# On Timing Offset and Frequency Offset Estimation in LTE Uplink[*]

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**Abstract.** In this paper, the timing offset and frequency offset estimation in LTE uplink is studied. The approaches proposed are based on the demodulation reference signals (DMRS) in PUSCH for FDD mode. With the channel estimation in the frequency domain at the receiver by using the two DMRS, i.e., DMRS located in two OFDM symbols, within one sub-frame, timing offset estimation is conducted by exploring the phase shift between different sub-carriers for each DMRS, while the frequency shift estimation is implemented by studying the phase rotation between the two DMRS for each sub-carrier. Statistical average is used to enhance the performance of estimation. Simulation results demonstrate that the proposed algorithms can offer satisfactory performance even at relatively low signal to noise ratios in the additive whiten Gaussian noise environments.

**Keywords:** Channel estimation, timing offset, frequency offset, LTE.

## 1 Introduction

By the end of 2004, the third Generation Partnership Project (3GPP) started the 3G long term evolution (LTE) project to ensure its long-term comparative advantage of wireless standards. In LTE systems, the uplink transmission scheme is based on single-carrier frequency division multiple access (SC-FDMA) transmission with cyclic prefix[1,2].Compared with orthogonal frequency division multiple access (OFDM) scheme, a prominent advantage of SC-FDMA over OFDM is that its transmitted signal has a lower peak-to-average power ratio (PAPR)[3]. Nominally SC-FDMA leads to a single-carrier transmit signal, in fact, the signal is based on multiple frequency bins for every symbol and is under the influence of frequency offset and timing errors. This is because that imperfect synchronization can generate inter-carrier interference (ICI) and inter-symbol interference (ISI), thus will induce both co-channel and inter-channel interference[4]-[6] . Moreover, carrier frequency synchronization for the uplink of SC-FDMA system is more difficult, since the frequency recovery for one user may result in the misalignment of the other synchronized users [7]-[9].

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Until now, several timing and frequency synchronization techniques for LTE uplink system have been studied [10]-[12]. [10] has proposed an accurate timing and frequency offsets estimation algorithm for SC-FDMA uplink, however, the work only considers the timing and frequency offsets of a new user entering the system and assume all the other users have already been perfectly synchronized. In our work, we should consider the timing and frequency offsets of all active users. In [11], a maximum-likelihood (ML) based joint channel and frequency offset estimation algorithm for SC-FDMA system is proposed using one training block. The algorithm has good estimation performance and fast convergence rate. However, the computational complexity is prohibitively high due to too much matrix inversion calculation, particularly when the number of sub-carrier is large. The timing synchronization method proposed in [12] is based on a raw channel estimation using the sounding reference signal (SRS) of the respective channels for each antenna and time slot in PUCCH, PUSCH. First the phase deviation between neighboring sub-carriers is calculated, then a weighted averaging in MAC layer is carried out to convert the phase into a time offset estimation value. This algorithm has good estimation performance over a large number of samples. However, it just considers the phase deviation between adjacent sub-carriers, while we will explore in this paper all the phase deviations between a sub-carrier couple with m (m is an integer larger or equal to 1) sub-carrier interspacing to achieve a more accurate time offset value by averaging the estimates corresponding to different and possible m values. The method to present in the following sections is based on channel estimation by using DMRS in PUSCH in LTE uplink. With efficiency in computation, the approach shows high accuracy in estimation even at a relatively low SNR.

## 2 System Model

### 2.1 DMRS Sequence in LTE Uplink

The DMRS positions in a sub-frame of a LTE system in the frequency division duplex (FDD) is given in Fig.1. In LTE FDD, each radio frame of 10ms consists of 20 slots of length 0.5ms, numbered from 0 to 19[13]-[14].A sub-frame is defined as two consecutive slots each consisting 12 or 14 OFDM symbols, depending on whether normal or extended cyclic prefix is used. In PUSCH, the pilot occupies the fourth SC-FDMA symbol in each slot of a sub-frame with normal cyclic prefix. The two pilot sequences in a sub-frame are the same.

