence of different elevation angle ground-to-air channel, the simulation results have verified improvement algorithm validity in advancing positioning precision.

Keywords: Ground-to-air channel, Direction-finding and positioning, Weighed value.

1 Introduction

In the interferometer direction-finding and positioning system, the ground-to-air channel transmission usually cause the parameter measure error of receiving signal, but [1][2] studied interferometer direction-finding and positioning based on the perfect electromagnetic wave transmission circumstances, it did not accord with the actual application, [3][4]proposed the method of join weighted value in order to improve positioning precision, but how to determine weighted value is not presented, [5][6]mentioned the weighted value determining method, but the concretely applied occasion is not distinguished.

In this paper, according to transmitting signal actual fading degree in the different elevation angle ground-to-air channel transmission process sorting channel type, aiming at specific circumstances of different interferometer carrier propose weighted value determining method, airborne interferometer and spaceborne interferometer direction-finding and positioning algorithm have improved under the influence of different elevation angle ground-to-air channel through parameter measure is affected by different Rice factor.

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2 Characteristics of Multipath Fading in Ground-to-Air Communications

In ground-to-air communications, the causes of multipath mainly from three aspects: ① scattering from the surface feature and topography near the ground station; ②aerial carrier reflection or scattering; ③ remote ground barrier reflection. Usually, the degree of multipath fading in ground-to-air communications has involved with relative elevation angle of the aerial carrier and ground station, according to the different elevation angle, ground-to-air channel can be described by four types, such as Fig. 1.

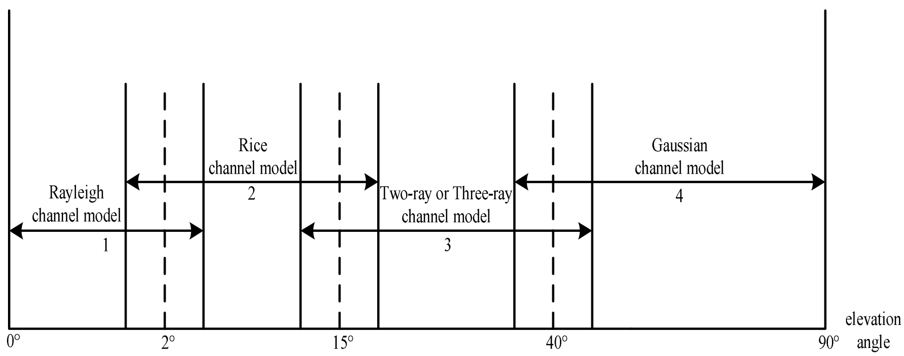

Fig. 1. Types of ground-to-air channel in different elevation angle

For the airborne interferometer, elevation angle of target radiation relative to aircraft will not be big in order to ensure enough distance of reconnaissance direction-finding and avoid the enemy fire. In Fig. 1, the ground-to-air channel is called Rice channel while the elevation angle is small. (Rayleigh channel is a special case of Rice channel), Rice factor K is defined as the ratio between the direct component power and all scattering wave power, which completely determines the degree of multipath fading. The direct component power and all scattering wave power can express respectively

$$2\sigma^2 = P\frac{1}{K+1} \text{ , } |\rho|^2 = P\frac{K}{K+1} \tag{1}$$

Where P is total power of receiving signal.

Rice factor of ground-to-air channel under the low elevation is usually in the interval of 2-15dB[7]. For the spaceborne interferometer, the elevation angle of target radiation relative to satellite is observable, Rice factor K is big, and ground-to-air channel will tend to Gaussian channel from the Rice channel.

