Connectivity of Vehicular Ad Hoc Networks in Downtown Scenarios

Shigeo Shioda

Graduate School of Engineering, Chiba University,
1-33 Yayoi, Inage, Chiba, 263-8522 Japan
shioda@faculty.chiba-u.jp
http://www.qos.tu.chiba-u.jp

Abstract. We study the connectivity in vehicular ad-hoc networks in a downtown scenario, where the mobility of vehicles is constrained on a lattice-shaped road network. We theoretically investigate the connectivity under the Poisson-positioning assumption, where vehicles are positioned according to a Poisson process on each road at any arbitrary instants. We find that the Poisson-positioning assumption allows the existence of the finite critical-vehicle density; that is, if (and only if) the density of vehicles is greater than the finite critical density, then there exists a large (theoretically infinite) cluster of vehicles and an arbitrary pair of vehicles in the cluster is connected in single or multiple hops. Under the Poisson-positioning assumption, we derive two approximation formulas for the critical density, which are given as a function of the transmission range of each vehicle and the distance between intersections. We also consider the connectivity under more realistic movement patterns of vehicles where the Poisson-positioning assumption does not hold. We numerically find that, even in non-Poisson-positioning cases, there exists the critical vehicle density, which is larger than the one under the Poisson-positioning assumption. The effectiveness of deploying roadside-relay stations to provide better connectivity between vehicles is also investigated.

Keywords: connectivity, VANET, critical density, bond percolation, Poisson, NETSIM.

1 Introduction

Vehicular ad hoc networks (VANETs) have recently received considerable attention for their potential of improving the safety and the comfort of drivers through infrastructure-less communications among vehicles. Since VANETs allow vehicles to propagate information about their speed and movement direction to vehicles in their vicinity, drivers can quickly detect potentially dangerous events such as lane changes or sudden slowdown [1,16,14]. VANETs also allow vehicles to inform approaching vehicles about situations of traffic congestion, drivers can take alternative routes to reach their destination when original routes are heavily congested.

The connectivity is a fundamental performance measure of ad-hoc networks or sensor networks. For VANETs, the connectivity is very important as a measure to see

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the reliability of information propagation from vehicles to vehicles. Most of previous works on the connectivity of sensor or ad-hoc networks [3,6,11,5] were conducted based on the assumption that the nodes are randomly deployed according to a Poisson process on free two-dimensional space. This assumption does not hold in VANETs in two following senses; first, in VANETs, the mobility of vehicles is constrained along roads. Second, vehicles would not be randomly positioned according to a Poisson process because of the nature of vehicle's mobility; for example, a cluster of vehicles emerges around an intersection when the signal is on red. These properties (constraint along roads and non-randomness) of vehicle positions in VANETs might yield conclusions different with previous works obtained on the connectivity of sensor or ad-hoc networks.

The aim of this work is to study the fundamental characteristics of the connectivity in VANETs. In particular, we investigate what influence is made on the connectivity by the constraint along road or non-randomness of vehicle positions. First, we theoretically investigate the connectivity under the assumption that vehicles are positioned on roads according to a Poisson process at any arbitrary instants, which we call the *Poisson positioning assumption* in this paper. We find that, under the Poisson-positioning assumption, there exists the finite critical-vehicle density; that is, if (and only if) the density of vehicles is greater than the critical density, then a large set (theoretically a set of infinite size) of vehicles would emerge almost surely and an arbitrary pair of vehicles in the set is connected in single or multiple hops. We derive two approximation formulas for the critical density, which are expressed by the transmission range of each vehicle and the distance between intersections.

Next, we consider the connectivity when vehicles move according to the NETSIM model, under which the Poisson-positioning assumption does not hold anymore. We numerically find that, even in non-Poisson-positioning cases, there still exists the critical vehicle density. The critical density in the NETSIM model is, however, larger than the one obtained under the Poisson-positioning assumption. We also investigate the efficiency of deploying roadside relay stations to provide better connectivity between vehicles.

Note that several studies have been made on the connectivity of VANETs [17,4,13] in highway scenarios, where vehicles are positioned along the one-dimensional space (line). Most of these studies attempt to provide better connectivity by deploying stationary or mobile gateways to the Internet, and their focuses are different with our study. We also note that finite critical density does not exist in one-dimensional case. This paper is the extended version of our previous work [15]; an approximation formula (4) for the critical density is newly derived through the notion of weakly open edge in the current manuscript and related numerical results are presented. The proofs of lemmas, used to derive main results, are also presented in the current manuscript.