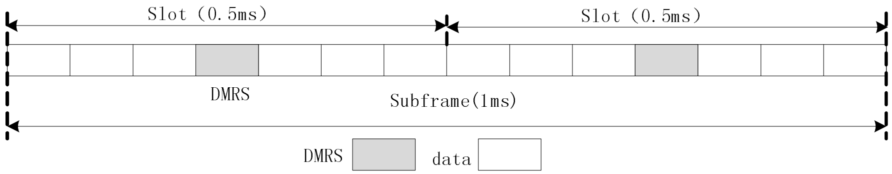

**Fig. 1.** DMRS in LTE uplink

The DMRS for PUSCH in the frequency domain will be mapped to the same set of physical resource blocks (PRB) used for the corresponding PUSCH transmission with the same length expressed by the number of sub-carriers.

### 2.2 Structure of Our Proposed Algorithm

The simplified block diagram of our proposed algorithm in LTE SC-FDMA system is illustrated in Fig.2. At the transmitter in base-band, the binary information bits are firstly grouped and mapped to the 16QAM symbol before the SC-FDMA modulation. To generate a SC-FDMA symbol, an M-point DFT is applied to a group of 16QAM symbols before mapping to consecutive M sub-carriers at the input of an N-point IFFT operator. Furthermore, Cyclic Prefix (CP) with a length larger than channel maximum delay is added.

At the receiver, the CP is removed from received symbol first. After N-point FFT and sub-carrier de-mapping, channel estimation is carried out to estimate time offset and frequency offset.

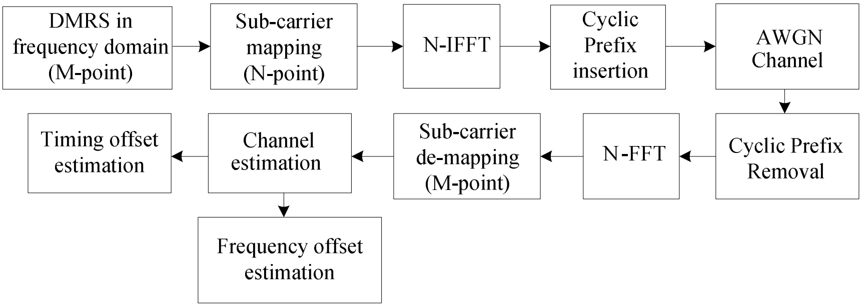

**Fig. 2.** System model of our proposed algorithm in base-band

## 3 Timing Offset Estimation

From Fig. 1, we can see that these are two DMRS symbols in each sub-frame. In the time-domain, each DMRS can be denoted by a $\mathrm{N_{RS}}$-dimensional sampling vector $x_i(n)=[x_i(1),...,x_i(N_{RS})]^T, i=1,2$, where $\mathrm{N_{RS}}$ is the length of the demodulation reference signal. If there is a timing offset $\tau_\mathrm{T}$, then the corresponding reference signal in the frequency-domain at the receiver can be written as

$$Y_i(k)=\exp(-j2\pi\tau_T k/N)H_i(k)X_i(k)+W_i(k), i=1,2\ ,\ k=0,\cdots,M-1 \tag{1}$$

where $H_i(k)$ is the channel frequency response of the *i*th DMRS without time offset, $W_i(k)=FFT(w_i(n))$, $w_i(n)$ denotes the additive whiten Gaussian noise (AWGN). In

(1), $\tau_T$ is the normalized timing offset, i.e., $\tau_T = \tau / T$ with $T$ being the sampling period.
From (1), the least square channel estimation based on DMRS can be implemented by

$$H^{TO}{}_i(k) = \frac{Y_i(k)}{X_i(k)}\ ,\ k = 0,...,M-1 \tag{2}$$

To present the proposed approach more explicitly, let us consider the received signal without AWGN. In this case, the received signal in the frequency-domain can be rewritten as

$$Y_i(k) = \exp(-j2\pi\tau_T k / N) H_i(k) X_i(k), i = 1,2 \tag{3}$$

For the signal model (3), we have

$$\begin{aligned} H_i^{TO}(k) &= \exp(-j2\pi\tau_T k / N) H_i(k) \\ &= A_i \exp(-j2\pi\tau_T k / N)\exp(j\theta_i), i = 1,2 \end{aligned} \tag{4}$$

where $A_i$ and $\theta_i$ denote the amplitude and phase of $H_i(k)$. Without AWGN, the frequency response of channels will be an accurate evaluation by using (4).

