# A Distributed OFDM Polarizing Transmission via Broadcast Switching

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**Abstract.** A simple polar-and-forward (PF) relay scheme, with source polar coding and relay polar coding, is proposed to provide an alternative solution for transmitting with high reliability. We analyze the bit error rate (BER) performance behaviors with the switching polar system equipped with four OFDM blocks, which is an idea approach to select OFDM symbols that tend to polarize in terms of the reliability under certain OFDM combining and splitting for the polarizing frequency selective fading (FSF) channels.

**Keywords:** Frequency selective fading, OFDM, bit error rate, polar codes.

## 1 Introduction

The channel polarization shows an attractive construction of provably capacity-achieving coding sequences [1, 2, 3, 4]. Recently, MIMO relay communications, together with orthogonal frequency division multiplexing (OFDM) techniques, have proposed an effective way of increasing reliability as well as achievable rates in next generation wireless networks. A usual approach to share information is to tune in the transmitted signals and process the whole (or partial) received information in regenerative [5] or non-regenerative way [6].

The problem with the previous relay system is the data rate loss as the number of relay nodes increases [7, 8]. This leads to the use of polar sequences in the MIMO-OFDM system, where relay nodes are allowed to simultaneously transmit multiple OFDM symbols over the FSF channels. In the light of superiority of these relay strategies with the availability of channel state information (CSI), we consider the relay scheme design for the FSF channels using the polar-and-forward (PF) technique, in which each relay node polarizes and retransmits the partial signals with the fixed power constraint. We consider a simple polar system that achieves the fascinating symmetric capacity of the FSF channels based on OFDM polarizing with a successive interference cancellation (SIC) decoder at

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destination node, which is motivated by Arikan's channel polarization that shows the occurrence of capacity-achieving code sequences for the binary-input discret memoryless channels. It is an extension of work where OFDM combining and splitting are used for recursive code construction with the SIC decoding, which are essential characters of the polar system.

Some notations are defined throughout this paper. $\mathbb{Z}_N$ denotes an integer set $\{0, 1, \cdots, N-1\}$. Superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, and $(\cdot)^*$ represent the transpose, complex conjugate transpose, and complex conjugate of a matrix. $\mathrm{diag}(\mathbf{d}_0, \cdots, \mathbf{d}_{N-1})$ is a diagonal matrix with diagonal entries $\mathbf{d}_0, \cdots, \mathbf{d}_{N-1}$.

## 2 Channel Polarization

We consider the distributed wireless system [7, 8] based on OFDM modulation with $N$ subcarriers. There is one source node $S$, one destination node $D$, and two relay nodes $R \triangleq \{R_1, R_2\}$, which are provided with one transmit antenna. The design of the relay scheme that can mitigate relay synchronization errors is considered. The $N_s$ independent OFDM symbols are transmitted simultaneously from source node $S$ to destination node $D$ in two stages. In the first stage the initial OFDM symbols are polarized and transmitted from source node $S$ to each relay node $R_k$, $\forall\ k \in \{1, 2\}$. In the second stage each relay node $R_k$ polarizes and forwards the (partial) symbols received from source node $S$ to destination node $D$ while source node $S$ keeps silent. We further assume that each single-link between a pair of transmit and receive antenna is frequency selective Rayleigh fading with $L$ independent propagation, which experiences quasi-static and remains unchanged in certain blocks. Denote the fading coefficients from source node $S$ to relay node $R_k$ as $\mathbf{h}_{SR_k} = \phi_k$ and coefficients from relay node $R_k$ to destination node $D$ as $\mathbf{h}_{R_k D} = \kappa_k$. Assume that $\phi_k$ and $\kappa_k$ are all independent zero mean complex Gaussian random variables. Channel impulse responses $\phi_k(t)$ from source node $S$ to destination node $R$ are

