Multichannel Opportunistic Spectrum Access in Fading Environment Using Optimal Stopping Rule

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Abstract. This paper studies the tradeoff between throughput and multichannel diversity in multichannel opportunistic spectrum access (OSA) systems. We explicitly consider channel condition as well as the activities of the primary users. We assume that the primary users use the licensed channel in a slotted fashion and the secondary users can only explore one licensed channel at a time. The secondary users then sequentially explore multiple channels to find the best channel for transmission. However, channel exploration is time-consumed, which decreases effective transmission time in a slot. For single secondary user OSA systems, we formulate the channel exploration problem as an optimal stopping problem with recall, and propose a myopic but optimal approach. For multiple secondary user OSA systems, we propose an adaptive stochastic recall algorithm (ASRA) to capture the collision among multiple secondary users. It is shown that the proposed solutions in this paper achieve increased throughput both the scenario of both single secondary user as well as multiple secondary suers.

Keywords: opportunistic spectrum access, multichannel diversity, optimal stopping rule.

1 Introduction

Recently, opportunistic spectrum access (OSA) has drawn great attention [1-4], for it has been regarded as a promising solution to the problem of spectrum shortage that wireless communication systems are facing today. In OSA, there are two types of users. One is the primary user (PU) which is the licensed owner of the spectrum, and the other is the secondary user (SU) which is allowed to transmit on the licensed spectrum at a particular time and location when and where the PUs are not active [5].

We consider an OSA system consisting multiple licensed channels and multiple SUs. Due to hardware limitation, the SUs can only sense one channel at a time. The key concern in such a system is designing a spectrum decision rule with which the SUs use the licensed channels. An intuitive approach is that the SUs sense the licensed channels according to the decent order of channel idle

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probabilities [6]. However, only considering the presence of the primary users is not enough in practical systems. The reason is that different channels always have different quality, which eventually results in different transmission rate. To capture the variation in channel conditions, a more reasonable approach is considering channel condition as well as the activities of PUs when designing spectrum decision rule in OSA systems. To achieve this, the SUs are required to detect the activities of PUs and estimate channel conditions. For simplicity, we use the term channel exploration to indicate the tasks of detecting the activities of the PUs and estimating channel conditions.

In this paper, the SUs are assumed to have slotted transmission structure, and can explore multiple licensed channels according to a particular order in a slot. Specifically, the SUs have to spend time to perform efficient and reliable channel exploration before they transmit data. Obviously, as the number of explored channels increases, the multichannel gain increases. However, since channel exploration is time-consumed, the effective transmission time in a slot decreases. Thus, there is a fundamental tradeoff between multichannel diversity and throughput. To address this tradeoff, we formulate the channel exploration problem as an optimal stopping problem with recall. The key idea of our approach can be described as follows. After exploring each licensed channel, each secondary user decides (1) to continue to explore the residual licensed channels or (2) to stop channel exploration and choose one previous explored idle channel with the highest channel condition to transmit. The goal of SUs is to choose a time to stop to maximize their expected throughput.

It should be mentioned that optimal stopping rule has been widely applied to OSA systems. For instance, in [7], the authors formulated the spectrum sensing as an optimal stopping problem by taking the hardware constraint into consideration. In [8], the authors presented a sequential spectrum sensing algorithm, which explicitly takes into account the sensing overhead. The considered problem can be solved by the method of optimal stopping rule. In [9], optimal strategies for determining which channels to probe, in what sequence and which channel to use for transmission have been investigated. In addition, they considered both constant access time (CAT) and constant data time (CDT) policy. Most of existing approaches have assumed that the SUs are not allowed use the previous explored channels. The given reason is that recalling a previous explored channel may leads to collision since the channel may be occupied by other secondary users. However, this consideration is limited. In an OSA system with multiple SUs, the unexplored channels may also be occupied by other secondary users. In this sense, both recalling a previous explored channel and exploring a residual channel may result in collision. Thus, the optimal design for multichannel multiuser OSA systems is not achieved in most existing work.

In summary, we make the following contributions in this paper:

- For single user OSA systems, we formulate the channel exploration problem as an optimal stopping problem with recall and propose a myopic but optimal approach. It is shown that the proposed approach achieves an increased thought.

– For multiuser OSA systems, we propose an adaptive stochastic recall algorithm (ASRA). It is also shown that the proposed ASRA achieves higher throughput.