Let us define

$$\begin{aligned} R_i(k_1,k_2) &= (H_i^{TO}(k_1))^*(H_i^{TO}(k_2) \\ &= A_i \exp(j2\pi\tau_T m / N), i = 1,2 \end{aligned} \tag{5}$$

And

$$\varphi_{m,i} = 2\pi\tau_T m / N, i = 1,2 \tag{6}$$

where $m = k_1 - k_2$ is the interspacing between sub-carrier $k_1$ and sub-carrier $k_2$. Then, we can get

$$\phi_{m,i} = a\tan 2\left(\frac{\sum \ \mathrm{Im}\{R_i(k_1,k_2)\}}{\sum \ \mathrm{Re}\{R_i(k_1,k_2)\}}\right) = 2\pi\tau_T m / N, i = 1,\ 2 \tag{7}$$

where the function atan2(*) is defined by

$$a\tan 2(y,x)=\begin{cases}\arctan(\frac{y}{x}) & x>0\\ \pi-\arctan(\frac{y}{x}) & y\geq 0, x<0\\ -\pi+\arctan(\frac{y}{x}) & y<0.x<0\\ \frac{\pi}{2} & y>0, x=0\\ -\frac{\pi}{2} & y<0, x=0\\ \textit{undefined} & y=0, x=0\end{cases} \tag{8}$$

The normalized time offset can be calculated as

$$\tau_T=\frac{\phi_{m,i}N}{2\pi m} \tag{9}$$

The actual time offset is given by

$$\tau=\tau_T T=\frac{\phi_{m,i}NT}{2\pi m}=\frac{\phi_{m,i}}{2\pi m\Delta} \tag{10}$$

where $\Delta$ represents the interspacing between each two neighboring SC-FDMA sub-carriers in the frequency domain. For a system with 20MHZ bandwidth, $\Delta$ has a value of 15KHZ defined by 3GPP LTE specifications [1].

The timing offset calculated by using (10) for model (3) is an accurate evaluation value. In practice, however, AWGN should be considered and (10) is an estimate for corresponding channel estimates by using (2). In this case, a second average can be further used over different $m$ to acquire a more precise timing offset estimate.

## 4 Fractional Frequency Offset Estimation

In this section, a channel estimation based fractional frequency offset estimation approach in PUSCH is presented. It is well known that a frequency shift is equivalent to a phase rotation in time domain by assuming that the channel during a sub-frame is fixed. This assumption is ideal but is reasonable for the very short sub-frame duration in LTE. In ideal AWGN environments, the received signals in time-domain corresponding to the two DMRS in PUSCH, $x_1(n)$ and $x_2(n)$, can be expressed by

$$y_1^{FO}(n) = \exp\{-j2\pi\delta n / N\} x_1(n) + v_1(n) \tag{11}$$

$$y_2^{FO}(n) = \exp\{-j2\pi\delta(n+Q) / N\} x_2(n) + v_2(n) \tag{12}$$

Where $n = 0, \cdots, N_{RS} - 1$, $\delta$ is a fractional frequency offset normalized by the sub-frame interspacing $\Delta$, $v_1(n)$ and $v_2(n)$ denote AWGN introduced in the received signals.

To present the proposed approach fractional frequency offset estimation, let us ignore the AWGN component for an ideal wireless channel with no any noise. In this case, the received signal corresponding to the two DMRS in a sub-frame can be expressed, respectively, by (13) and (14).

$$x_1^{FO}(n) = \exp\{-j2\pi\delta n / N\} x_1(n) \tag{13}$$

$$x_2^{FO}(n) = \exp\{-j2\pi\delta(n+Q) / N\} x_2(n), n = 1, 2, ..., N_{RS} \tag{14}$$

In (14), $Q$ denotes the integral sampling point deviation between the first sampling points of the two reference signals. The value of $Q$ is decided by the bandwidth of LTE system. For bandwidths of 5MHz, 10MHz and 20MHz, $Q$ are 3840, 7680 and 15360, respectively.