$$\phi_k(t) = \sum_{l=0}^{L-1} \alpha_{sk}(l)\delta(t - \tau_{l,sk}),$$

where $\alpha_{sk}(l)$ represents the channel coefficient of the $l^{th}$ path of the FSF channels, and $\tau_{l,sk}$ is the corresponding path delay. Each channel coefficient $\alpha_{sk}(l)$ is modelled as zero mean complex Gaussian random variables with variance $\sigma_{l,sk}^2$ such that $\sum_{l=0}^{L-1} \sigma_{l,sk}^2 = 1$. Similarly, other channel impulse responses $\kappa_k(t)$ from $R_k$ to $D$ are

$$\kappa_k(t) = \sum_{l=0}^{L-1} \alpha_{rk}(l)\delta(t - \tau_{l,rk}),$$

where $\alpha_{rk}(l)$ represents the channel coefficient, and $\tau_{l,rk}$ is the corresponding path delay. In addition, we denote the average power of each relay $R_k$ as $p_r$. The average transmit power at source node $S$ is $p_t$. The constraint on the total network power is $p = p_t + 2p_r$. We adopt the power allocation strategy suggested in [9], i.e.,

$$p_t = 2p_r = p/2. \tag{1}$$

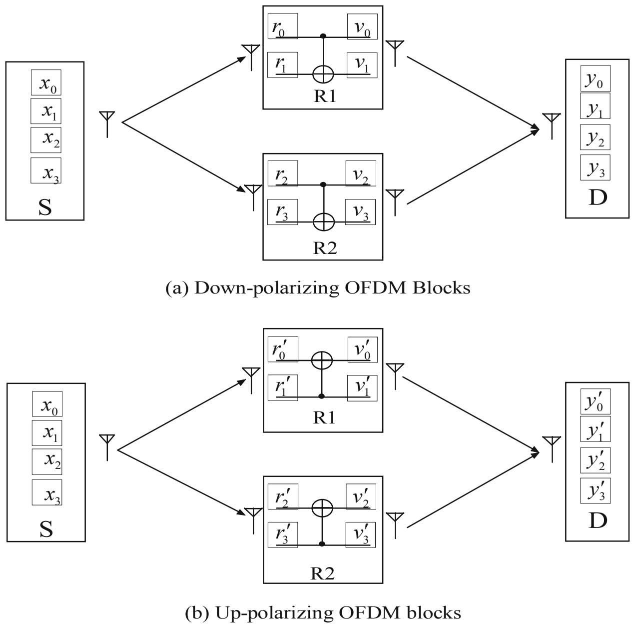


**Fig. 1.** The polar MIMO-OFDM relay system based on the OFDM polarizing for the FSF channels

At source node $S$ the transmitted information are modulated into complex symbols $x_{ij}$ and then each $N$ modulated symbols as a block are poured into an OFDM modulator of $N$ subcarriers. Denote four consecutive OFDM symbols by $x_i = (x_{i,0}, \cdots, x_{i,N-1})^{\mathrm{T}}$, $\forall\ i \in \mathbb{Z}_4$. We define $x_i + x_j = (x_{i,0} + x_{j,0}, \cdots, x_{i,N-1} + x_{j,N-1})^{\mathrm{T}}$, $\forall\ i, j \in \mathbb{Z}_4$, for polarization calculation.

The four consecutive OFDM symbols are processed with the *down-polarizing* $4 \times 4$ matrix $\mathbf{Q}_4$ at source node $S$, i.e., $\mathbf{U} = \mathbf{X}\mathbf{Q}_4$, where $\mathbf{U} = (u_0, u_1, u_2, u_3)$ denotes the polarizing matrix of size $N \times 4$, $\mathbf{X} = (x_0, x_1, x_2, x_3)$ denotes the signal matrix of size $N \times 4$ corresponding to four OFDM blocks, the source polar matrix $\mathbf{Q}_4$ is given by $\mathbf{Q}_4 = \mathbf{I}_2 \otimes \mathbf{Q}_2$, where $\otimes$ denotes the Kronecker product and $\mathbf{Q}_2$ is the Arikan *down-polarizing* matrix [1], i.e., $\mathbf{Q}_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Namely, we have $u_{2k-2} = x_{2k-2}$ and $u_{2k-1} = x_{2k-2} + x_{2k-1}$, $\forall\ k \in \{1, 2\}$.