This paper is organized as follows; in Section 2, we explain the model of road maps, mobility model, and channel model. In Section 3, we investigate the connectivity of VANETs under the Poisson-positioning assumption. In particular, we prove the existence of the critical density and derive two approximation formulas for the critical density; one gives the strict upper bound of the critical density and the other gives estimates lower than the first one. In Section 4, we numerically investigate the connectivity of VANETs when vehicles move according to the NETSIM model by simulation. In

Section 5, we study how the deployment of the roadside relay stations would improve the connectivity. Finally, we conclude this article in Section 6.

2 Network and Channel Model

2.1 Road Network Model

In this paper, we assume a downtown scenario where roads cross each other in a lattice shape. In the following, we refer a section of road between neighboring intersections to as *edge*. Each edge is assigned an index, and let d_i denote the length of edge i. The lengths of edges are not necessarily the same but its maximum is bounded from above; that is $d_{sup} \stackrel{\text{def}}{=} \sup\{d_i,\ i = 1, 2, \ldots\} < \infty$. We also assign an index to each road.

The mobility of vehicles is constrained along roads. We neglect the width of roads, but we assume that two vehicles facing in the opposite direction on a road can move without crash. In this paper we consider the single-lane case, but the extension to multiple-lane cases is possible.

2.2 Channel Model

In this paper, we use the following simplified channel model; when a vehicle transmits a message, all nodes within distance r from the sender correctly receives the message with positive probability, while any nodes out of distance r from the sender cannot receive the message at all. Although the above channel model may look too simple, it would capture the most relevant feature of wireless transmission. This channel model is often called “Boolean Model” [12,3].

We refer the circular region of radius r centered at a vehicle to as the *transmission area* and refer the circular region of radius $r/2$ centered at a vehicle to as the *communication area* (Fig. 1). If the communication areas of two vehicles have an intersection, then two vehicles are directly reachable from each other. We refer a connected region which is made of communication areas to as *cluster* (Fig. 2). Vehicles contained in a common cluster are mutually reachable by single or multiple hops at the physical (MAC) layer.

3 Connectivity under Poisson Positioning Assumption

3.1 Connectivity

In this work, we use the term “connectivity” in the sense of reachability at the physical (MAC) layer. Thus, as explained in Section 2.2, two vehicles are “connected”, if and only if they are contained in the common cluster. The connectivity at the physical layer is a necessary condition for the connectivity at the network (and higher) layer. We are interested in the fundamental characteristics of physical-layer connectivity, such as how it depends on the density of vehicles or the structure of the underlying road networks, and we do not consider the influence of routing protocols or transport protocols on the connectivity. The overheads caused by routing protocols are also neglected.

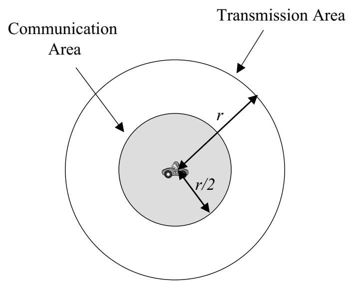

Fig. 1. Transmission area and communication area

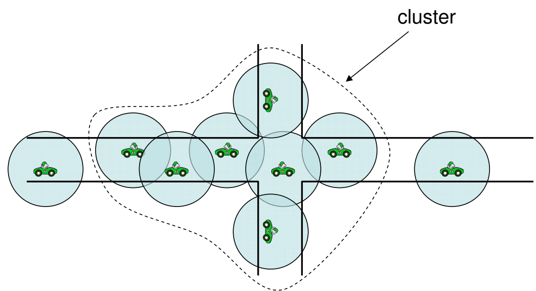

Fig. 2. Cluster

3.2 Poisson-Positioning Assumption

In this section, we investigate the connectivity under the following *Poisson-positioning assumption*:

(1) Vehicles are positioned according to a Poisson process on a road at any arbitrary instants.
(2) The density of vehicles on each road is constant.
(3) The minimum density of vehicles among roads is positive; that is

$$\lambda \stackrel{\text{def}}{=} \inf\{\lambda_s,\ s = 1, \ldots\} > 0,$$

where λ_s is the vehicle density on road s.

3.3 Critical Density

The connectivity is closely related to how large clusters are composed in the network. It follows from the percolation theory of Boolean models that, if vehicles are freely positioned according to a Poisson process in a two-dimensional infinite area, then there exists the finite critical density of vehicles, above which the unique infinite-size cluster emerges almost surely [12]. This fact has been used for analyzing the coverage property

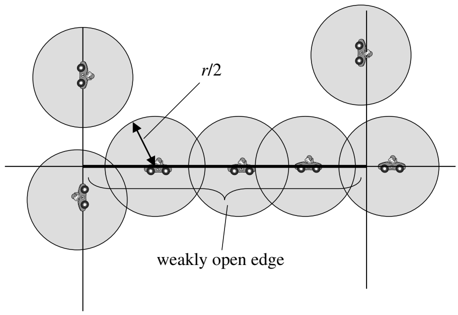

Fig. 3. Weakly open edge

of sensor networks or the connectivity of ad-hoc networks where mobile nodes are randomly positioned in the network [3,6,11]. The existence of the finite critical density was also proved for ad-hoc networks with more complicated channel models [5].

