# Probabilistic Distance Estimation in Wireless Sensor Networks 

Ge Huang ${ }^{1}$, Flávia C. Delicato ${ }^{2}$, Paulo F. Pires ${ }^{2}$, and Albert Y. Zomaya ${ }^{1}$<br>${ }^{1}$ Centre for Distributed and High Performance Computing<br>School of Information Technologies<br>The University of Sydney<br>NSW 2006, Australia<br>${ }^{2}$ DIMAp, Federal University of Rio Grande do Norte, Brazil


#### Abstract

Since all anchor-based range-free localization algorithms require estimating the distance from an unknown node to an anchor node, such estimation is crucial for localizing nodes in environments as wireless sensor networks. We propose a new algorithm, named EDPM (Estimating Distance using a Probability Model), to estimate the distance from an unknown node to an anchor node. Simulation results show that EDPM reaches a slightly higher accuracy for distance estimation than the traditional algorithms for regularly shaped networks, but reveals significantly higher accuracy for irregularly shaped networks.


Keywords: Probability Model, Estimating Distance, Wireless Sensor Networks.

## 1 Introduction

As an essential aspect in wireless sensor networks (WSNs), localization has attracted much research attention over the years. Proposed WSN localization algorithms fall into two main categories: range-based and range-free. Range-free approach does not rely on characteristics of the wireless signal, and is commonly employed in large scale networks, where energy efficiency is a crucial issue.

Being an important class of range-free localization algorithms, DV-Hop algorithms have the advantages of being simple but providing reasonably high accuracy. However, one of the major shortcomings of DV-hop algorithms is that they are not suitable for irregularly shaped networks, which largely affects its potential applicability in a wide range of WSNs domains. The reason lies in the following. Like all range-free localization algorithms, DV-hop algorithms face the problem of estimating the distance from an unknown node to an anchor node. For any given anchor node Q and unknown node P , in order to achieve a high accuracy in distance estimation, DV-Hop algorithms not only require that there is no large obstacle between P and Q , but also require that there are no large obstacles between all other participating anchor nodes(at least two) and Q.

Our proposed EDPM algorithm only requires that there are no large obstacles between P and Q to have a good estimation of the distance from P to Q . Hence, compared to DV-Hop algorithms our algorithm works more effectively in irregularly shaped networks, where presence of large obstacles is more likely.

## 2 Related Work

In all the DV-Hop algorithms, each anchor node A needs to calculate the average hop distance $\delta_{\mathrm{A}}$. DV-Hop algorithms that use formula (2.1) to calculate $\delta_{\mathrm{A}}$ are categorized as unbiased DV-Hop algorithms such as in [1, 4,5], while others that use the formula (2.2) are categorized as least mean square DV-Hop algorithms such as in [3].

$$
\begin{align*}
\delta_{A} & =\frac{\sum_{Q}\|A-Q\|_{2}}{\sum_{Q} h(A, Q)}  \tag{2.1}\\
\delta_{A} & =\frac{\sum_{Q} h(A, Q)\|A-Q\|_{2}}{\sum_{Q} h^{2}(A, Q)} \tag{2.2}
\end{align*}
$$

Besides, DV-Hop algorithms that require all anchor nodes participate in average hop distance calculation are designated as nonselective DV-Hop algorithms such as in [1, 3, 5], while others that select certain anchor nodes to participate are designated as selective DV-Hop algorithms such as in [4, 6].

Most DV-Hop algorithms are unbiased and nonselective with their distance estimation method named as UNDE (Unbiased Nonselective Distance Estimation). UNDE can achieve reasonable accuracy in isotropic and regularly shaped density WSNs, but has relatively large error in randomly distributed or irregularly shaped networks. Ji and Liu [3] proposed a nonselective least mean square DV-Hop algorithm with its distance estimation method named as BNDE (Biased Nonselective Distance Estimation). BNDE can reach reasonably high accuracy in regularly shaped WSNs, but it generates relatively large error in irregularly shaped WSNs. Authors in [4] proposed a so-called "convex hull test method", to select anchor nodes to participate in the average hop distance calculation. We name the distance estimation algorithm used in [4] as CHTDE (Convex Hull Test Distance Estimation). In Cshaped and O-shaped WSNs, CHTDE reaches better accuracy, while in star-shaped WSNs, the error is quite large. Moreover, CHTDE has a high computational complexity, and does not work as effectively as other DV-hop algorithms in large scale regularly shaped networks.