In the OFDM modulator, the four consecutive blocks are modulated by the $N$-point FFT. Then each block is precoded by a cyclic prefix (CP) with length

$l_{cp}$. Thus each OFDM symbol consists of $L_s = N + l_{cp}$ samples, which are broadcasted to two relay nodes. Denote by $\tau_{sd2}$ the overall relative delay from source node $S$ to relay node $R_2$, and then to destination node $D$, which is relative to relay node $R_1$. In order to combat against timing errors, we assume that $l_{cp} \geq \max_{l,k}\{\tau_{l,sk}+\tau_{l,rk}+\tau_{sd2}\}$. Denote four consecutive OFDM symbols by $\check{u}_i$, $\forall\ i \in \mathbb{Z}_4$, where $\check{u}_i$ consists of FFT$(u_i)$ and the CP.

At each relay $R_k$, the received noisy signals will be polarized, processed and forwarded to destination node $D$. We define two processed vectors $\check{\mathbf{u}}_1 = (\check{u}_0^{\mathrm{T}}, \check{u}_2^{\mathrm{T}})^{\mathrm{T}}$ and $\check{\mathbf{u}}_2 = (\check{u}_1^{\mathrm{T}}, \check{u}_3^{\mathrm{T}})^{\mathrm{T}}$, which are polarized at $R_1$ and $R_2$, respectively. Therefore, the received signals at each relay node $R_k$ for four successive OFDM symbol durations can be given by

$$\begin{aligned} \check{r}_{k0} &= \sqrt{p_t}\check{u}_0 \circledast \phi_k + \check{\check{n}}_{k0},\ \check{r}_{k1} = \sqrt{p_t}\check{u}_1 \circledast \phi_k + \check{\check{n}}_{k1}, \\ \check{r}_{k2} &= \sqrt{p_t}\check{u}_2 \circledast \phi_k + \check{\check{n}}_{k2},\ \check{r}_{k3} = \sqrt{p_t}\check{u}_3 \circledast \phi_k + \check{\check{n}}_{k3}, \end{aligned} \tag{2}$$

where $\phi_k$ is an $L \times 1$ vector defined as $\phi_k = (\alpha_{sk}(0), \cdots, \alpha_{sk}(L-1))$, $\circledast$ denotes the linear convolution, and $\check{\check{n}}_{ki}$, $\forall\ i \in \mathbb{Z}_4$, denotes the additive white Gaussian noise (AWGN) at $R_k$ with zero-mean and unit-variance.

**Table 1.** Implementation of the PF scheme for the down-polarized system at relay nodes. OM$_i$ denote the $i^{th}$ OFDM block.

| | Polar $R_1$ | Polar $R_2$ | Process $R_1$ | Process $R_2$ |
|---|---|---|---|---|
| OM$_0$ | $\check{r}_{10}$ | $\check{r}_{20}$ | $\zeta(\check{r}_{10})$ | 0 |
| OM$_1$ | $\check{r}_{11}$ | $\check{r}_{21}$ | 0 | $\check{r}_{21}^*$ |
| OM$_2$ | $\check{r}_{10}+\check{r}_{12}$ | $\check{r}_{22}$ | $\zeta(\check{r}_{10}+\check{r}_{12})$ | 0 |
| OM$_3$ | $\check{r}_{13}$ | $\check{r}_{23}+\check{r}_{21}$ | 0 | $(\check{r}_{23}+\check{r}_{21})^*$ |