The aim of this section is to prove the existence of the finite critical density under the road topology-limited positions of vehicles. We also attempt to obtain some analytical formulas giving an estimate of the critical density. Note that few results have been obtained concerning analytical formula of the critical density in general Boolean percolation models.

3.4 Existence of the Critical Density for VANETs

We first consider a simple road network, where all edges have the same length d and the densities of vehicles on roads are all equal to λ. We call this road network model the *basic model*. Meanwhile, road network models, which are not the basic model but satisfy the conditions mentioned in Sec. 3.2, are called *general models*.

Definition 1. An edge is called *weakly open* if and only if it is completely covered by a cluster of communication areas (Fig. 3). An edge is called *strongly closed* if and only if it is not weakly open.

Definition 2. An edge is called *strongly open* if and only if it is completely covered by a cluster of communication areas, each of which is produced by a vehicle positioned on the edge itself (Fig. 4). An edge is called *weakly closed* if and only if it is not strongly open.

An edge is weakly open if it is strongly open; the converse is false. Figure 3 shows an example of the edge, which is weakly open but is not strongly open. Note that each edge is strongly open or not, independently of all other edges. An edge being weakly open or not is however associated (positively correlated) with the statuses (weakly open or not) of neighbor edges.

Let p_w (p_s) denote the probability that an edge is weakly (strongly) open. The theory of coverage process [8] on a one-dimensional space leads to the following lemma.

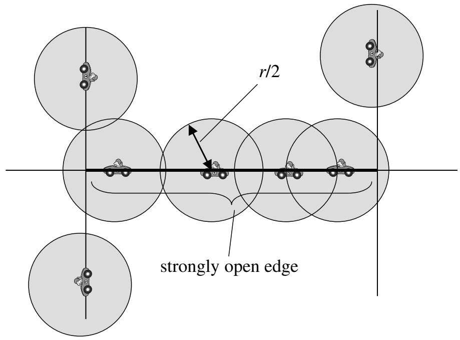

Fig. 4. Strongly open edge

Lemma 1.

$$p_w \geq p_1(\lambda; d, r) \stackrel{\text{def}}{=} 1 + \sum_{j=1}^{\lfloor (d/r)+1 \rfloor} \frac{(-1)^j}{j!} \{\lambda(d-(j-1)r)\}^{j-1} \times e^{-jr\lambda} \{\lambda(d-(j-1)r)+j\}, \tag{1}$$

where $\lfloor x \rfloor$ denotes the largest integer not greater than x.

Proof. See Appendix.

Lemma 2.

$$p_s \geq p_2(\lambda; d, r) \stackrel{\text{def}}{=} \begin{cases} (1-e^{-\lambda r/2})^2 p_1(\lambda; d-r, r) & \text{for } d > r, \\ 1-e^{-(r-d)\lambda} + e^{-(r-d)\lambda}(1-e^{-(d-\frac{r}{2})\lambda})^2 & \text{for } r \geq d > r/2, \\ 1-e^{-d\lambda} & \text{for } r/2 \geq d. \end{cases} \tag{2}$$

Proof. See Appendix.

The well known result of the homogeneous and independent bond percolation on a square lattice (Fig. 5) gives a simple proof of the existence of the finite critical density. In a homogeneous and independent bond percolation model, each edge is open with probability p and closed otherwise, independently of all other edges. A finite or infinite connected sequence of edges is referred to as *path*. It is known that if $p > 0.5$, then there exists the unique infinite-length path using open edges only almost surely [10,7].

The basic model can be considered as a homogeneous and independent percolation model on the square lattice. Note that the Poisson positioning assumption is very essential because, only under this assumption, each edge is strongly open or not, independently of all other edges.

Lemma 3. If $p_2(\lambda; d, r) > 0.5$ in the basic model, then the unbounded cluster emerges almost surely.