The rest of this paper is organized as follows. In section II, we present the system model and define the channel exploration problem. In section III, we develop an optimal stopping problem with recall for the channel exploration problem, and propose efficient approaches for single secondary user and multiple secondary users. In section IV, simulation results and discussions were presented. Finally, we make conclusion in section V.

2 System Model and Problem Formulation

2.1 System Model

We consider an OSA system involving M SUs and N licensed channels. The PUs are assumed to use the licensed channels in a slotted fashion. We assume that the activities of the primary users are independent from channel to channel and from slot to slot. Furthermore, we assume that each licensed channel is not occupied with probability θ_n in each slot. We denote the state of each channel in a slot as $S_n, n \in \mathcal{N}$, where $\mathcal{N} = \{1, 2, ..., N\}$ is the set of licensed channels. Specifically, $S_n = 1$ indicates that licensed channel n is idle while $S_n = 0$ indicates that it is occupied. For simplicity of analysis, we assume that the spectrum sensing is perfect in this paper[1].

We consider Rayleigh fading environment in this paper; moreover, each channel undergoes block-fading in each slot. That is, the channel gain of each channel is fixed in a slot and changes randomly in the next slot. Thus, the channel gain of the nth SU, $G_n, \forall n \in \mathcal{N}$, is identical independent distributed (i.i.d.) exponential random variables with mean unit. Namely, the common probability density function for all $G_n, \forall n \in \mathcal{N}$ is given by $f_g(x) = e^{-x}, x \geq 0$.

2.2 Problem Formulation

The SUs are assumed to employ constant transmitting power policy. Thus, the instantaneous received signal-to-noise-ratio (SNR) on channel n is given by $\gamma_n = Pg_n/\sigma^2$, where P represents the peak power of the SU, σ^2 represents the variance of white Gaussian noise which is set to be one for simplicity of analysis, and g_n represents the instantaneous channel gain.

The transmission process of the SUs is shown in Fig. 1. Let T and τ denote the length of the slot and the length of required time for reliable channel exploration respectively. Suppose that a SU stops channel-exploration after exploring the first n licensed channels, and chooses one of the explored idle channels for transmission. Then the obtained throughput for this SU is given by:

[1] Although the analysis in this paper is mainly for the scenario of perfect sensing, it can easily be extended to the scenario of imperfect sensing.

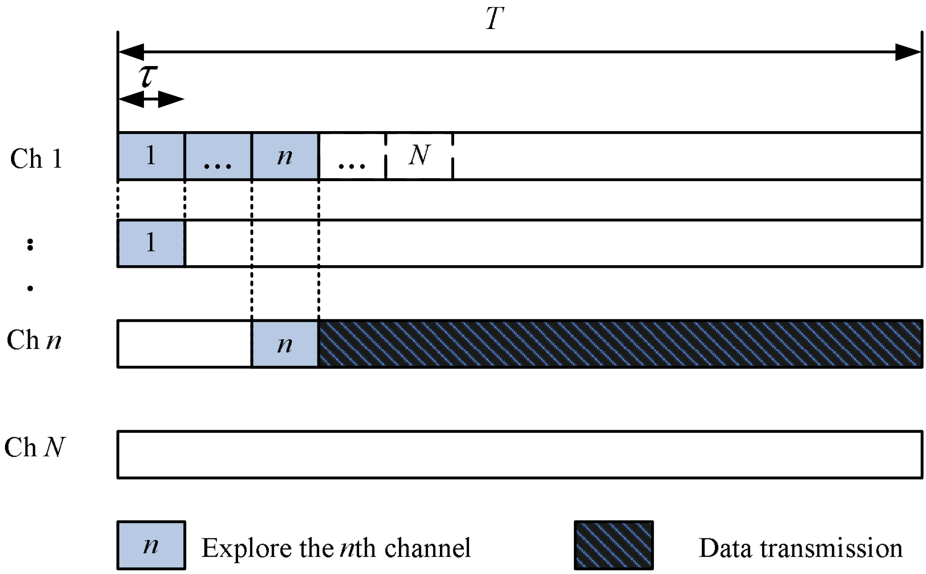

Fig. 1. The diagram of the transmission of the secondary users in a slot

$$R_n = c_n F\big((s_1, g_1), ...(s_n, g_n)\big) \tag{1}$$

where $c_n = T - n\tau$ represents the effective transmission time in a slot, (s_n, g_n) represent the realizations of random variables (S_n, G_n) and F represents the capacity function which is determined by specific transmission policy.