Then each relay node $R_k$ polarizes, processes and forwards the received noisy signals as shown in Table I, where $\zeta(\cdot)$ denotes the time-reversal of the signals [5], i.e., $\zeta(\check{r}_{ki}(\epsilon)) \triangleq \check{r}_{ki}(L_s - \epsilon)$, $\forall\ \epsilon \in \mathbb{Z}_{L_s}$, $\forall\ k \in \{1, 2\}$ and $\forall\ i \in \mathbb{Z}_4$. Denote by $\check{v}_0 \triangleq \zeta(\check{r}_{10})$, $\check{v}_1 \triangleq \zeta(\check{r}_{10} + \check{r}_{12})$, $\check{v}_2 \triangleq \check{r}_{21}^*$ and $\check{v}_3 \triangleq (\check{r}_{21} + \check{r}_{23})^*$. For the $\epsilon^{th}$ subcarrier of $\check{v}_i$ we also take the notations $\check{v}_{i,\epsilon} \triangleq \check{v}_i(\epsilon)$, $\forall\ \epsilon \in \mathbb{Z}_N$.

After the above-mentioned processing, each relay node $R_k$ amplifies the yielded symbols with a scalar $\lambda = \sqrt{p_r/(p_t + 1)}$ while remaining the average transmission power $p_r$. At destination node $D$, the CP is removed for each OFDM symbol. We note that relay node $R_1$ implements the time reversions of the noisy signals including both information symbols and CP. What we need is that after the CP removal, we obtain the time reversal version of only information symbols, i.e., $\zeta(\text{FFT}(u_i))$, $\forall\ i \in \mathbb{Z}_4$. Then by using some properties of FFT/IFFT, we achieve the feasible definition as follows.

*Definition 2.1 [5]:* According to the processed four OFDM symbols at relay node $R_1$ we can obtain $\zeta(\phi_1')\circledast\zeta(\text{FFT}(u_i))$ at destination node $D$ if we remove the CP as in a conventional OFDM system to get an $N$-point vector and shift the last

$\tau_1' = l_{cp} - \tau_1 + 1$ samples of the $N$-point vector as the first $\tau_1'$ samples. Here $\phi_1'$ is an equivalent $N \times 1$ channel vector defined as $\phi_1' = (\alpha_{s1}(0), \cdots, \alpha_{s1}(L-1), 0, \cdots, 0)$, and $\tau_1$ denotes the maximum path delay of channel $\phi_1$ from source node $S$ to relay node $R_1$, i.e., $\tau_1 = \max_l\{\tau_{l,s1}\}$. In a similar way, we define another equivalent $N \times 1$ channel vector $\kappa_1' = (\alpha_{r1}(0), \cdots, \alpha_{r1}(L-1), 0, \cdots, 0)$.

At destination node $D$, after the CP removal the received four successive OFDM symbols can be written as

$$\begin{aligned} y_0 &= \lambda(\sqrt{p_t}\zeta(\mathrm{FFT}(u_0)) \circledast \zeta(\phi_1') + \bar{n}_{10}) \circledast \kappa_1' + n_0 \\ y_1 &= \lambda(\sqrt{p_t}\zeta(\mathrm{FFT}(u_0+u_2)) \circledast \zeta(\phi_1') + \bar{n}_{10} + \bar{n}_{12}) \circledast \kappa_1' + n_1 \\ y_2 &= \lambda(\sqrt{p_t}(\mathrm{FFT}(u_1))^* \circledast t_{sd2} \circledast t_1' \circledast \phi_2' + \bar{n}_{21}^*) \circledast \kappa_2' + n_2 \\ y_3 &= \lambda(\sqrt{p_t}(\mathrm{FFT}(u_3+u_1))^* \circledast t_{sd2} \circledast t_1' \circledast \phi_2' + \bar{n}_{21}^* + \bar{n}_{23}^*) \\ &\quad \circledast \kappa_2' + n_3, \end{aligned} \tag{3}$$

