Compression of Gray Scale Images Using Linear Prediction on Wavelet Coefficients

P.S. Arya Devi and M.G. Mini

Dept. of Electronics, Model Engineering College
Kochi, India
aryabinoy@gmail.com, mininair@mec.ac.in

Abstract. Each year, terabytes of image data- both medical and non medical- are generated which substantiates the need of image compression. In this paper, the correlation properties of wavelets are utilised in linear predictive coding to compress images. The image is decomposed using a one dimensional wavelet transform. The highest level approximation and a few coefficients of details in every level are retained. Using linear prediction on these coefficients the image is reconstructed.With less predictors and samples from the original wavelet coefficients compression can be achieved. The results are appraised in objective and subjective manner.

Keywords: Image Compression, bits per pixel (bpp), wavelet transform, linear predictive coding (LPC), correlation.

1 Introduction

Image compression is a key issue to be addressed in the area of transmission and storage of images. The storage and transmission of large volumes of image data is a challenging task owing to limited storage space and bandwidth. With the emerging technologies, there are promises of unlimited bandwidth. But the need and availability for images outgrow the increase in network capacity. The high costs involved in providing large bandwidth and huge storage space further necessitates the need for image compression. Image compression finds its application in various fields ranging from satellite imaging, medical imaging to teleconferencing, HDTV etc. Compressing an image is the process of reducing the size of the image, without degrading the image quality by exploiting its redundancy and irrelevancy. Even as many techniques available and emerging in the field of image compression, the demand for digital image transmission indicate that there is always room for better and novel methods.

Wavelet Transform (WT) is a tool that allows multi resolution analysis of an image. It can extract relevant information from an image and can adapt to human visual characteristics. WT decomposes an image into a set of different resolution sub-images corresponding to the various frequency bands and gives a multi-resolution representation of the image with localization in both spatial and frequency domains.

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Images can be modeled using homomorphic systems, in which a logarithmic transformation is used to convert multiplicative superposition to additive superposition of signals. In such systems linear prediction can be compatible [1]. Linear prediction model optimally extract information about a current sample from a neighborhood of samples in its causal past [2]. The process of signal or system modeling removes redundancy, which is the essence of data compression [3].

In our proposed method the image is decomposed into approximation and details at multi-scale. The highest level approximation and details are retained. In addition to that a few coefficients of details in every level are also retained. Using linear prediction on these coefficients the details are reconstructed. The approximation of a lower level is reconstructed from higher level coefficients using inverse wavelet transform (IWT). The predicted coefficients are used here. Finally the original image is reconstructed in this manner.

2 Theory

A brief revision of theory associated with the proposed method is given below. The areas dealt here are image compression fundamentals, wavelet transform, linear prediction and correlation properties of wavelet.

2.1 Image Compression Fundamentals

In most of the images, the neighboring pixels are correlated, and image contains redundant information. By compressing an image we should find a less correlated representation of the image. Image compression relies on reduction of redundancy and irrelevancy. Redundancy reduction removes duplication from image, and irrelevancy reduction omits parts of the signal that will not be noticed by Human Visual System (HVS). The redundancies in an image can be identified as spatial redundancy, coding redundancy and inter pixel redundancy. Image compression aims at reducing the number of bits needed to represent an image by removing the redundancies as much as possible. [4]

Image compression methods can mainly be classified as lossy compression and lossless compression. In lossless compression the decompressed image will be an exact replica of the original image without any loss in data. It offers a compression ratio of around 2 to 4. Lossy compression will not result in exact recovery of original image. Some fine details are sacrificed to obtain better compression ratio. The compression ratio in this case can exceed 100[5]. Wavelet compression methods are one of the most popular compression methods. Due to symmetric impulse response bior9/7 wavelet is suitable from compression perspective [6].

2.2 Wavelet Transform

Discrete dyadic wavelet transforms have been successfully applied to solve different problems in many fields, owing to the good spatial localization and fairly good frequency localization of their bases [7], [8]. They are invertible and efficient. When applied to an image, the image is split into its details at different scales, orientations and positions. The transformed image is de-correlated [9].