After exploring the first n channels, the US observes the sequence $\{o\}_{n=1}^{N}$, where $o_n = (s_n, g_n)$. Based on the observations and the achieved throughput R_n specified by (1), the SU decides whether to stop channel exploration with receiving the throughput given by R_n or to proceed to explore the remaining channels.

The goal of the each SU is to choose a time to stop to maximize the expected throughput. Our problem belongs to the optimal stopping problem with finite horizon [11], since the SU must stop at the last channel. Generally, we can define the following function at each stage n:

$$V_n^{(N)} = \begin{cases} R_n(o_1, ..., o_n), & n = N \\ \max\{R_n(o_1, ..., o_n), E[V_{n+1}^{(N)}|o_1, ..., o_n]\}, 1 \le n < N \end{cases} \tag{2}$$

where $E[V_{n+1}^{(N)}|o_1, ..., o_n]$ represents the expected throughput that the SU can achieve starting from stage n having observed the sequence of $\mathbf{O}_n = (o_1, ..., o_n)$. Thus, the optimal stopping time for the channel exploration problem is given by:

$$N^* = \min\{n \ge 1 : R_n(o_1, ..., o_n) \ge E[V_{n+1}^{(N)}|o_1, ..., o_n]\} \tag{3}$$

In other words, it is optimal to stop if the current obtained throughput is no less than the expected throughput of stopping at a future channel.

2.3 Optimal Stopping Rule without Recall

In [10], the channel exploration problem was formulated as the problem of optimal stopping without recall. Specifically, the SU is one allowed to use the current channel and is not allowed to use a previously explored channel, which leads to the following defined reward function:

$$R_n^{norec}(o_1, ..., o_n) = R_n^{norec}(o_n) = c_n \log(1 + s_n g_n P) \tag{4}$$

It is seen that the reward in each stage is only a function of current observation, then the expected throughput of proceeding exploration can be reduced to $E[V_{n+1}^{(N)}|o_n]$. Moreover, by the method of of backward induction, it is given by:

$$E[V_n^{(N)}] = \begin{cases} c_n\theta_n E_x[\log(1+xP)], n = N \\ \big(1 - \theta_n + \theta_n E[V_{n+1}^{(N)}] E_x\big[1|\log(1+xP) < E[V_{n+1}^{(N)}]\big]\big) + \\ \ c_n\theta_n E_x\big[\log(1+xP)|\log(1+xP) \geq E[V_{n+1}^{(N)}]\big], 1 \leq n < N \end{cases} \tag{5}$$

where $E_x[\,]$ represents taking expectation over x.

Based on (4) and (5), the optimal stopping rule for channel exploration problem can be easily described as follows. After exploring each channel, the SU compares the current obtained throughput specified by (4) and the expected throughput of proceeding exploration specified by (5). The SU stops channel exploration if the former is no less than the later, otherwise proceeds exploring residual channels.

3 Optimal Stopping Rule with Recall for Channel Exploration

Since we do not focus on designing optimal sensing order in this paper, we assume that all licensed channels have the same idle probability, i.e., $\theta_n = \theta, \forall n \in \mathcal{N}$. As a result, the SUs explore the licensed channels in a random order.

It is known that the SUs in existing work is not allowed to use a previously explored channel, which leads to a conservative design. To improve the throughput performance, the SUs should be allowed to use the previous explored idle channels. First, for single user OSA systems, we formulate the channel exploration problem as an optimal stopping problem with recall and propose a myopic but optimal rule. Secondly, for multiuser OSA systems, we investigate the impact of interactions among users and propose an adaptive stochastic recall algorithm.

3.1 Single SU Scenario

Unlike existing work, we assume that the SU is allowed to use a previously explored channel for transmission, which motivates us to define the current obtained throughput as follows:

$$R_n^{rec}(o_1, ..., o_n) = c_n \log(1 + \eta_n^{\max} P) \tag{6}$$

where $\eta_n^{\max} = \max\limits_{1 \le i \le n} \{s_i g_i\}$ is defined as the maximum effective channel gain among the first n explored channels.