where $t_{sd2}$ is an $N \times 1$ vector that represents the timing errors in the time domain denoted as $t_{sd2} = (\mathbf{0}_{\tau,sd2}, 1, 0, \cdots, 0)^{\mathrm{T}}$, and $\mathbf{0}_{\tau_{sd2}}$ is a $1 \times \tau_{sd2}$ vector of all zeros, and $t_1'$ is the shift of $\tau_1'$ samples in the time domain defined as $t_1' = (\mathbf{0}_{\tau_1'}, 1, 0, \cdots, 0)^{\mathrm{T}}$. Since the signals transmitted from $R_2$ will arrive at the destination $\tau_{sd2}$ samples later and after the CP removal, the signals are further shifted by $\tau_1'$ samples. The total number of shifted samples is denoted by $\tau_2 = \tau_{sd2} + \tau_1'$. Here $\bar{n}_{ki}$ is the AWGN at relay node $R_k$ and $n_i$ denotes the AWGN at destination node $D$ after the CP removal.

After that the received OFDM symbols are transformed by the $N$-point FFT. As mentioned before, because of timing errors, the OFDM symbols from relay node $R_2$ arrive at destination node $D$ $\tau_{sd2}$ samples later than that of symbols from relay node $R_1$. Since $l_{cp}$ is long enough, we can still maintain the orthogonality between subcarriers. The delay $\tau_{sd2}$ in the time domain corresponds to a phase change in the frequency domain, i.e., $f^{\tau_{sd2}} = (1, e^{-\iota 2\pi\tau_{sd2}/N}, \cdots, e^{-\iota 2\pi\tau_{sd2}(N-1)/N})^{\mathrm{T}}$, where $f = (1, e^{-\iota 2\pi/N}, \cdots, e^{-\iota 2\pi(N-1)/N})^{\mathrm{T}}$ and $\iota = \sqrt{-1}$. Similarly, the shift of $\tau_1'$ samples in the time domain also corresponds to a phase change $f^{\tau_1'}$, and hence the total phase change is $f^{\tau_2}$.

Denote by $\check{y}_i = (\check{y}_{i0}, \check{y}_{i1}, \cdots, \check{y}_{i(N-1)})$, $\forall\ i \in \mathbb{Z}_4$, the received four consecutive OFDM symbols at destination node $D$ after the CP removal and the $N$-point FFT transformations. Therefore, we have

$$\begin{aligned} \check{y}_0 &= \lambda[\sqrt{p_t}\mathrm{FFT}(\zeta(\mathrm{FFT}(u_0))) \circ \check{\phi}_1 \circ \check{\kappa}_1 + \check{\bar{n}}_{10} \circ \check{\kappa}_1] + \check{n}_0 \\ \check{y}_1 &= \lambda[\sqrt{p_t}\mathrm{FFT}(\zeta(\mathrm{FFT}(u_0 + u_2))) \circ \check{\phi}_1 \circ \check{\kappa}_1 \\ &\quad +(\check{\bar{n}}_{10} + \check{\bar{n}}_{20}) \circ \check{\kappa}_1] + \check{n}_1 \\ \check{y}_2 &= \lambda[\sqrt{p_t}\mathrm{FFT}((\mathrm{FFT}(u_1))^*) \circ f^{\tau_2} \circ \check{\phi}_2 \circ \check{\kappa}_2 + \check{\bar{n}}_{21} \circ \check{\kappa}_2] + \check{n}_2 \\ \check{y}_3 &= \lambda[\sqrt{p_t}\mathrm{FFT}((\mathrm{FFT}(u_3 + u_1))^*) \circ f^{\tau_2} \circ \check{\phi}_2 \circ \check{\kappa}_2 \\ &\quad +(\check{\bar{n}}_{21}^* + \check{\bar{n}}_{23}^*) \circ \check{\kappa}_2] + \check{n}_3, \end{aligned} \tag{4}$$

where $\circ$ denotes the Hadamard product, $\check{\phi}_1 = \mathrm{FFT}(\zeta(\phi_1'))$, $\check{\kappa}_1 = \mathrm{FFT}(\kappa_1')$, $\check{\phi}_2 = \mathrm{FFT}((\phi_1')^*)$, $\check{\kappa}_2 = \mathrm{FFT}(\kappa_2')$, $\check{\bar{n}}_{ki} = \mathrm{FFT}(\bar{n}_{ki})$, and $\check{\bar{n}}_i = \mathrm{FFT}(\bar{n}_i)$, $\forall\ k \in \{1,2\}$ and $\forall\ i \in \mathbb{Z}_4$.