The Power Spectrum of Angle of Arriving φ (AOA) submit to three special distribution: even distributing, Gauss distribution and Laplace distribution. Due to the von Mises distribution can approach else distribution, so expression of von Mises distribution is

$$P_A(\varphi) = \frac{1}{2\pi I_0(u)} e^{u\cos(\varphi - \bar{\varphi})}, \ \varphi \in [-\pi, \pi] \tag{2}$$

where I_0 is the zero-order modified Bessel function, parameter $u \geq 0$ control the shape of Power Spectrum of Angle. When $u = 0$, $P_A(\varphi) = \frac{1}{2\pi}$; When $u = \infty$, $P_A(\varphi) = \delta(\varphi)$,where $\delta(\varphi)$ is Dirac function; when u is small, von Mises distribution approach cosine distribution; when u is large, von Mises distributing approach the Gauss distribution of mean $\bar{\varphi}$ and variance $\frac{1}{u}$.

3 Improved Direction-Finding Positioning Algorithm Used in Airborne Interferometer

Under the low elevation, the height of aircraft can be ignored, Namely that the aircraft and ground target radiation are horizontal, aircraft will measure the azimuth of the target radiation in different locations, and achieve positioning using the intersection of azimuth line[1].

3.1 Weighted Least Squares Estimates (WLSE) for Positioning

In order to improve the positioning accuracy, usually Least Squares Estimates (LSE) is used to process azimuth measure data, however, the error variance of azimuth measure is different in diverse channel state, so the location of target radiation can be estimated by Weighted Least Squares Estimates (WLSE). On the assumption that the actual position coordinates of target radiation is (x, y), Coordinates of ith observation point of aircraft is (x_i, y_i), measure azimuth is α_i, then α_i can be calculated as follow,

$$\alpha_i = h_i(x, y) + v_i \tag{3}$$

where

$$h_i(x, y) = \arctan\left(\frac{y - y_i}{x - x_i}\right),$$

v_i is ith measure error. The initial estimate point (x_0, y_0) of formula (3) in the target position (x, y) is linearized by Taylor series expansion, and retain the first and second items, then formula（3） can be changed into formula (4),

$$\alpha_i - h_i(x_0, y_0) = \left[\frac{\partial h_i}{\partial x} \quad \frac{\partial h_i}{\partial y}\right]\Bigg|_{(x,y)=(x_0,y_0)} \times \begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix} + v_i \tag{4}$$

where $\left.\frac{\partial h_i}{\partial x}\right|_{(x,y)=(x_0,y_0)} = -\frac{y_0 - y_i}{r_{0i}^2}$, $\left.\frac{\partial h_i}{\partial y}\right|_{(x,y)=(x_0,y_0)} = \frac{x_0 - x_i}{r_{0i}^2}$, $r_{0i}^2 = (x_0 - x_i)^2 + (y_0 - y_i)^2$ 。

All observation points are put into a matrix, observation equation can be obtained

$$\mathbf{Z} = \mathbf{H} \bullet \mathbf{W} + \mathbf{V} \tag{5}$$

where

$$\mathbf{Z} = \begin{bmatrix}\alpha_1 - h_1 \\ \alpha_2 - h_2 \\ \vdots \\ \alpha_n - h_n\end{bmatrix}\Bigg|_{(x,y)=(x_0,y_0)}$$

$$\mathbf{H} = \begin{bmatrix}\frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} \\ \frac{\partial h_2}{\partial x} & \frac{\partial h_2}{\partial y} \\ \vdots & \vdots \\ \frac{\partial h_n}{\partial x} & \frac{\partial h_n}{\partial x}\end{bmatrix}\Bigg|_{(x,y)=(x_0,y_0)}$$

$$\mathbf{W} = \begin{bmatrix}\Delta x \\ \Delta y\end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix}v_1 \\ v_2 \\ \vdots \\ v_n\end{bmatrix}$$

Cost function of WLSE can be obtained as

$$\boldsymbol{\varepsilon} = \mathbf{V}^{\mathrm{T}}\mathbf{P}\mathbf{V} \tag{6}$$

where $\mathbf{P}$ is weighted matrix. （6）can also express

$$\boldsymbol{\varepsilon} = [\mathbf{Z} - \mathbf{H}\bullet\mathbf{W}]^{\mathrm{T}}\mathbf{P}[\mathbf{Z} - \mathbf{H}\bullet\mathbf{W}] \tag{7}$$