## 3 Estimating Distance Using Probability Model (EDPM)

In the formulating of our solution, we assume that every node in the WSN has the same communication radius $r$. For any nodes P and Q and considering the positive real number a, we use $R(P, Q, a)$ to represent the rectangle with line segment $\overline{P Q}$ as its middle line, and width a. For any two nodes P and Q , we call P and Q line-of-sight connected, if there exists a hop path from P to Q in $R(P, Q, r / 2)$. Note that, if there is a large propagation obstruction between an anchor node Q and an unknown node P , by intuition we can tell that we cannot reach ideal accuracy for distance estimation no matter how efficient the estimation method is. Thus, we only estimate the distance between such a pair of an unknown node and an anchor node that are line-of-sight connected. For lack of space, the proof of the following theorem is omitted.

Theorem 3.1. Suppose the anchor node Q and the unknown node P are line-of-light connected, and the node density within $R(P, Q, r / 2)$ is $\rho$. Let $\delta=2 / \sqrt{\pi \rho}$ and n be the smallest integer not less than $\|Q-P\|_{2} / \delta$. Suppose $\mu=\mathrm{r} / \delta$ is an integer. Let $\mathrm{Q}_{1}, \ldots, \mathrm{Q}_{\mathrm{n}}$ be n geometric points on the ray $\overrightarrow{Q P}$ such that the distance from $\mathrm{Q}_{\mathrm{m}}$ to Q is $(2 \mathrm{~m}-1) \delta / 2$. Let $\mathrm{C}_{\mathrm{m}}$ be close disk centered at $\mathrm{Q}_{\mathrm{m}}$ and with radius $\delta / 2$ (according to the definition of $\delta$, there must exist nodes inside). Let $M=\cup C_{m}$. Tag $Q$ as 0 , the nodes in M and within the communication range of Q as 1 , and the untagged nodes in M and within the communication range of some nodes tagged by 1 as 2 . Repeat this process until P is tagged. Suppose P is tagged as h. Let $\mathrm{k}(0)=0$. For every $1<\mathrm{s}<\mathrm{h}$, let $\mathrm{k}(\mathrm{s})=\max \left\{\mathrm{m} \mid\right.$ there exists node tagged as s in $\left.\mathrm{C}_{\mathrm{m}}\right\}$. Then,
(1) For every $1<s<h$, there exists $\mathrm{P}_{\mathrm{s}} \in \mathrm{C}_{\mathrm{k}(\mathrm{s})}$ such that $\mathrm{Q}=\mathrm{P}_{0}, \ldots, \mathrm{P}_{\mathrm{h}}=\mathrm{P}$ is the smallest hop count path from Q to P .
(2) For every $1 \leq \mathrm{s}<\mathrm{h}$, let $\omega_{\mathrm{s}}=\delta[\mathrm{k}(\mathrm{s})-\mathrm{k}(\mathrm{s}-1)]$, then $\omega_{\mathrm{s}}=\mathrm{r}$ or $\mathrm{r}-\delta$.
(3) The probability that $\omega_{1}$ has value $r$ is 1 ; if the value of $\omega_{s}(1 \leq s<h)$ is $r$, then the probabilities that $\omega_{s+1}$ takes value r and $\mathrm{r}-\delta$ are both likely to be $1 / 2$; if the value of $\omega_{\mathrm{s}}(1 \leq \mathrm{s}<\mathrm{h})$ is $\mathrm{r}-\delta$, then the probability that $\omega_{\mathrm{s}+1}$ has value r is quite high, and that of value $r-\delta$ is very low.
(4) Let $\zeta=\delta[\mathrm{n}-\mathrm{k}(\mathrm{h}-1)]$, then $\zeta$ has a distribution which is close to normal distribution.