According to the properties of the well-known $N$-point FFT transforms for an $N \times 1$ vector $x$, we have $(\mathrm{FFT}(x))^* = \mathrm{IFFT}(x^*)$ and $\mathrm{FFT}(\zeta(\mathrm{FFT}(x))) = \mathrm{IFFT}(\mathrm{FFT}(x)) = x$ [5]. Therefore, the formulas in (4) can be written in the polar form for each subcarrier $\epsilon$, $\forall\ \epsilon \in \mathbb{Z}_N$, as follows

$$\begin{pmatrix} y_{0\epsilon} \\ y_{1\epsilon} \\ y_{2\epsilon} \\ y_{3\epsilon} \end{pmatrix} = \lambda\sqrt{p_t} \begin{pmatrix} \check{\phi}_{1\epsilon}\check{\kappa}_{1\epsilon} & 0 & 0 & 0 \\ \check{\phi}_{1\epsilon}\check{\kappa}_{1\epsilon} & \check{\phi}_{1\epsilon}\check{\kappa}_{1\epsilon} & 0 & 0 \\ 0 & 0 & \Phi_{2\epsilon} & 0 \\ 0 & 0 & \Phi_{2\epsilon} & \Phi_{2\epsilon} \end{pmatrix} \begin{pmatrix} u_{0\epsilon} \\ u_{2\epsilon} \\ u_{1\epsilon}^* \\ u_{3\epsilon}^* \end{pmatrix} + \mathbf{e}_{0\epsilon} \tag{5}$$

which can be rewritten as

$$\begin{aligned} \begin{pmatrix} y_{0\epsilon} \\ y_{1\epsilon} \\ y_{2\epsilon}^* \\ y_{3\epsilon}^* \end{pmatrix} &= \lambda\sqrt{p_t} \left(\begin{array}{cc|cc} \check{\phi}_{1\epsilon}\check{\kappa}_{1\epsilon} & 0 & 0 & 0 \\ \check{\phi}_{1\epsilon}\check{\kappa}_{1\epsilon} & 0 & \check{\phi}_{1\epsilon}\check{\kappa}_{1\epsilon} & 0 \\ \Phi_{2\epsilon}^* & \Phi_{2\epsilon}^* & 0 & 0 \\ \Phi_{2\epsilon}^* & \Phi_{2\epsilon}^* & \Phi_{2\epsilon}^* & \Phi_{2\epsilon}^* \end{array}\right) \begin{pmatrix} x_{0\epsilon} \\ x_{1\epsilon} \\ x_{2\epsilon} \\ x_{3\epsilon} \end{pmatrix} + \mathbf{e}_{\epsilon} \\ &= \mathcal{H}_I \mathbf{x}_{I\epsilon} + \mathcal{H}_F \mathbf{x}_{F\epsilon} + \mathbf{e}_{\epsilon}, \end{aligned} \tag{6}$$

where $\Phi_{2\epsilon} \triangleq f_\epsilon^{\tau_2}\check{\phi}_{2\epsilon}\check{\kappa}_{2\epsilon}$, $\Phi_{2\epsilon}^* \triangleq (f_\epsilon^{\tau_2}\check{\phi}_{2\epsilon}\check{\kappa}_{2\epsilon})^*$, $f_\epsilon^{\tau_2} = \exp(-\iota 2\pi\epsilon\tau/N)$, $\mathbf{x}_{I\epsilon} = (x_{0\epsilon}, x_{1\epsilon})^{\mathrm{T}}$, $\mathbf{x}_{F\epsilon} = (x_{2\epsilon}, x_{3\epsilon})^{\mathrm{T}}$, $x_{i\epsilon}$ is the $\epsilon^{th}$ element of $x_i$, $\check{\kappa}_{k\epsilon}$ and $\check{\phi}_{k\epsilon}$ denote the $\epsilon^{th}$ element of $\check{\kappa}_k$ and $\check{\phi}_k$, $\forall\ k \in \{1,2\}$ and $\forall\ i \in \mathbb{Z}_4$. Two vectors $\mathbf{e}_{0\epsilon}$ and $\mathbf{e}_{\epsilon}$ denote the corresponding polarized noises.