Order $\frac{\partial \boldsymbol{\varepsilon}}{\partial \mathbf{W}} = 0$, so

$$\hat{\mathbf{W}} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{P}\mathbf{Z} \tag{8}$$

On the assumption that measure error of each observation point is independent and mean is null, variance is σ_i^2 $(i = 1, 2, ..., n)$, then the matrix $\mathbf{P}$ meet the following condition,

$$\mathbf{P} = \mathbf{J}^{-1} \tag{9}$$

where

$$\mathbf{J} = diag\left[\sigma_1^2 \quad \sigma_2^2 \quad \cdots \quad \sigma_n^2\right], (i = 1, 2, ..., n)$$

On the assumption that the actual position coordinates and estimation position coordinates of target radiation is $\mathbf{X} = [x, y]^{\mathrm{T}}$ and $\tilde{\mathbf{X}} = [\tilde{x}, \tilde{y}]^{\mathrm{T}}$ respectively, so

$$\mathrm{E}\left[\hat{\mathbf{X}} - \mathbf{X}\right] = \mathrm{E}\left[\hat{\mathbf{W}} - \mathbf{W}\right] = \mathrm{E}\left[\left(\mathbf{H}^{\mathrm{T}}\mathbf{J}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{J}^{-1}\mathbf{V}\right] = \mathbf{H}^{-1}\mathrm{E}[\mathbf{V}] = 0 \tag{10}$$

The expression（10）prove that WLSE is agonic estimation algorithm when the mean of measure error equal to zero.

3.2 Weighted Value Determination under the Influence of the Ground-to-Air Channel

Measure error under the influence of Rice channel exist CRAMER-RAO bound, for the uniform linear array, The CRAMER-RAO bound is[8]

$$\mathrm{var}(\alpha) \geq \frac{12}{(2\pi)^2 K \times \frac{M+1}{M-1}(\frac{L}{\lambda})^2 \sin^2\alpha} \tag{11}$$

Where K is Rice factor; M is the number of antenna array elements; L is the total length of the line array; λ is the wavelength of the received wave; α is the azimuth of the received wave (the angle between the received wave and the direction–finding baseline). If using two mutual vertical linear arrays, the lesser one can be selected as the CRAMER-RAO bound of the actual direction–finding system. In formula (11), the weighted value of WLSE for positioning under the influence of the ground-to-air channel can be obtained by estimating the Rice factor K.

The Rice factor K can be estimated by the lookup table of the first and second-order moment information, not only the estimation precision is well, but also real time can be met. The expressions of Rice distribution random order moment is[9]

$$\mu_n = \mathrm{E}[r^n] = (2\sigma^2)^{n/2} \exp(-K)\Gamma(1+n/2)\,{}_1F_1(1+n/2;1;K) \quad (12)$$

where $\Gamma(\bullet)$ is Gamma function, ${}_1F_1(\bullet)$ is Confluent hypergeometric function. The following formula can be obtained according to first and second-order moment information

$$\frac{\mu_1^2}{\mu_2} = \frac{\pi e^{-K}}{4(K+1)}[(K+1)\,\mathrm{I}_0(\frac{K}{2}) + K\,\mathrm{I}_1(\frac{K}{2})]^2 \quad (13)$$

where $\mathrm{I}_0(\bullet)$ and $\mathrm{I}_1(\bullet)$ are the first and the zero-order modified Bessel function respectively.

Because the Rice factor K of ground-to-air channel under the low elevation is usually in a certain range, it is easier to obtain the value of K by lookup table than by equation (13), namely that μ_1 and μ_2 are obtained by calculating the formula (13) with the value of K in a certain range, and compare them with the measured μ_1 and μ_2, then take the most similar value of K in the table as estimation result.