Under the framework of optimal stopping with recall, it is noted from (2) that at each stage n, we have to calculate $E[V_{k+1}^{(N)}|\mathbf{O}_k]$ backward from stage N to stage $n+1$. However, such a backward induction solution is a type of dynamic programming, which has exponential complexity. Furthermore, each $g_n, n \in \mathcal{N}$, is a continuous random variable which results in an un-trackable calculating space for the backward induction solution. Thus, such a backward induction approach is not feasible in practice.

A feasible approach is considering a truncated version of the problem. The simplest version of such truncation is the 1-stage ahead rule (1-SLA), with which the stopping time is determined by:

$$N_1 = \min\{n \ge 1 : R_n^{rec}(o_1, ..., o_n) \ge E[V_{n+1}^{(1)}|o_1, ..., o_n]\} \tag{7}$$

In other words, 1-SLA calls for stopping at the first n channels for which the throughput for stopping is at least as great as the expected throughput of continuing one stage and then stopping. In this sense, the 1-SLA rule is also called as myopic rule. In our problem, we have

$$E[V_{n+1}^{(1)}|o_1, ..., o_n] = c_{n+1} \int_0^\infty \log(1 + \max\{\eta_n^{\max}, x\}P) h_g(x) dx \tag{8}$$

where $h_g(x)$ is the auxiliary probability distribution function defined as follows:

$$h_g(x) = (1 - \theta)\delta(x) + \theta f_g(x) \tag{9}$$

where $\delta(x)$ is the impulse function specified $\delta(x) = 0, x \ne 0$ by and $\int_{-\infty}^{\infty} \delta(x) dx = 1$.

3.2 The Optimality of 1-SLA Rule

In general, 1-SLA is not optimal. However, when the following condition is satisfied, the 1-SLA rule is optimal.

Definition1[11]: Let A_n denote the events $\{R_n(\mathbf{O}_n) \ge E[V_{n+1}^{(1)}|\mathbf{O}_n]\}$. We say that the stopping problem is monotone if

$$A_1 \subset A_2 \subset ... \subset A_N \tag{10}$$

Namely, if the 1-SLA rule calls for stopping at stage n, then it will call for stopping at all future stages no matter what the future observations turn out to be.

Theorem 1. *In a finite horizon monotone stopping rule problem, the 1-SLA rule is optimal.*

Proof. Refer to [11].

Theorem 2. *The 1-SLA rule is optimal to the channel exploration problem.*

Proof. We define the following function:

$$\begin{aligned} F_n(\eta_n^{\max}) &= R_n^{rec} - E(V_{n+1}^{(1)}|\mathbf{O}_n) \\ &= c_n \log(1+\eta_n^{\max}P) - c_{n+1}\int_0^\infty \log(1+\max\{\eta_n^{\max},x\}P)h_g(x)dx \end{aligned}, \quad (11)$$

and the following auxiliary function:

$$\begin{aligned} G_n(\eta_n^{\max}) &= \tau\int_0^\infty \log(1+\max\{\eta_n^{\max},x\}P)h_g(x)dx \\ &-(T-n\tau)\int_0^\infty [\log(1+\max\{\eta_n^{\max},x\}) - \log(1+\eta_n^{\max})]h_g(x)dx \end{aligned} \quad (12)$$

It is noted that $G_n(\eta_n^{\max})$ is a transformation of $F_n(\eta_n^{\max})$, and they exhibit the following properties:

$$\begin{cases} G_n(\eta_n^{\max}) = 0 \Leftrightarrow F_n(\eta_n^{\max}) = 0 \\ G_n(\eta_n^{\max}) > 0 \Leftrightarrow F_n(\eta_n^{\max}) > 0 \end{cases} \quad (13)$$

We then compare $G_{n+1}(\eta_{n+1}^{\max})$ and $G_n(\eta_n^{\max})$ as follows:

$$\begin{aligned} & G_{n+1}(\eta_{n+1}^{\max}) - G_n(\eta_n^{\max}) \\ & = \tau\int_0^\infty \log(1+\max\{\eta_{n+1}^{\max},x\}P)h_g(x)dx - \tau\int_0^\infty \log(1+\max\{\eta_n^{\max},x\}P)h_g(x)dx \\ & + c_n\int_0^\infty [\log(1+\max\{\eta_n^{\max},x\}P) - \log(1+\eta_n^{\max}P)]h_g(x)dx \\ & - c_{n+1}\int_0^\infty [\log(1+\max\{\eta_{n+1}^{\max},x\}P) - \log(1+\eta_{n+1}^{\max}P)]h_g(x)dx \end{aligned} \quad (14)$$