We note that sub-vector $\mathbf{x}_{I\epsilon}$ serves as the *information* vector while sub-vector $\mathbf{x}_{F\epsilon}$ as the *frozen* vector for the *down-polarizing* MIMO relay system, which can be derived from the Bhattacharyya parameter vector for the derivation of the reliability of the FSF channels calculated in next section.

## 3 Depolarizing MIMO-OFDM Relay System

So far we have established the polar system based on the OFDM polarizing for the FSF channels. Next, we analyze the reliability of the the FSF channels with transmission probabilities $W_4^{(i)}$ for the $i^{th}$ OFDM symbol based on the Bhattacharyya parameter vector $\mathbf{z}_4 = (z_{4,0}, z_{4,1}, z_{4,2}, z_{4,3})$, which can be calculated from the recursion formula [1], i.e.,

$$z_{2k,j} = \begin{cases} z_{k,j}^2, & \text{for } 0 \le j \le k-1; \\ 2z_{k,j-k} - z_{k,j-k}^2, & \text{for } k \le j \le 2k-1, \end{cases} \tag{7}$$

for $\forall\ k \in \{1,2\}$ starting with $z_{1,0} = 1/2$. From scratch, we form a permutation $\pi_4 = (i_0, i_1, i_2, i_3)$ of $(0,1,2,3)$ corresponding to entries of $\mathbf{x} = (x_0, x_1, x_2, x_3)^{\mathrm{T}}$ so that the inequality $z_{4,i_j} \le z_{4,i_k}$, $\forall\ 0 \le j < k \le 3$, is true. Thus we have the reliability of OFDM splitting for the FSF channels given by

$z_4 = (1/16, 7/16, 9/16, 15/16)$, which creates a permutation $\pi_4 = (0, 1, 2, 3)$. It implies that for each subcarrier of the source OFDM symbols $\mathbf{x}_\epsilon$, the first two signals $\{x_{0,\epsilon}, x_{1,\epsilon}\}$ can be transmitted with higher reliability than that of the last two signals $\{x_{2,\epsilon}, x_{3,\epsilon}\}$. Therefore, for the reliable transmission of signals, we let $\{x_{0,\epsilon}, x_{1,\epsilon}\}$ to be the *information* bits that are required to be transmitted from relay nodes, and $\{x_{2,\epsilon}, x_{3,\epsilon}\}$ to be *frozen* bits that provide assistance for transmissions. In practice, the *frozen* bits $\{x_{2,\epsilon}, x_{3,\epsilon}\}$ are always be set zeros for simplicity, i.e., $\{x_{2,\epsilon} = 0, x_{3,\epsilon} = 0\}$. This property can be utilized for the flexible transmission of signals on the FSF channels with high reliability [1].

In the similar way, we can derive the reliability of *up-splitting* system for the FSF channel $W'^{(i)}_4$ based on the Bhattacharyya parameter vector $\mathbf{z}'_4 = (z'_{4,0}, z'_{4,1}, z'_{4,2}, z'_{4,3})$, which can be calculated from [1], i.e.,

$$z'_{2k,j} = \begin{cases} 2z'_{k,j-k} - z'^2_{k,j-k}, & \text{for } 0 \le j \le k-1; \\ z'^2_{k,j}, & \text{for } k \le j \le 2k-1, \end{cases} \tag{8}$$

The reliability of the FSF channels can be derived as $z_4 = (15/16, 9/16, 7/16, 1/16)$. Therefore, for the reliable transmission of signals over each subcarrier, we select the *information* bits $\{x_{2,\epsilon}, x_{3,\epsilon}\}$ and the *frozen* bits $\{x_{0,\epsilon}, x_{1,\epsilon}\}$.