4 Improved Positioning Algorithm Used in Spaceborne Interferometer

For the spaceborne interferometer, positioning can be achieved by measuring the azimuth and the elevation of target radiation on the ground. The positioning geometry model is shown in Fig. 2.

In Fig. 2, r is the line of sight(connection between the receiving antenna O and the target radiation T on the ground)distance, β_x and β_y is the angle between the line of sight with the X-axis and Y-axis respectively, h is the height of the satellite, (x, y) is the coordinates of the target radiation on the ground, and the length of the two direction-finding baselines $d_x = d_y = d$, then

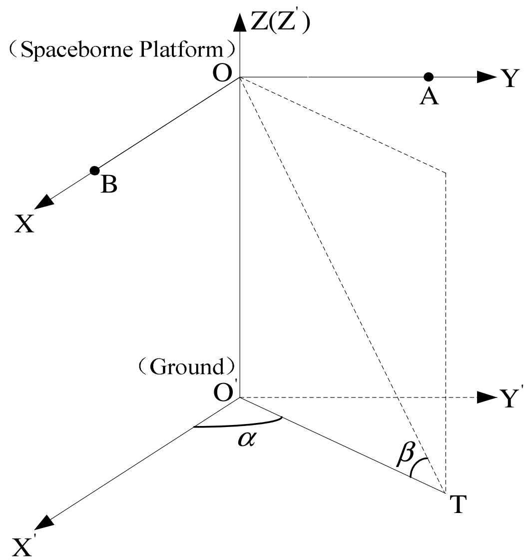

Fig. 2. Spaceborne interferometer positioning sketch map

$$\begin{cases} \varphi_x = 2\pi d \cos\beta_x / \lambda \\ \varphi_y = 2\pi d \cos\beta_y / \lambda \\ x = r\cos\beta_x \\ y = r\cos\beta_y \\ r = \sqrt{x^2 + y^2 + h^2} \end{cases} \tag{14}$$

The target radiation can obtain from formula (14)

$$\begin{cases} x = \left[\dfrac{h^2 k_x^2}{1 - k_x^2 - k_y^2} \right]^{1/2} \\ y = \left[\dfrac{h^2 k_y^2}{1 - k_x^2 - k_y^2} \right]^{1/2} \end{cases} \tag{15}$$

where

$k_x = \cos\beta_x = \lambda\varphi_x / 2\pi d$, $k_y = \cos\beta_y = \lambda\varphi_y / 2\pi d$

On the assumption that the satellite gets n positioning points (x_i, y_i) $(i = 1, 2, ..., n)$ during the move, generally the n positioning points are fused by the average method [2], that is,

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = \frac{1}{n} \sum_{i=1}^{n} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \tag{16}$$

4.1 The Influence of Ground-to-Air Channel on the Average Fusion Method of the Positioning Points

When fusing the positioning points by the average method, if the distribution of the positioning point (x_i, y_i) is agonic, the positioning point estimation $(\tilde{x}, \tilde{y})$ is agonic too, the estimation result is optimal, whereas, if the distribution of the positioning point is biased, a great error will exist in the fusion of the several positioning points by the formula (16). The demonstration about biased distribution of the positioning point under the influence of the ground-to-air channel is present as following.