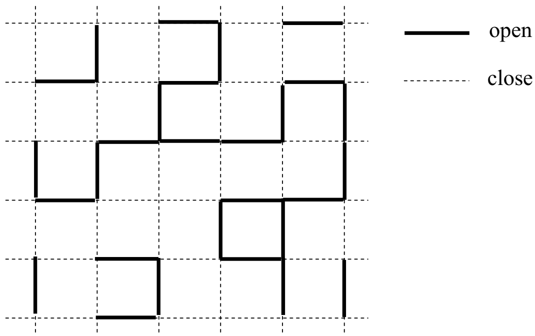

Fig. 5. Bond percolation on a square lattice

Proof. According to the theory of the bond percolation on the square lattice, if $p_s \geq p_2(\lambda; d, r) > 0.5$, then there exists the unique infinite-length path using strongly open edges only almost surely. This infinite-length (strongly) open path is completely covered by the unbounded cluster, which completes the proof.

Lemma 2 reveals that, if $p_2(\lambda; d, r) > 0.5$, then there exists the set of infinite number of vehicles, mutually connected at physical layer, almost surely. Since $p_2(\lambda; d, r)$ is a strictly increasing function of λ, we can define

$$p_2^{-1}(0.5; d, r) \stackrel{\text{def}}{=} \inf\{\lambda; p_2(\lambda; d, r) \geq 0.5\} < \infty.$$

The above discussion shows that, if the density of vehicles is larger than $p_2^{-1}(0.5; d, r)$, then the set of infinite number of vehicles, mutually connected at physical layer, emerges almost surely. This fact proves the existence of the finite critical density in the basic model.

The existence of the finite critical density in the basic model readily proves the existence of the finite critical density in general models.

Lemma 4. If $\lambda_{inf} > p_2^{-1}(0.5; d_{sup}, r)$, then the set of a large (theoretically infinite) number of vehicles, which are mutually connected at physical layer, emerges almost surely in general models.

Proof. See Appendix.

3.5 Approximation Formulas for the Critical Density

Since the phase transition occurs if $\lambda_{inf} > p_2^{-1}(0.5; d_{sup}, r)$ according to Lemma 4, $\lambda_c^{(2)}(d_{sup}, r)$, defined by

$$\lambda_c^{(2)}(d_{sup}, r) \stackrel{\text{def}}{=} p_2^{-1}(0.5; d_{sup}, r), \tag{3}$$

gives an upper bound of the critical density. We have found through simulation experiments that $\lambda_c^{(2)}(d_{sup}, r)$ yields the vehicle density, around which the connection probability defined in Sec. 3.6 reaches one (Sec. 4).

If $\lambda_{inf} > p_1^{-1}(0.5; d_{sup}, r)$, each edge is weakly open with probability larger than 0.5. If there exists an infinite-length path using weakly open edges only, the set of an infinite number of vehicles, mutually connected at physical layer, also emerges. Unfortunately, an edge being weakly open or not depends on whether neighbor edges are weakly open or not. The critical probability of the dependent bond percolation on the square lattice, where the states of different edges are not independent, is not always equal to 0.5. In this paper, however, we conjecture that $\lambda_c^{(1)}(d_{sup}, r)$, defined by

$$\lambda_c^{(1)}(d_{sup}, r) \stackrel{\text{def}}{=} p_1^{-1}(0.5; d_{sup}, r), \tag{4}$$

would yield a good estimate of the critical density. Note that $\lambda_c^{(1)}(d_{sup}, r) \leq \lambda_c^{(2)}(d_{sup}, r)$. We have found through simulation experiments that $\lambda_c^{(1)}(d_{sup}, r)$ gives the vehicle density around which the connection probability exceeds 0.5 (Sec. 4).

The two formulas, $\lambda_c^{(1)}(d, r)$ and $\lambda_c^{(2)}(d, r)$, are for the unit-length-wise critical density, which is defined as the average number of vehicles in an interval of unit length on road for phase transition. In some cases, the transmission-area-wise critical density, defined as the average number of vehicles in a square with sides of length r for phase transition, are useful. The following formulas give the transmission-area-wise critical densities;

$$\alpha_c^{(1)}(d, r) = \frac{2\lambda_c^{(1)}(d, r)r^2}{d}, \quad \alpha_c^{(2)}(d, r) = \frac{2\lambda_c^{(2)}(d, r)r^2}{d}.$$

It is easy to see that $\alpha_c^{(1)}(d, r)$ and $\alpha_c^{(2)}(d, r)$ depend only on d/x. Figure 6 show the estimates of the critical density by $\alpha_c^{(1)}(d, r)$ and $\alpha_c^{(2)}(d, r)$ as well as the critical density when vehicles are freely positioned on a two-dimensional space. It were numerically shown that the critical density for free two-dimensional space is equal to 1.4125 [6,11,9]. Comparing $\alpha_c^{(1)}(d, r)$ (or $\alpha_c^{(2)}(d, r)$) with the critical density on free two-dimensional space in Fig. 6, we see that the road constraint on vehicle positions significantly reduces the critical density especially when $d \gg r$. When d is close to r, however, the road constraint on vehicle positions makes the critical density even larger. The road constraint reduce the possibilities for having different paths between two vehicles, which would increase the critical density especially when d is close to r.