## 4 Simulation Results

According to the OFDM depolarizing algorithm with the SIC decoder for the polar system, we present some simulation results and compare their BER performance behaviors. We present the BER performance as functions of the transmit power $p_t$. We deploy the Alamouti code while implementing the OFDM depolarizing techniques for the FSF channels. We can also use the ML symbol-wise decoding, as well as the OFDM depolarizing in four time slots, where the data symbols in $\mathcal{A}$ are drawn from BPSK constellation.

In Fig. 2, we present the BER curves of the stacked Alamouti code for four OFDM symbols transmitted at source node $S$. We consider the polar systems provided with transmission power $p_t$ for reference in terms of the fixed power allocation strategy in (1). For the present polar system, it shows that the slope of the BER performance curve of the proposed PF scheme with the stacked Alamouti code for the polar system via the OFDM depolarizing algorithm approaches the direct transmitting system when power $p_t$ increases. It implies that the PF scheme can achieve full diversity with the depolarizing algorithm. Furthermore, the BER performance behavior of the present polar system outperforms that of the direct transmission approach which verifies our analysis of the transmission reliability of the FSF channels. Simulations demonstrate that the proposed PF scheme has a similar performance as that of the Alamouti scheme with the ML decoding when the depolarizing is applied at the receiver.

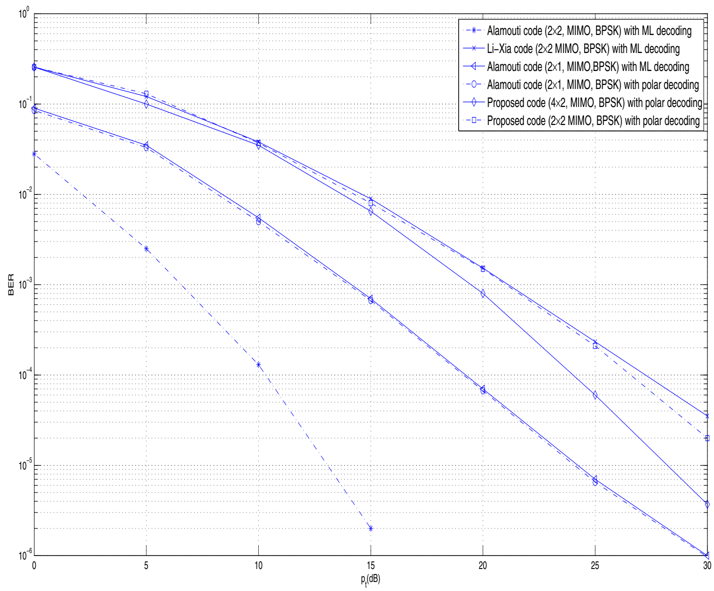


**Fig. 2.** BER performance behaviors with the depolarizing receiver

## 5 Conclusion

In this paper, we have presented a simple design of the PF scheme based on the switching polar systems over the FSF channels, i.e., the *down-polarizing* system and the *up-polarizing* system using two polarizing operations $\mathbf{Q}_2$ and $\mathbf{Q}_2'$ first suggested by E. Arikan. The present polar wireless system has a salient recursiveness feature and can be decoded with the SIC decoder, which renders the PF relay scheme analytically tractable and provides a low-complexity coding algorithm while multiple OFDM symbols are equipped and broadcasted from source node $S$. We analyze the BER performance and diversity of this system based on the Alamouti code with the fixed transmit power using over the FSF channels, which tend to polarize with respect to the increasing reliability under certain OFDM combining and splitting operations. Simulations demonstrate that the proposed polar system has the similar BER performance behaviors as that of the stacked Alamouti code with the ML decoding, but outperforms this direct transmission method in terms of the BER performance for large transmission power $p_t$ when the OFDM depolarizing algorithm is applied at destination node $D$.

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