One-dimensional wavelet theory defines a function ψ, the wavelet, and its associated scaling function φ, such that the family of functions $\{ \psi^j(x) \}$ $j \in Z$, are orthonormal , where

$$\psi^j(x) = \sqrt{2^j}\psi(2^j x) \tag{1}$$

The wavelet transform can be implemented by quadrature mirror filters $G = (g(n))$ and $H = (h(n))$, $n \in Z$, where

$$h(n) = 1/2\,\langle\phi(x/2), \phi(x-n)\rangle;\ g(n) = (-1)^n h(n) \tag{2}$$

(< >denotes L^2 inner product). H corresponds to a low-pass filter, and G is an octave wide high-pass filter. The reconstruction filters have impulse responses [10]

$$h^*(n) = h(l - n)\ ;\ g^*(n) = g(l - n) \tag{3}$$

A group of transform coefficients resulting from the same sequence of low pass and high pass filtering operations gives approximation and detail coefficients respectively.

Two dimensional wavelet transform is performed by applying a separable filter bank to the image. Applying the one dimensional transform in each row, we get two approximation and details coefficients in each row (*L* and *H* subbands). Then applying one dimensional DWT column-wise on these *L* and *H* subbands, four subbands *LL*, *LH*, *HL*, and *HH* are obtained. *LL* is a coarser version of the original input signal called approximation image. *LH*, *HL*, and *HH* are the high frequency sub-bands containing the detail information (vertical, horizontal and diagonal details images). The number of decompositions performed on original image to obtain sub-bands is called sub-band decomposition [11]. Fig. 1 shows 2-Dimensional DWT (2D DWT) performed in separable form on an image and the two level decomposition of the image.

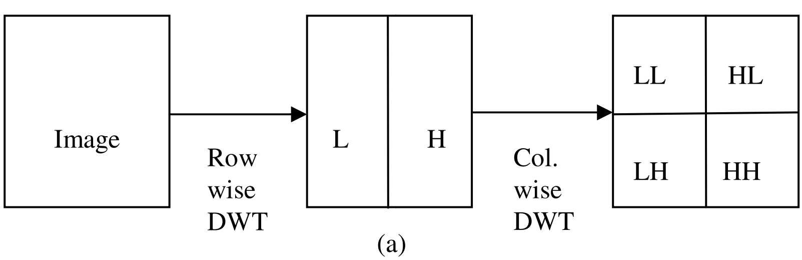
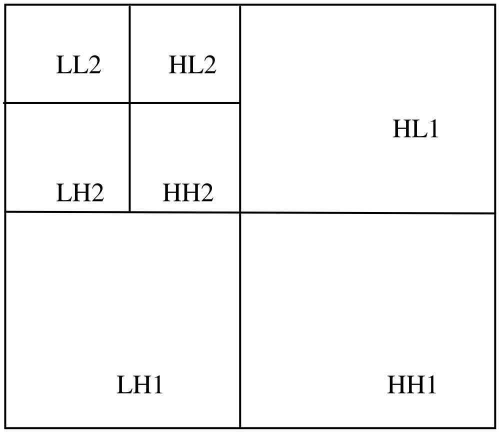


Fig. 1. 2D DWT performed in separable form on an image a) First level decomposition b) Second level decomposition



(b)

Fig. 1. *(continued)*

2.3 Linear Prediction

Linear Predictive Coding (LPC) is a popular and efficient technique mainly used in signal compression and speech processing. The signal is modeled as a linear combination of its past values and a hypothetical input to a causal system whose output is the given signal. In the frequency domain, this is equivalent to modeling the signal spectrum by a pole zero model [3]. The sample of the signal is predicted and if the prediction is done from weighted sum of other samples of the signal, the linear predictive model is auto regressive (AR) model [12]. The model parameters are obtained by a least squares analysis in the time domain and frequency domain [3]. The AR model of a process expresses it as finite linear aggregate of its previous values. Let us denote the values of stochastic process at equally spaced times, n,(n-1),(n-2)... by $y(n)$, $y(n$-1),$y(n$-2),.....

$$y(n) = \alpha_1\, y(n-1) + \alpha_2\, y(n-2) + .. + \alpha_p\, y(n-p) \tag{4}$$

Exploiting autocorrelation values the Yule-Walker equations can be arrived at.