3.6 The Connection Probability

The emergence of the unbounded cluster does not ensure that all vehicles are mutually connected because some of vehicles are not necessarily contained in the unbounded cluster. However, we can conjecture that the probability of two arbitrary vehicles being connected at an arbitrary instant, which is called the *connection probability*, would rapidly change from 0 to 1 around the critical density. We have confirmed this conjecture by simulation, which will be shown in Sec. 4.

From the viewpoint of the bidirectional communications between vehicles, the *persistent connection probability*, which is defined by the probability that two arbitrary vehicles are continuously connected for some duration, is also important. As shown in Sec. 4, the *persistent connection probability* shows similar dependence on the vehicle density with the connection probability. That is, the persistent connection probability rapidly changes from 0 to 1 around some positive vehicle density.

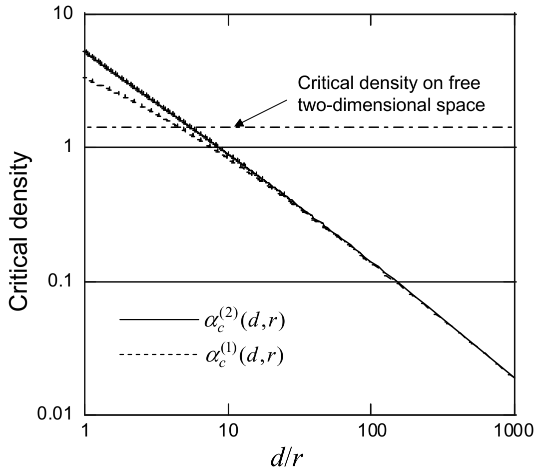

Fig. 6. Critical densities

4 Connectivity in Non-poisson-Positioning Cases

4.1 Mobility Model

Next, through simulation experiments, we investigate the connectivity under more realistic movement patterns of vehicles. In simulations, we emulated movement patterns of vehicles by NETSIM. NETSIM simulates the mobility of vehicles based on the car-following logic [2]. In NETSIM, vehicles probabilistically turn left or right at intersections, and stop at intersections when the signal is on red. The Poisson-positioning assumption obviously does not hold under the NETSIM model. For reference, we also run simulations under the fixed-speed model, where all vehicles were initially deployed on roads according to a Poisson process and moved on a road with fixed speed, which ranged from 20 km/h to 60 km/h. The speeds of vehicles on a given road were all the same[1], and vehicles did not turn left or right at intersections. The Poisson-positioning assumption seems to hold in the fixed-speed model.

4.2 Simulation Condition

In the simulation, we used a 10 km × 10 km square area, where 11 vertical and 11 horizontal roads are crossing in a lattice shape. The distances between intersections are all 1 km. We run simulations with different vehicle densities and with different transmission ranges of vehicles.

4.3 Connection Probability

Figure 7 shows the relationship between the connection probability and the vehicle density when the transmission range of vehicles r is 200 m. In the fixed-speed model,

[1] The speeds of vehicles on different roads are different.

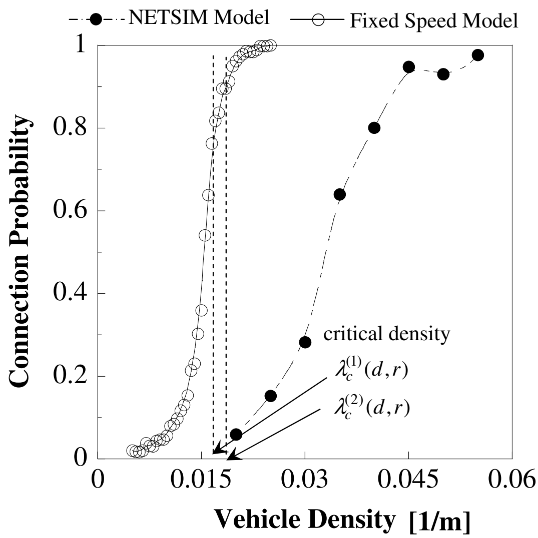

Fig. 7. Connection probability (r = 200m)