For simplicity, only show demonstration that whether x is biased, and the demonstration of y biased result is similar. Set $f_x(k_x, k_y) = x$, which is in second order Taylor series expansion, then

$$\begin{aligned} f_x(k_x,k_y) = {} & f_x(k_{x_0},k_{y_0}) + \left.\frac{\partial f_x(k_x,k_y)}{\partial k_x}\right|_{(k_x,k_y)=(k_{x_0},k_{y_0})} (k_x - k_{x_0}) + \left.\frac{\partial f_x(k_x,k_y)}{\partial k_y}\right|_{(k_x,k_y)=(k_{x_0},k_{y_0})} (k_y - k_{y_0}) \\ & \left.\frac{\partial^2 f_x(k_x,k_y)}{2\cdot \partial k_x^2}\right|_{(k_x,k_y)=(k_{x_0},k_{y_0})} (k_x - k_{x_0})^2 + \left.\frac{\partial^2 f_x(k_x,k_y)}{2\cdot \partial k_y^2}\right|_{(k_x,k_y)=(k_{x_0},k_{y_0})} (k_y - k_{y_0})^2 \end{aligned} \tag{17}$$

In the case of the large line of sight component, the phase difference $\varphi_x \sim N(\varphi_{x_0}, \sigma_{\varphi_x}^2)$ and $k_x \sim N(k_{x_0}, \sigma_{k_x}^2)$ gain mean by formula (17) is

$$\mathrm{E}(x) = f_x(k_{x_0}, k_{y_0}) + a\mathrm{E}[(k_x - k_{x_0})^2] + b\mathrm{E}[(k_y - k_{y_0})^2] \tag{18}$$

In formula (18), a and b is the second-order item coefficient respectively, because the mean of first-order item is zero and the mean of second-order item is biased, so the distribution of x is biased.

4.2 Improved Fusion Algorithm of the Positioning Points

Due to the measured position points distribution is biased under the influence of ground-to-air channel, therefore, two issues need to be solved when fusing the positioning points in order to improve positioning accuracy: ①How to select the appropriate positioning point in fusion; ②How to determine the effective ways to fuse.

Due to the satellite during the move would not gain too many positioning points of target, in the case of the positioning points distribution is biased, it is not the more positioning points are involved in fusion, the error variance is smaller after fusion, if there is a positioning point with great error variance(maybe the biased degree is much larger than the variance value), the total positioning accuracy will be worse after fusion. So the positioning points should be filtered according to the criterion that "the muster of the points whose total positioning error variance are minimum", the "total error variance" formula which contain m positioning points can be expressed as:

$$T_m = \frac{1}{m^2}\sum_{i=1}^{m}\sigma_i^2 , \quad m = 1, 2, ..., n \tag{19}$$

where n is the total number of the positioning points, σ_i^2 is the variance of the ith positioning point in the muster. In the calculation process, all positioning points can be sorted according to error variance from small to large, then calculate T_m of the former m positioning points, the positioning points involved in fusion can be obtained through n times calculation only. For the fixed target radiation in the ground, σ_i can be substituted for the square root of the positioning error summation in horizontal and vertical direction(namely $\text{GDOP}=(\sigma_{xi}^2+\sigma_{yi}^2)^{1/2}$), the positioning points which are involved in fusion can be obtained by gaining the muster which have the minimum value of T_m.

After obtaining positioning points involved in fusion, for each positioning point (x_i, y_i), the positioning errors in the direction of x and y axis are composed of two parts respectively, one is the variance $(\sigma_{1i}^2, \sigma_{2i}^2)$ formed by the systematic bias between estimate and true values, the other is the variance $(\sigma_{xi}^2, \sigma_{yi}^2)$ formed by the spreading positioning points, therefore, the positioning error in the direction of two axis is $\tilde{\sigma}_{xi}^2 = \sigma_{1i}^2 + \sigma_{xi}^2$ and $\tilde{\sigma}_{yi}^2 = \sigma_{2i}^2 + \sigma_{yi}^2$ respectively. The positioning error variance after weighted fusion is minimum, through the determining method for optimized weighted value which based on the distribution of the positioning error variance, the method of weighted value determination is:

$$\omega_i = \frac{1}{(\tilde{\sigma}_{xi}^2 + \tilde{\sigma}_{yi}^2)\bullet\sum_{j=1}^{n} 1\big/(\tilde{\sigma}_{xj}^2 + \tilde{\sigma}_{yj}^2)} \tag{20}$$

Therefore, the result after fusion is:

$$\text{Loc}=\sum_{i=1}^{l}\omega_i \begin{bmatrix} x_i^{'} \\ y_i^{'} \end{bmatrix} \tag{21}$$

where l is the number of the positioning points after filter, $\begin{bmatrix} x_i^{'} \\ y_i^{'} \end{bmatrix}$ is the ith positioning point after filter.