In the remainder of this section we suppose that Q is an anchor node and P is an unknown node which is line-of-light connected with Q , and $\mathrm{Q}=\mathrm{P}_{0}, \ldots, \mathrm{P}_{\mathrm{h}}=\mathrm{P}$ is the smallest hop count path from Q to P. Let $A N(Q, P)=\left(\sum_{i=0}^{h-1} N\left(P_{i}\right)\right) / h$, where $N\left(P_{i}\right)$ is the number of the neighbours of $\mathrm{P}_{\mathrm{i}}(0 \leq i<h)$. It is obvious that the node density within $R(P, Q, r / 2) \quad \rho$ is approximately $(A N(Q, P)+1) / \pi r^{2}$. Let $\delta=2 / \sqrt{\pi \rho}$. Suppose $\tau$ is a random variable with ( $0-1$ ) distribution such that the probability that it is 1 is $1 / 2$. Suppose $\zeta$ is a random variable that follows a normal distribution with mean $r / 2$, and standard deviation $(r-2 \delta) / \sqrt{70}$. By theorem 3.1, we propose the following algorithm.

## Algorithm EDPM (Estimating Distance Using Probability Model Method)

```
flag }\leftarrow\mathrm{ true,sum }\leftarrow0
    for i=1 to h-1 step 1 do
        if flag==true then
        sum}\leftarrowsum+r
        flag \leftarrow\tau;
        if flag==true then
            sum}\leftarrow\mathrm{ sum- </3;
        else
```

9:
$10:$
11:
12:
13:
14: end if
15: end for
16: sum $\leftarrow \operatorname{sum}+\zeta$

## 4 Simulation Experiments

To analyse the performance of EDPM experimentally, we use C++ language to develop a simulator, which can generate various scale and distribution of networks. In experiments 1 to 4 we simulate respectively one type of regularly shaped networks (a rectangular network) and three types of irregularly shaped networks: O-shaped, Cshaped, and star-shaped networks, and compare the performance of EDPM under such network topologies with the algorithms UNDE, BNDE and CHTDE. In every experiment, we randomly select one anchor node and ten unknown nodes. Results of experiments 1 and 2 are shown in Figure 1 (a) and (b), and of experiments 3 and 4 are shown in Figure 2 (a) and (b). Overall, the experiments demonstrated that our EDPM algorithm works effectively under all topologies. Moreover, the more complex the topology is the more our algorithm outperforms other algorithms. Results also confirm the dependence of DV-hop algorithms on the uniformity and regularity of the network.


Fig. 1. Distance estimation accuracy in (a) rectangular (b) O-shaped network


Fig. 2. Distance estimation accuracy in (a) C-shaped (b) star-shaped network

From experiments 1-4 we can tell that when the number of anchor nodes is no greater than one thousandth of the total number of nodes, DV-hop algorithms do not perform well, which is the same conclusion as [2].

Since UNDE requires all anchor nodes to participate in the calculation of $\delta_{A}$, the more irregular the topology is, the worse UNDE performs. A similar behavior occurs for BNDE. CHTDE, which only selects a portion of anchor nodes to participate in the calculation of $\delta_{\mathrm{A}}$, has better performance than UNDE and BNDE under irregular shaped topology, but performs worse than both UNDE and BNDE in regularly shaped networks because of its selection strategy. Both UNDE and BNDE that have poor performance in irregularly shaped networks can reach similar accuracy in regularly shaped networks.

## 5 Conclusion

Our experiments show that the proposed distance estimation method can reach much higher accuracy compared to popular distance estimation methods based on the average hop-distance UNDE, BNDE and CHTDE. We demonstrated that even if exists only one anchor node in the network, it won't affect EDPM accuracy, while distance estimation methods based on the average hop-distance almost have no practical use when only few anchor nodes exist. Moreover, EDPM has higher energy efficiency, since it can save the communication cost that anchor nodes use to broadcast the computed average hop distance to the entire network, while such cost is necessary for all DV-Hop algorithms.

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