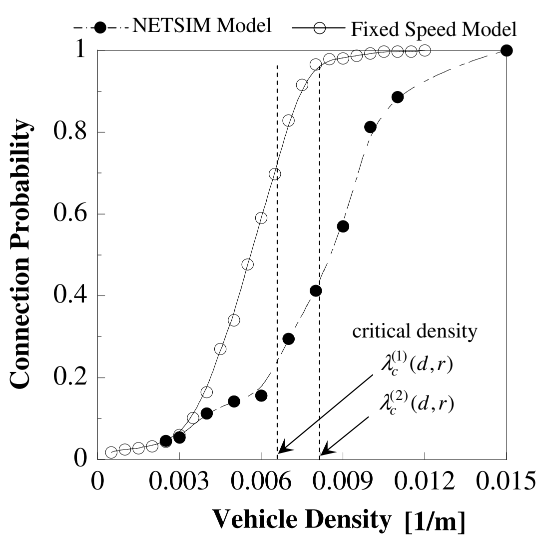

Fig. 8. Connection probability (r = 400m)

the connection probability rapidly tends from 0 to 1 around two estimates of the critical density, $\lambda_c^{(1)}$ (0.017 1/m) and $\lambda_c^{(2)}$ (0.019 1/m), and the connection probability is almost equal to 1 when vehicle density exceeds the two estimates. In particular, $\lambda_c^{(2)}$ gives the vehicle density, at which the connection probability is about to reach one, and $\lambda_c^{(1)}$($< \lambda_c^{(2)}$) gives the vehicle density around which the connection probability exceeds 0.5.

This result agrees with the theoretical consequence obtained in Sec. 3. We also observe that the dependence of the connection probability on the vehicle density in the NETSIM model is very similar with that in the fixed-speed model; the connection probability in the NETSIM model also shows rapid increase when the vehicle density is around 0.02 1/m and it is close to 1 when the vehicle density exceeds 0.05 1/m. This fact suggests that the critical density still exists even in non-Poisson-positioning cases. Note that the critical density in the NETSIM model is larger than that in the fixed-speed model. We show the results when $r = 400$ m in Fig. 8, where the similar dependence of the connection probability on the vehicle density is observed, but the critical density when $r = 400$ m is smaller than that when $r = 200$ m. In the fixed-speed model vehicles were randomly positioned along the road, while in the NETSIM model a cluster emerged around the intersection along the horizontal road. The cluster in the NETSIM model was made by the action of traffic signal; vehicles on the horizontal road were waiting for the light turn to green. In the NETSIM model, clusters of vehicles often emerged at intersections and thus vehicle positions did not follow the Poisson process, which requires higher vehicle density to ensure the connectivity between vehicles.

4.4 Persistent Connection Probability

Figures 9 and 10 show the persistent connection probabilities during 10 minutes when the transmission ranges of vehicles are 200 m and 400 m, respectively. The dependence of the persistent connection probability on the vehicle density is very similar with that of the connection probability; we also see the rapid increase of the persistent connection probability as the vehicle density increases. Note that higher vehicle density is required for the phase transition (rapid increase from zero to one) in terms of the persistent connection probability compared with the connection probability. For example, in the fixed-speed model, the persistent connection probability begins to increase when the vehicle density exceeds the estimate of critical density by $\lambda_c^{(2)}$ (0.019 1/m) while the connection probability is about to reach one at the same vehicle density.

5 Deployment of Roadside Relay Stations

Deploying message relay stations (RSs) on roadside seems to improve the connectivity between vehicles. Finally, we theoretically investigate the effectiveness of deploying the roadside RSs. For simplicity, we assume that the densities of vehicles on roads are all equal to λ and the lengths of edges are all equal to d. First observe that if

$$\lambda \geq \lambda_c^{(2)}(d, r) = p_2^{-1}(0.5; d, r), \tag{5}$$

then the deployment of the RSs is not necessary because the vehicle density is over the critical density and thus the connectivity between vehicles is almost ensured without RSs. We call the region satisfying (5) the *RS-unnecessary region*.

Meanwhile, if $\lambda < \lambda_c^{(2)}(d, r) = p_2^{-1}(0.5; d, r)$, then the deployment of the roadside RSs may improve the connectivity. Now let L_{rs} be the deployment interval of RSs. Since the intensity of RSs is given by $1/L_{rs}$, the connection probability would be almost equal to 1

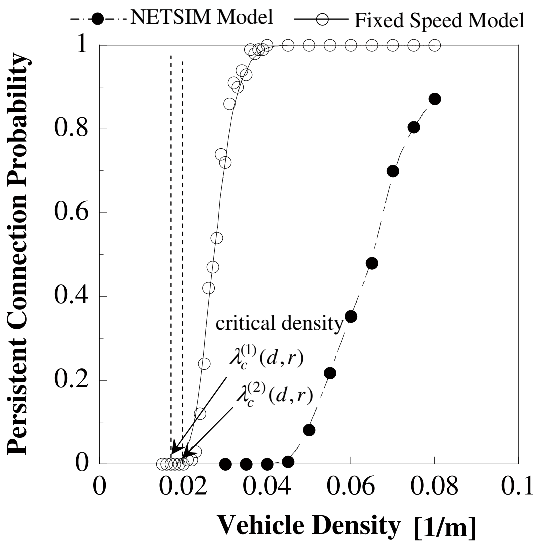

Fig. 9. Persistent Connection probability (r = 200m)