After $N$ -point FFT operation at the receiver, the output of the reference signal in frequency-domain corresponding to (13) and (14) can be written as

$$X_1^{FO}(k) = \sum_{n=0}^{N-1} \exp\{-j2\pi(k+\delta)n / N\} x_1(n)$$

$$= X_1\,(k+\delta) \tag{15}$$

And

$$X_2^{FO}(k) = \exp\{-j2\pi\delta Q / N\} X_2\,(k+\delta) \tag{16}$$

In practice, the received signals will suffer from the influence of the multipath propagation. Hence, the received signal components in the frequency-domain corresponding to (15) and (16) after the FFT conversion can be further expressed as

$$X_1^{FO}(k) = H_1(k) X_1(k+\delta) \tag{17}$$

$$X_2^{FO}(k) = \exp\{-j2\pi\delta Q / N\} H_2(k) X_2(k+\delta) \tag{18}$$

Since the transmitted two DRMS are the same in one sub-frame, and we can also assume that the channel during a sub-frame is fixed, the fractional frequency offset value can be obtained by calculating

$$\delta = a\tan 2([\frac{1}{N_1}\sum_{k=0}^{N_1-1}[H_2^{FO}(k)]^* H_1^{FO}(k)]) \bullet \frac{N}{2\pi Q} \tag{19}$$

$$df = \delta\Delta = \frac{\Delta N}{2\pi Q} \bullet a\tan 2([\frac{1}{N_1}\sum_{k=0}^{N_1-1}[H_2^{FO}(k)]^* H_1^{FO}(k)]) \tag{20}$$

Where

$$H_i^{FO} = \frac{X_i^{FO}(k)}{X_i(k+\delta)}, \quad i = 1,2 \tag{21}$$

In practice, we do not know $X_i(k+\delta)$ for each DMRS. In addition we have to consider the effects of AWGN. Therefore, we use the following estimates in (20) to get the fractional frequency offset estimates, i.e.

$$\hat{H}_i^{FO} = \frac{Y_i^{FO}(k)}{X_i(k)}, \quad i = 1,2 \tag{22}$$

For a system with a bandwidth of 20MHZ, $\Delta = 15KHz$. The range of fractional frequency offset estimation algorithm can reach $[-\frac{1}{15},\frac{1}{15}]$ and the corresponding range of the actual frequency offset is $[-1KHz, 1KHz]$.

## 5 Numerical Examples

In this section, performance of the proposed timing-offset and frequency offset estimation algorithms for LTE system is evaluated through simulations. The ideal AWGN channel is assumed. A 16QAM-SC-FDMA system with a sampling frequency of 30.72MHZ is considered. 2048-point FFT/IFFT with sub-carrier interspacing 15KHz is used. The lengths of CP are 160 samples in the first SC-FDMA symbol and 140 samples in the rest six symbols, respectively. During the simulations, 12 SC_FDMA symbols and 2 reference signals in each sub-frame are generated. 100 resource blocks (RB) are adopted for PUSCH data transmission. That means that during the 2048 sub-carriers, 1200 sub-carriers are used for pilots and data transmission.

In Fig.3, a timing offset of length 100 sampling periods are set, the root mean square error (RMSE) of the SC-FDMA symbol timing offset estimation measured in samples is plotted against the signal to noise ratio (SNR). As for the low SNR case the estimation is very accurate too, the estimation error is as low as 0.043 sampling periods in SNR of 0dB. Then in a high SNR of 30dB, the error is reduced to 0.001 sampling periods, almost to zero. This means that proposed timing offset estimation algorithm can offer a very excellent performance in AWGN channel.