$$R_{yy}(k) = \sum_{m=1}^{p} \alpha_k\ R_{yy}(m-k) \tag{5}$$

Using Levinson- Durbin algorithm [13] the above equations can be solved. Levinson Durbin algorithm is as follows:

1. Initialise the recursion

$$\alpha_0(0) = 1$$
$$\varepsilon_0 = r_{yy}(0) \tag{6}$$

2. For k=0,1,...M

$$\gamma_{p-1} = r_{yy}(p) + \sum_{k=1}^{p-1} \alpha_p(k) r_{yy}(p-k) \tag{7}$$

$$\Gamma_p = -\frac{\gamma_{p-1}}{\epsilon_{p-1}} \tag{8}$$

$$\alpha_p(k) = \alpha_{p-1}(k) + \Gamma_p \alpha_p^*(p-k) \tag{9}$$

$$\alpha_p(k+1) = \Gamma_p \tag{10}$$

$$\varepsilon_{p+1} = \varepsilon_p[1 - |\Gamma_p|^2] \tag{11}$$

Thus we can obtain the predictor coefficients. Using these coefficients, the signal can be predicted using (4).

2.4 Correlational Properties of Wavelet Coefficients

Let $f(t)$ be a signal whose WT is calculated. Let CA_j and CD_j be the approximation and details coefficients at decomposition level j. We assume the signal to be stationary and so transform coefficients also should have same property. The expectation of approximation coefficients at the j^{th} level can be given as

$$E[CA_j] = \sqrt{2^{-j}} \int f(t)\, E_n[\Phi(2^{-j}t - n)]dt \quad \approx \frac{\sqrt{2^{-j}}}{N} \int f(t) \tag{12}$$

where E_n is an average operator while N is the length of support range of $[\Phi(2^{-j}\, t\text{-}n)]$. The expectation of approximation coefficients is proportional to the average of the original signal. It can also be found that detail coefficients have zero mean.

$$E[CD_j] = \sqrt{2^{-j}} \int f(t)\, E_n[\Psi(2^{-j}t - n)]dt = 0 \tag{13}$$

The autocorrelation of CA_j and CD_j are proportional to autocorrelation of original signal. CAj and CDj are decorrelated. The detail coefficients at different levels are also decorrelated. The property of localized WT coefficients is exploited in our method. This property will lead to more correlation of wavelet coefficients in the same prediction channel. The higher the correlation, the more is the scope for redundancy removal in prediction filtering [14]

3 The Proposed Method

In the proposed method one dimensional (1D) DWT is performed on each row of the original image. The decomposition up to four levels is done and the approximation and detail coefficients of fourth level are retained. Predictor coefficients for each row in detail coefficients at $(i\text{-}1)^{th}$ level is calculated. Few prediction coefficients and that much detail coefficients, at each level along with retained coefficients from fourth level forms compressed image.

In the decompression, with a few predictor coefficients and that much detail coefficients in the i^{th} level, all the details coefficients in the i^{th} level are reconstructed. Approximation coefficients for i^{th} level is obtained from inverse DWT on $(i+1)^{th}$ level of wavelet coefficients. Using approximation and detail coefficients of i^{th} level approximation of $(i\text{-}1)^{th}$ level is obtained. Thus the whole image is reconstructed.

4 Evaluation Criteria

For assessing fidelity of reconstructed image, there are two classes of criteria; objective fidelity criteria and subjective fidelity criteria. In objective fidelity criteria the level of information loss is expressed as a function of input image, compressed image and subsequently decompressed image. Some of the objective fidelity criteria are normalized mean square error (NMSE), normalized absolute error (NAE), and peak signal to nose ratio (PSNR) [11]. The measures like compression ratio (CR) and bit per pixel (bpp) also evaluate a compression method.