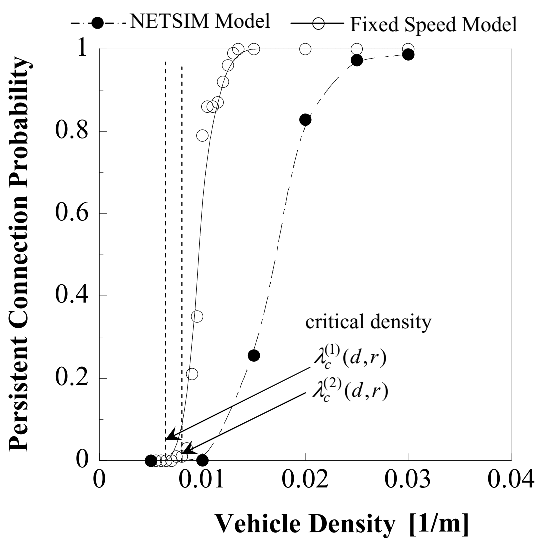

Fig. 10. Persistent Connection probability (r = 400 m)

when $\lambda + 1/L_{rs} \geq \lambda_c^{(2)}(d,r)$. This consideration yields the following guideline for the RS deployment interval in order to have the full (100-percentile) inter-vehicle connectivity:

$$L_{rs} \leq \frac{1}{\lambda_c^{(2)}(d,r) - \lambda} = \frac{1}{p_2^{-1}(0.5; d,r) - \lambda}. \tag{6}$$

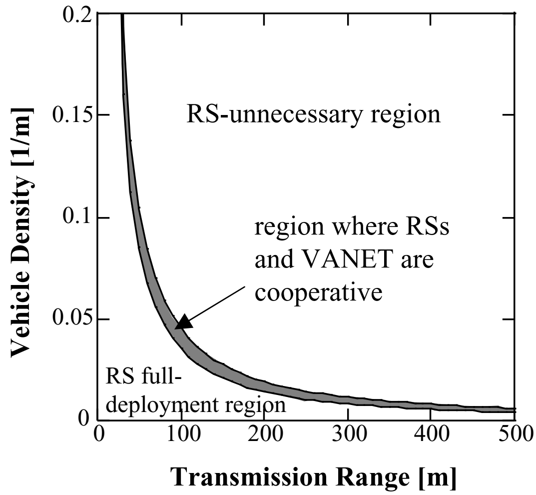

Fig. 11. RS-unnecessary and VANET-unnecessary regions

Here we note that if

$$\frac{1}{p_2^{-1}(0.5; d, r) - \lambda} < r, \tag{7}$$

then the guideline (6) requires $L_{rs} < r$, under which RSs are mutually connected and thus vehicles are also mutually connected via RSs without message relay of vehicles. In other words, in the region satisfying (7), VANET does not essentially contribute the inter-vehicle connectivity. We call the region satisfying (7) the *RS-full-deployment region*.

The above consideration clarifies that RSs and VANET are cooperative only the outside of the union of RS-unnecessary and RS-full-deployment regions, which is given below

$$p_2^{-1}(0.5; d_{sup}, r) - \frac{1}{r} \leq \lambda < p_2^{-1}(0.5; d_{sup}, r).$$

Figure 11 shows the region (hatched region) where the RS-deployment and VANET are cooperative when $d = 1$ km. We see that this region is so small. This simply illustrates that deploying roadside-relay stations to provide better connectivity between vehicles is not so effective in fixed-speed model.

6 Conclusion

In this paper, we study the fundamental characteristics of physical-layer connectivity in VANETs. We theoretically find that, under the Poisson-positioning assumption, there exists the finite critical-vehicle density, above which a large (theoretically infinite-size) set of vehicles emerges and an arbitrary pair of vehicles in the set is connected in single or multiple hops. We also consider the connectivity under the NETSIM-vehicle-mobility model where the Poisson-positioning assumption does not hold anymore.

We numerically find that, even in such non-Poisson-positioning cases, there still exists the critical vehicle density. The critical density in non-Poisson-positioning cases is, however, larger than that expected under the Poisson-positioning assumption.

We find that deploying roadside relay stations is not so effective in providing better connectivity under the Poisson-positioning assumption. However, this is not always the case if the Poisson-positioning assumption does not hold. In addition to this, in this work, we do not consider the effect that the radio-wave-propagation is obstructed by buildings at corners of intersections. If we take into account of the obstruction by the buildings, we may have different conclusions concerning the effectiveness of deploying roadside relay stations. This remains as one of future works.