It is seen that $\int_0^\infty \log(1+\max\{\eta_n^{\max},x\}P)h_g(x)dx$ is a strictly monotone increasing function of $\eta_n^{\max}$, $c_n\int_0^\infty [\log(1+\max\{\eta_n^{\max},x\}P) - \log(1+\eta_n^{\max}P)]h_g(x)dx\}$ is a strictly monotone decreasing function of $\eta_n^{\max}$ and $c_n > c_{n+1}$ is always true for all $n \in \mathcal{N}$. Moreover, recall the transmission policy of the secondary users, we have $\eta_{n+1}^{\max} = \max\{\eta_n^{\max}, s_{n+1}g_{n+1}\} \geq \eta_n^{\max}$. Thus, the following inequality can be obtained immediately:

$$G_{n+1}(\eta_{n+1}^{\max}) - G_n(\eta_n^{\max}) > 0 \quad (15)$$

Let us re-write A_n as follows:

$$A_n = \{\eta_n^{\max} : F_n(\eta_n^{\max}) \geq 0\} = \{\eta_n^{\max} : G_n(\eta_n^{\max}) \geq 0\} \quad (16)$$

Using (16) and (14), we have:

$$F_{n+1}(\eta_{n+1}^{\max}) > 0, \eta_n^{\max} \in A_n \quad (17)$$

which is equivalent to $A_n \subset A_{n+1}$. Finally, the following can be inductively obtained:

$$A_1 \subset A_2 \subset ... \subset A_N \quad (18)$$

By definition 1 and Theorem 1, Theorem 2 follows.

Theorem 2 characterizes the optimality of 1-SLA rule for the channel exploration problem. In the following, we investigate the structure of the 1-SLA rule.

Lemma 1. *Denote a_n as the solution of the following equation,*

$$F_n(\eta_n^{\max}) = 0 \tag{19}$$

Then a_n is unique for each $n \in \mathcal{N}$ and $a_{n+1} < a_n$.

Proof. It is known that $G_n(\eta_n^{\max})$ is a strict monotone increasing function of $\eta_n^{\max}$. In addition, the following always holds:

$$\begin{cases} G_n(0) = -c_{n+1} \int_0^{\infty} \log(1 + xP) h_g(x) dx < 0 \\ \lim\limits_{\eta_n^{\max} \to \infty} G_n(\eta_n^{\max}) = \tau \log\limits_{\eta_n^{\max} \to \infty} (1 + \eta_n^{\max} P) dx > 0 \end{cases}, \forall n \in \mathcal{N} \tag{20}$$

Hence, $G(\eta_n^{\max}) = 0$ has unique root, which means that $F_n(\eta_n^{\max}) = 0$ also has unique root. Now, suppose $F_n(a_n) = 0$ and $F_{n+1}(a_{n+1}) = 0$, then the following equation can be obtained from (13) and (15):

$$G_{n+1}(a_n) > G_n(a_n) = G_{n+1}(a_{n+1}) = 0 \tag{21}$$

We then have $a_{n+1} < a_n, \forall n \in \mathcal{N}$, where we use the fact that G_{n+1} is a monotone increasing function. Therefore, Lemma 1 is proved.

Lemma 2. *The optimal stopping rule for the channel exploration problem is described as follows:*

$$N^* = \min\{n \geq 1 : \eta_n^{\max} \geq a_n\} \tag{22}$$

Proof. Straightforward obtained from Theorem 2 and Lemma 1.

Remark 1: a_n can be regarded as the threshold of each stage. A closed-form expression of a_n is unavailable and we can resort to numeric method.

Remark 2: According to Lemma 2, the optimal stopping rule for the channel exploration problem is simple and can be described as follows. After exploring each channel, the SU compares $\eta_n^{\max}$ with the threshold a_n. It stops exploration and chooses the explored idle channel with the highest channel gain for transmission if $\eta_n^{\max}$ is no less than the threshold; otherwise, it proceeds to explore residual channels.

Remark 3: $a_{n+1} < a_n$ can be interpreted as follows. At the early stage, the probability of obtaining a higher throughput in a future stage is high since there are a number of unexplored channels. However, as n increases, the number of unexplored channels decreases and the exploration overhead increases. Thus, the SU may perform more aggressively in the early stage while more conservatively in the later stage.