Let $x(m,n)$ be the original M×N pixel image and $x^*(m,n)$ be the reconstructed image

$$NMSE = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}[x(m,n) - x^*(m,n)]^2}{\sum_{m=1}^{M}\sum_{n=1}^{N}[x(m,n)]^2} \tag{14}$$

$$NAE = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}[x(m,n) - x^{*}(m,n)]}{\sum_{m=1}^{M}\sum_{n=1}^{N}[x(m,n)]} \tag{15}$$

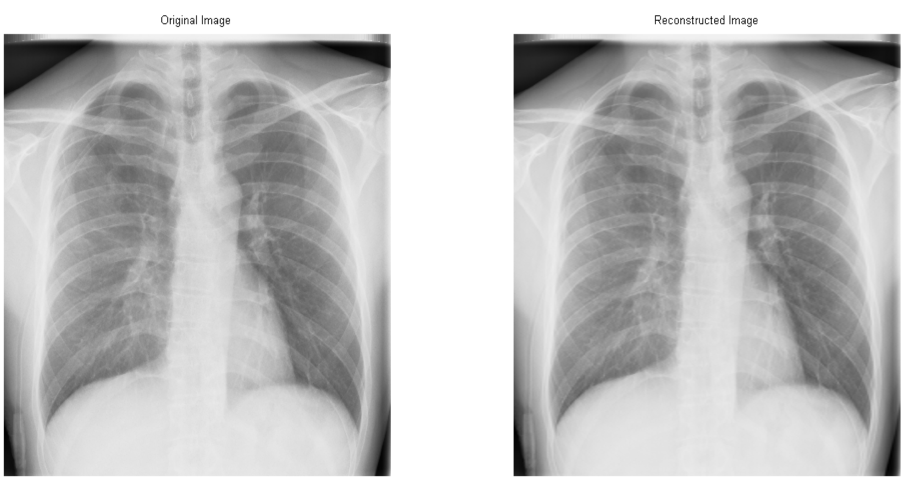
$$PSNR = 10\log_{10}\frac{255}{MSE} \tag{16}$$

The subjective fidelity criteria measure the quality of an image by human evaluation using an absolute rating scale. One possible rating scale is as shown below. {-3, -2, -1, 0, 1, 2, 3} which represent {much worse, worse, slightly worse, the same, slightly better, better, much better} [15].

5 Results and Discussions

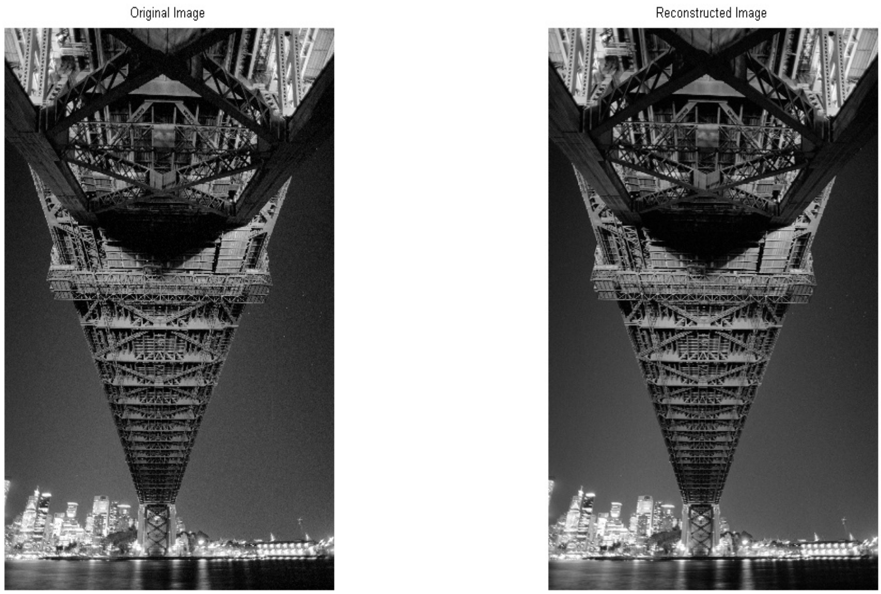
The images with large dimensions (1024 x 1024) are taken as test images. The test images constitute both medical and natural gray scale images. The wavelet used is bior4.4. The original and reconstructed images are shown in the fig. 2 and fig 3. The magnified versions of original and decompressed images are shown for comparison in fig. 4 and fig. 5.

The subjective assessment of the reconstructed images is also done. The original and decompressed images were shown to different viewers and there were asked to rate the images. As per the feedback this method is found to be effective in compressing the images without losing their vital information.