Appendix

A.1 Proof of Lemma 1

An edge (say edge i) is weakly open if edge i is completely covered by a cluster of communication areas of vehicles on the straight road including edge i. The probability of the latter is known as *the complete coverage probability* in the theory of the one-dimensional coverage process. According to formula (2.23) of [8], which gives the analytical expression of the complete coverage probability, $p_1(\lambda; d, r)$ is the probability that the interval of length d is completely covered by clusters made of segment with fixed-length r. That is, $p_1(\lambda; d, r)$ is equal to the probability that edge i is completely covered by a cluster of communication areas of vehicles on the straight road including edge i, which completes the proof.

A.2 Proof of Lemma 2

If $d \le r/2$, an edge is strongly open if at least one vehicle exists on the edge, whose probability is equal to $1 - e^{-d\lambda}$.

If $r/2 < d \le r$, an edge is strongly open if at least one vehicle exists in interval B in Fig. 12. Even if there is no vehicle in interval B, an edge is strongly open if intervals A and C in Fig. 12 respectively contain at least one vehicle. The probability that one vehicle exists in interval B is equal to $1 - e^{-(r-d)\lambda}$, and the probability that no vehicle exists in interval B but intervals A and C respectively contain at least one vehicle is equal to $e^{-(r-d)\lambda}(1 - e^{-(d-\frac{r}{2})\lambda})^2$. Thus, (2) also holds when $r/2 < d \le r$.

If $r < d$, an edge is strongly open if at least one vehicle exists in intervals A and C in Fig. 12 respectively and interval B in Fig. 12 is completely covered by one cluster. The probability that at least one vehicle exists in intervals A and C respectively is equal to $(1 - e^{-\lambda r/2})^2$, and the probability that interval B is completely covered by one cluster is equal to $p_1(\lambda; d, r)$. The event that at least one vehicle exists in intervals A and C respectively and the one that interval B is completely covered by one cluster are associated, so (2) also holds when $r < d$.

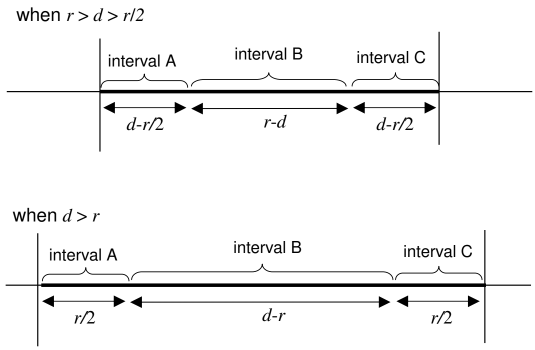

Fig. 12. Partitioning an edge into three intervals

A.3 Proof of Lemma 4

Let $p_s(i)$ be the probability that edge i is strongly open. Since $p_2(\lambda; d, r)$ is strictly increasing in terms of λ and strictly decreasing in terms of d,

$$p_2(\lambda_s; d_i, r) \geq p_2(\lambda_{inf}; d_{sup}, r).$$

Thus, if $\lambda_{inf} > p_2^{-1}(0.5; d_{sup}, r)$, then

$$p_s(i) \geq p_2(\lambda_s; d_i, r) \geq p_2(\lambda_{inf}; d_{sup}, r) > 0.5.$$

meaning that each edge is strongly open with probability larger than 0.5. Note that the general model corresponds to an inhomogeneous bond percolation because the probabilities of edges being open are different from each other. Thus, we need to prove that, if each edge is open with probability larger than 0.5, the unbounded cluster emerges almost surely even in inhomogeneous (and independent) bond percolation. To this end, let $\{X_i\}_{i\in\mathbb{N}}$ be a family of independent random variables indexed by the edge, where X_i is uniformly distributed on [0, 1]. We also define

$$\eta_i(p_i) = \begin{cases} 1 & \text{if } X_i > p_i, \\ 0 & \text{if } X_i \leq p_i. \end{cases}$$

We can say that edge i is open if $\eta_i(p_s(i)) = 1$. Now choose p_0 so that $p_2(\lambda_{inf}; d_{sup}, r) > p_0 > 0.5$. It is clear that $\eta_i(p_s(i)) \geq \eta_i(p_0)$ whenever $p_s(i) \geq p_0$. That is, we can couple the general model with a homogeneous bond percolation model where the edge is open with probability $p_0 > 0.5$. Since the homogeneous bond percolation model has the unique infinite-size cluster with probability one when $p_0 > 0.5$, the general model does so, which completes the proof.

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