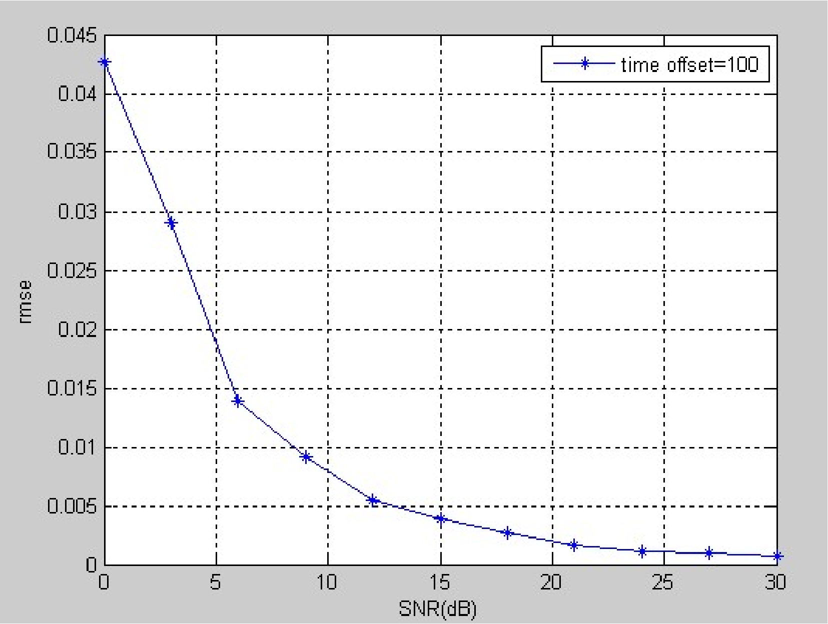

**Fig. 3.** Timing offset in terms of the RMSE vs. the SNR, measured in numbers of sampling periods

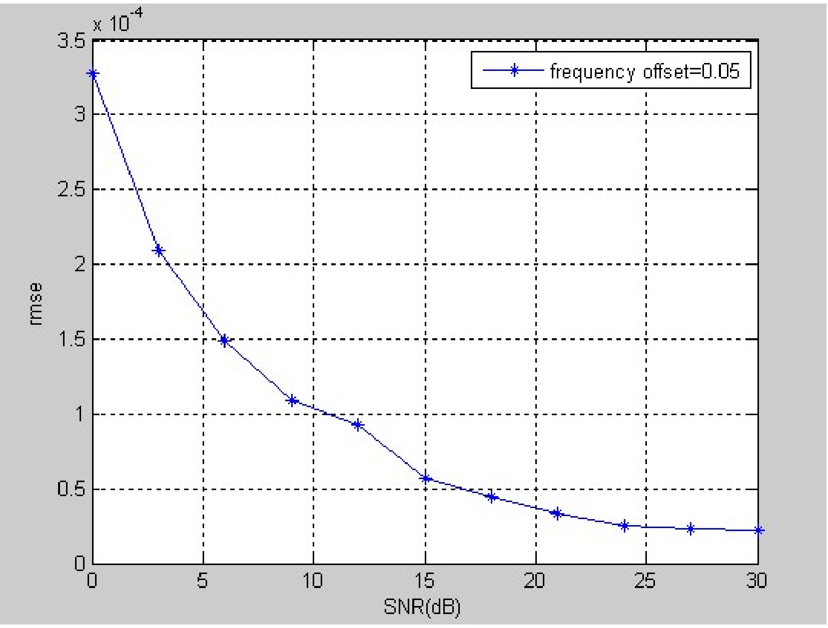

**Fig. 4.** Frequency offset in terms of the RMSE vs. the SNR, measured in numbers of sub-carrier spacing

In Fig.4, a frequency offset of 0.05 sub-carrier spacings are set in this simulation example. The RMSE of the SC-FDMA symbol frequency offset estimation measured in sub-carrier spacing is plotted against the SNR. It can be seen that the proposed fractional frequency offset estimation approach also demonstrates satisfied performance in AWGN even at relatively low SNR values.

## 6 Conclusion Remarks

The paper deal with the timing and frequency offset estimation in PUSCH of LTE systems. A timing offset algorithm and a frequency offset estimation approach are presented. Both the approaches are based on the channel estimation in the frequency-domain and exhibit low costs in computation. The proposed algorithm show excellent performance in ideal AWGN. In addition, the approaches can be also applied to the sounding channel and PUCCH channel for timing error estimation and frequency offset estimation.

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