5 Simulation Results

5.1 Power Spectrum of Angle of von Mises Distributing

When $\overline{\varphi} = 0^{\circ}$, $v = 200m/s$, carrier wave frequency $f_c = 450\text{MHz}$, Power Spectrum of Angle of von Mises distribution with different u is shown in Fig.3.

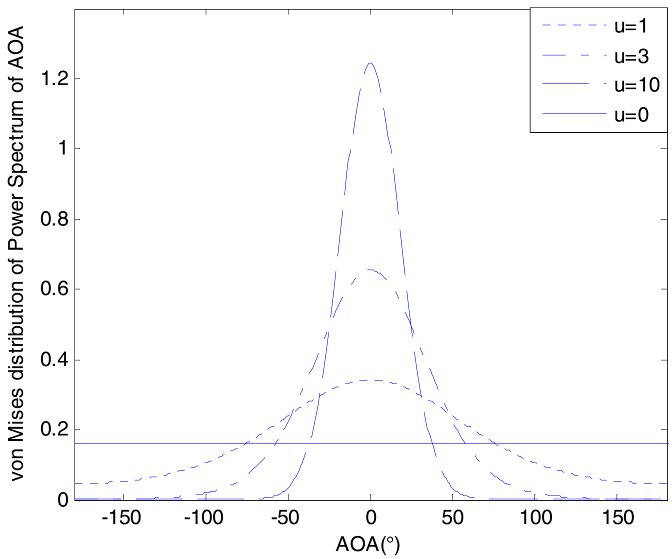

Fig. 3. von Mises distribution of Power Spectrum of AOA in Ground-to-Air communications

It can be seen from Fig.3 that, the AOA is changing more and more centralized with the u increasing.

5.2 Comparison of Simulation Results for Different Direction-Finding Positioning Algorithm Used in Airborne Interferometer

Taking the starting point of the aircraft as origin and the direction of aircraft flight as x axis, the rectangular coordinates can be established. The real position for target radiation has a coordinate of (150km, 150km), the aircraft flies along the horizontal direction, same as the positive direction of x coordinate axis, and performs direction-finding and localization for the target radiation. One dimension uniform linear array is employed in the airborne interferometer direction-finding system and the ratio between the interval distance d of linear array and the wavelength λ of the received wave is 0.5. The Rice factor K in measurement of the azimuth angle for the observation point submits to a random distribution in the interval of [10, 20]. The position of the target radiation is estimated by 200 iteration computation using the WLSE and LSE respectively. The simulation results are shown in Fig.3.

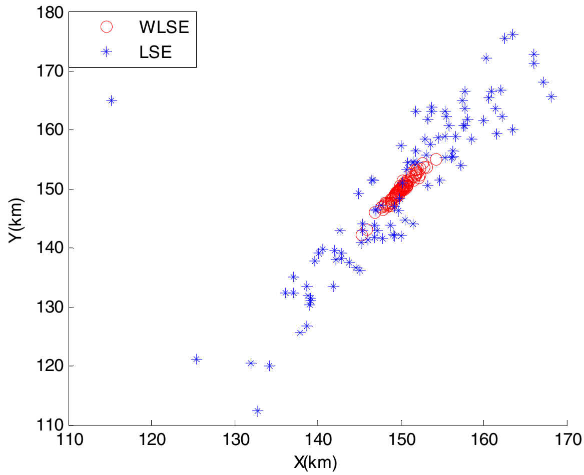

Fig. 4. Simulation results for position of the target radiation using the WLSE and the LSE

It can be seen from Fig.4 that, compared to the estimation results when using the LSE algorithm, a faster convergence speed and a smaller variance can be obtained when using the method of WLSE algorithm which determining the weighted value based on the estimation of channel Rican factor K.