3.3 Multiple SUs Scenario

In the last subsection, we formulated the channel exploration problem for single SU scenario as an optimal stopping problem with recall. In this section, we

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Adaptive stochastic recall algorithm (ASRA)

Step 1: 1. After exploring the first n licensed channels, the SUs recall with probability $p_{recall}(n, M)$. Thus, the maximum effective channel gain is calculated in a stochastic manner, i.e.,

$$\Pr(\eta_n^{\max} = \max_{1 \le i \le n} \{s_i g_i\}) = 1 - \Pr(\eta_n^{\max} = s_n g_n) = p_{recall}(n, M) \tag{23}$$

where $p_{recall}(n, M) = (\frac{1}{N-n+1})^{M-1}$.
Step 2: After obtaining $\eta_n^{\max}$, the SUs perform channel decision according to Lemma 2.

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consider the multiple SUs scenario. It is seen that in an OSA systems with multiple SUs, a collision occurs when more than one SU access the same channel at the same time. Thus, traditional optimal stopping rule will not lead to optimal design in the scenario of multiple SUs. To address this problem, new methods that consider the interactions among SUs is needed.

Normally, the optimal design for multiple SUs scenario is hard to obtain. We then seek for a heuristic method with which the SUs stochastically recall a previously explored channel. Specifically, in stage n, the SUs stochastically do not always recall the explored channels; instead, it recall with probability $p_{recall}(n, M)$. Instinctively, the probability $p_{recall}(n, M)$ should have the following properties:

1. Increases when n increases while decreases when M increases. In other words, the SUs are encouraged not to recall in the early stage while are encouraged to recall in the last stage.
2. $p_{recall}(n, 1) = 1, \forall n$. That is, if there is only one SU then it always recalls.

Based on the above, we propose an adaptive stochastic recall algorithm (ASRA) as described at the top of this page, for multiple SU OSA systems. We choose a simple $p_{recall}(n, M)$ in this paper, but other expressions can also be used.

4 Simulation Results and Discussion

In the simulation study, the common simulation parameters are as follows: $T = 100ms, \tau = 5ms, \theta = 0.5, P = 10dB$.

First, we evaluate the throughput performance of single SU OSA systems. Fig. 2 shows the expected throughput versus the number of licensed channels, for recall approach and no recall approach respectively. It is shown that the expected throughput of recall approach is higher than that of no recall approach. Furthermore, it is shown that the expected throughput of both approaches grows as the number of licensed channels increases, but it is saturated when the number of licensed channels becomes sufficiently large.

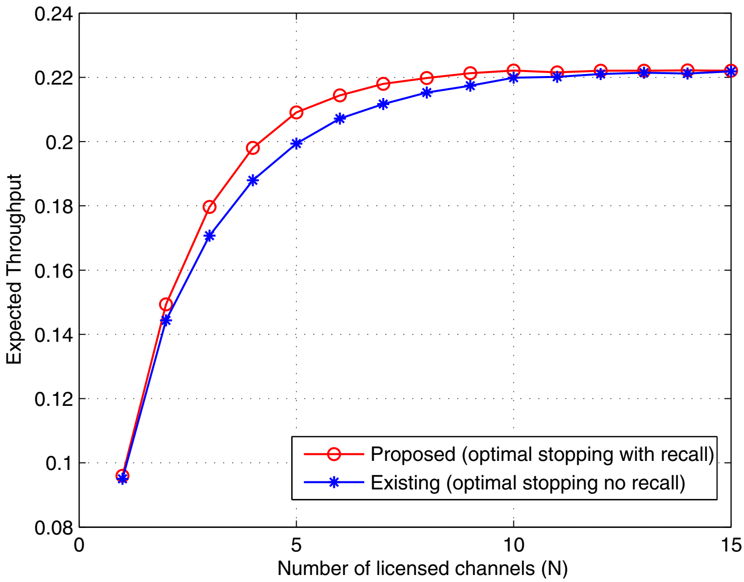

Fig. 2. Expected throughput for single SU systems using recall and no-recall approaches respectively