(a) (b)

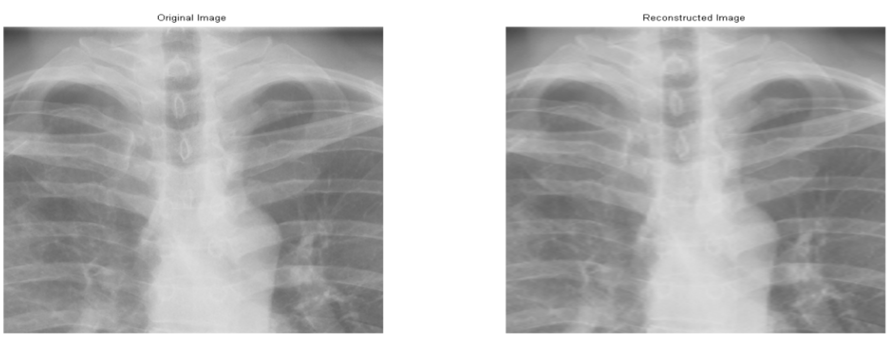
Fig. 2. Decompressed biomedical image at 7:1 a) Original image b) decompressed image



(a) (b)

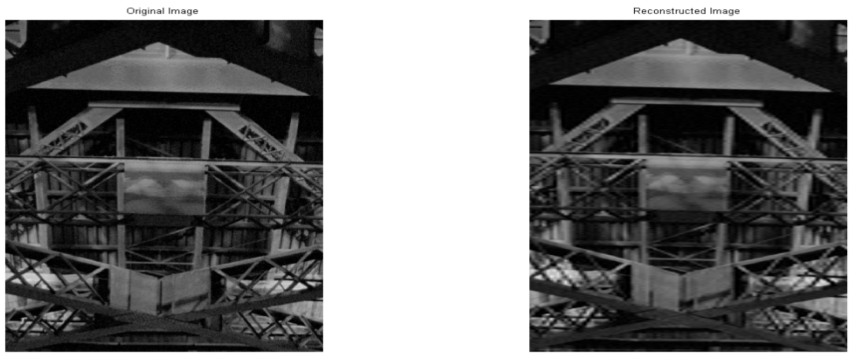
Fig. 3. Decompressed natural image at 6.7:1 a) Original image b) decompressed image

The performance of the algorithm is validated using objective criteria like PSNR, NMSE, NAE and BPP and various performance curves are shown in fig. 6. It is found that as compression ratio increases PSNR decreases. At higher compression ratio (CR), PSNR decreases at slower rate. This method gives PSNR higher than 45dB for some images.



(a) (b)

Fig. 4. Zoomed medical image -zoomed by 2 Original image b) decompressed image



(a) (b)

Fig. 5. Zoomed image - zoomed by 4 a) Original image b) decompressed image

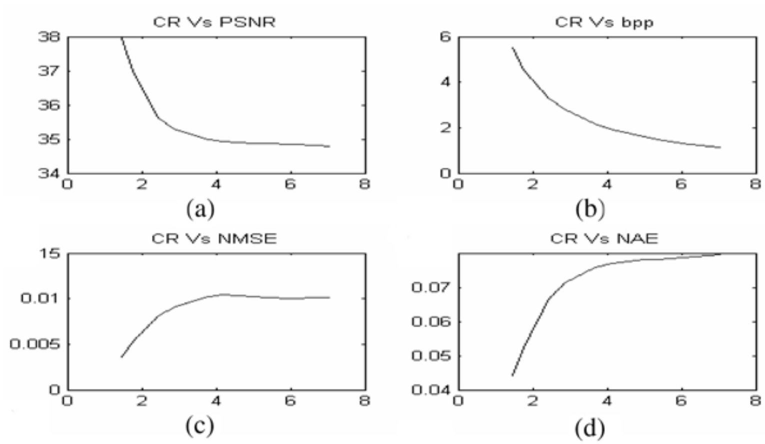


Fig. 6. Performance Curves a) PSNR b) NMSE c) bpp d) NAE

6 Conclusion

The proposed method works well with images of large dimensions. This method gives same compression ratio and bpp on images of same size. Here the correlation of each row in the image with its wavelet coefficients exploited to predict the next wavelet coefficients. The prediction can be done with number of coefficients as small as five. The performance method is evaluated on the basis of subjective criteria and parameters like PSNR, NAE, NMSE and bpp. The experimental results show that the proposed method, in addition to achieving good compression gives a better performance.

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