5.3 Comparison of Simulation Results for Different Direction-Finding Positioning Algorithm Used in Spaceborne Interferometer

Assuming that the Rican factor K submits to a random distribution in the interval of [50,100], and the height of the satellite is 150km, the coordinate can be established as shown in Fig 2. The satellite move along the positive direction of Y coordinate axis, and generates a positioning point every 1000m. The total number of positioning point is 20. The azimuth angle of target radiation relative to the starting point of satellite $\alpha = 30^\circ$, and the elevation angle $\beta = 45^\circ$ (that is to say the real position of target is (130000m, 75000m)). 100 iteration Monte Carlo experiments are performed on the average fusion and weighted fusion, and the simulation results are shown in Fig. 4.

It can be seen from Fig.4 that, the positioning precision of weighted fusion method when taking the influences of different Rican factor is better than that of average fusion method, and the weighted fusion method is validated. Table 1 lists the average positioning error when positioning different target points using the average fusion method and the weighted fusion method respectively, and it can be seen from Table 1 that the larger the distance of target position correspond the larger the average positioning error, and the advantages of weighted fusion method is more significant.

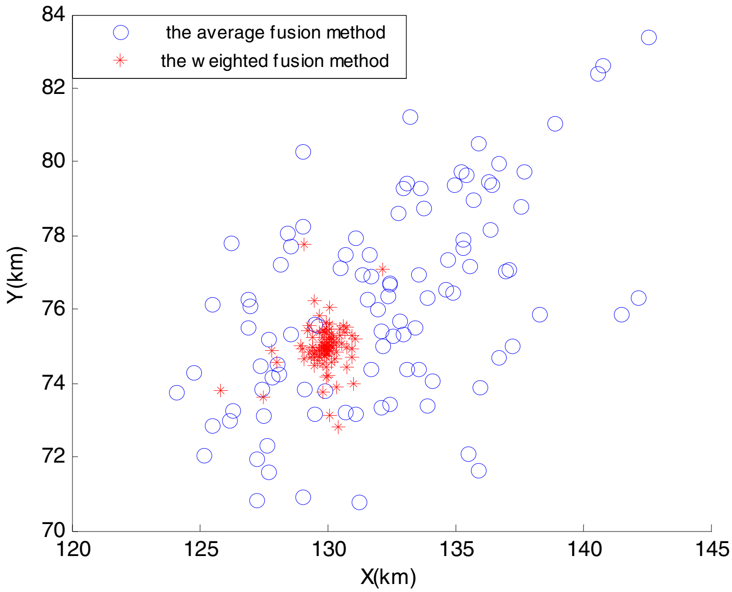

Fig. 4. Estimation results of target point using the average fusion method and the weighted fusion method

Table 1. The different target location points average position error using the average fusion method and the weighted fusion method respectively

Real Position of Target(m)	Position Error of Average Fusion Method(m)	Position Error of Weighted Fusion Method(m)
T_1(130000, 75000)	(4503.4, 3159.8)	(998.5, 774.8)
T_2(100000, 50000)	(3195.6, 2408.7)	(732.8, 646.5)
T_3(100000, 50000)	(5643.8, 4625.9)	(1117.2, 864.7)
T_4(100000, 50000)	(7986.2, 6699.1)	(1266.1, 965.6)

6 Conclusion

Aiming at parameter measure is affected by different channel type, this paper adopts Weighted Least Squares Estimates and Weighted positioning points fusion algorithm through Rice factor of different type channel determining weighted value in order to improve interferometer direction-finding positioning precision, the simulation results show that weighted algorithm of considering channel influence advance the position precision effectively.

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