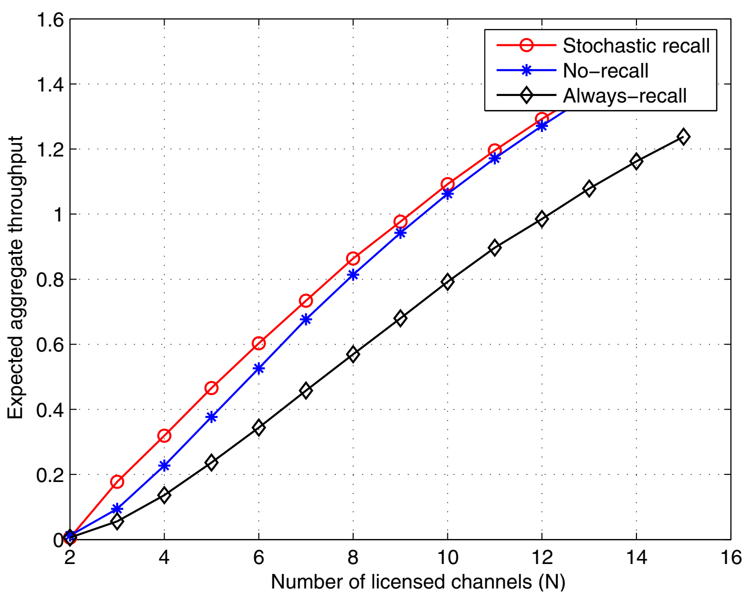

Fig. 3. Expected throughput versus the number of licensed channels for multiple SU systems (The number of SUs is set to $M = 10$)

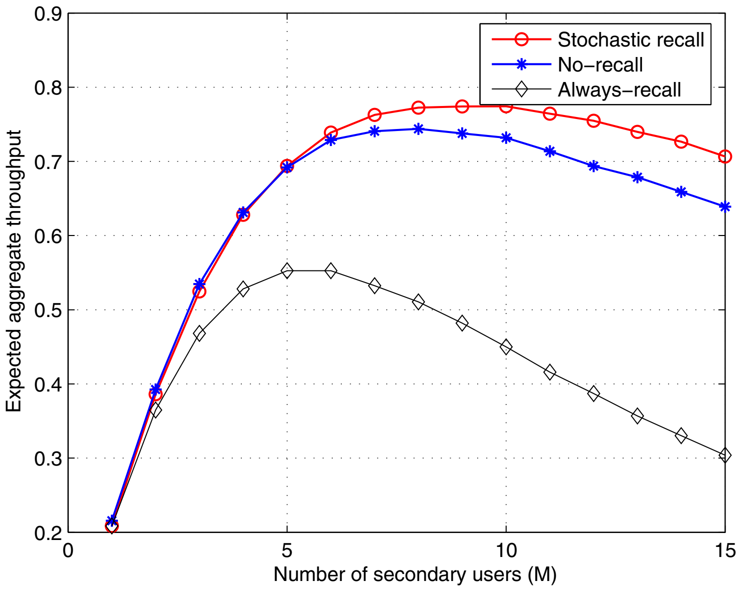

Fig. 4. Expected throughput versus the number of secondary users for multiple SU systems (The number of licensed channel is set to $N = 8$)

Second, we evaluate the throughput performance for multiple SU OSA systems. Specifically, we compare the obtained throughput of stochastic recall, always recall and no recall in multiple user scenario. Fig. 3 shows the expected throughput versus the number of licensed channels, for the above three approaches respectively. It is shown that the expected throughput of stochastic recall approach is always higher than those of no recall and always recall.

Third, we evaluate the throughput performance of three approaches when varying the number of SUs. Fig. 4 shows the expected throughput versus the number of SUs. It is noted from the figures that the expected throughput of stochastic recall outperforms other two approaches. In addition, the expected throughput grows as the number of secondary users increases for the scenario with small number of secondary users, but it decreases as the number of secondary users increases for the scenario with large number of secondary users.

5 Conclusion

We investigated the tradeoff of throughput and multichannel diversity for opportunistic spectrum access in fading environment. We considered the presence of primary users as well as the channel conditions when optimizing the expected throughput of secondary users. For single user systems, we formulated the channel exploration problem as an optimal stopping problem with recall and proposed a myopic but optimal approach. However, the approach presented for single secondary user is not fit for that of multiple secondary users, since the secondary users will collide multiple secondary users transmit on the same channel at the same time. Thus, we presented a stochastic recall approach for multiple users systems. Further work including re-establishing an optimal stopping framework for the multiple users systems is ongoing.

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