# **Image Deblurring Using Bayesian Framework**

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**Abstract.** This paper proposes a new method for identifying the blur model and its parameters to restore the image from the blurred image. This is based on the specific distortions caused by the distorting operator in the Fourier spectrum amplitude of an image. Due to the ill-posed nature of image restoration (IR) process, prior knowledge of natural images is used to regularize the IR problem. The Bayesian approach provides the means to incorporate prior knowledge in data analysis. The choice of prior is very important. A comparative analysis using various priors was studied qualitatively. The sparse and redundant prior method gives better results both subjectively and objectively when compared with other priors.

Keywords: Image Restoration, point spread function, image deblurring.

### 1 Introduction

The goal of image restoration (IR) is to reconstruct the original scene from a degraded observation. Images captured in uncontrolled environments invariably represent a degraded version of an original image due to imperfections in the imaging and capturing process. This degradation can be shift-variant or shift-invariant. This paper focuses on shift-invariant blurs. This degradation may be classified into two major categories: blur and noise. Images may be blurred due to atmospheric turbulence, defocusing of the lens, aberration in the optical systems, relative motion between the imaging system and the original scene. Automatic image deblurring is an objective of great practical interest for the enhancement of images in photo and video cameras, in astronomy, in tomography, in other biomedical imaging techniques, motion tracking applications, etc.

Image deblurring methods can be divided into two classes: *nonblind* and *blind*. In nonblind methods information about the blurring filter are known. Blind deconvolution refers to a class of problems when the original image is estimated from the degraded observations where the exact information about the degradation and noise is not available. The blind deconvolution problem is very challenging since it is hard to infer the original image and the unknown degradation only from the observed image. The method that is described here belongs to the latter class. In most situations of practical interest the blurring filter's impulse response, also called point spread function (PSF), is not known with good accuracy. Since nonblind deblurring methods

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are very sensitive to mismatches between the PSF used by the method and the true blurring PSF, a poor knowledge of the blurring PSF normally leads to poor deblurring results. In blind image deblurring (BID), not only the degradation operator is ill-conditioned, but the problem also is severely ill-posed as the uniqueness and stability of the solution is not guaranteed: there are infinite number of solutions (original image + blurring filter) that are compatible with the degraded image. An overview of BID methods can be obtained from [1].

In some cases, one has access to more than one degraded image from the same original scene, a fact which can be used to reduce the ill-posedness of the problem [13], [1]. In [13] they presented a novel filtering method for reconstructing an all-infocus image or an arbitrarily focused image from two images that are focused differently. Karen et. al[14] proposed a novel nonparametric regression method for deblurring noisy images based on the local polynomial approximation (LPA) of the image and the intersecting confidence intervals (ICI) that is applied to define the adaptive varying scales (window sizes) of the LPA estimators. Lokhande, Arya and Gupta [16] proposed a technique for restoring the motion blurred images.

In this work, first the blur model and its parameters are identified from the blurred image to restore the image. Here only Gaussian blur and Uniform blur are considered. A novel method has been proposed for this task. This is based on the specific distortions caused by the distorting operator in the Fourier spectrum amplitude of an image. A template based on several blurred images is constructed for both Gaussian and uniform blur. The Fourier spectrum amplitude of the noisy blurred image is compared with the templates to identify the blur model. The parameters of blur are also determined from the Fourier spectrum amplitude thresholded image. With this estimated blur model and parameter values, the blurring filter PSF is constructed, which is used for obtaining the image estimate (deblurred image) from the blurred image. Due to the ill-posed nature of IR, prior knowledge of natural images can be used to regularize the IR problem. The Bayesian approach [2] provides the means to incorporate prior knowledge in data analysis. The priors used here are Tikhonov (L2), Sobolev, Total Variational (TV) [3], Sparsity [4] and the Sparse and Redundant prior [5].

The system is implemented and experiments were conducted with blur parameter set as 7, 9 and 11 for Uniform blur and parameters 2.5, 2.9 and 3.3 for Gaussian blur. The performance measures used are Peak signal-to-noise ratio (PSNR) and Structural similarity index measure (SSIM). From the results obtained it can be inferred that the sparse and redundant prior (ASDS method) gives better results.

### 2 Image Deblurring Problem

The image degradation is modeled by

$$y = h * x + w \tag{1}$$

in which y and x are images which represent, respectively, the degraded image, the original image and w is the additive noise (white gaussian noise is taken); h is the PSF of the blurring operator, and \* denotes the mathematical operation of convolution.

An alternative way of describing (1) is through its spectral equivalence. By applying discrete Fourier transforms to (2), the following representation is obtained:

$$Y = HX + W \tag{2}$$

where capitals represent Fourier transforms of y, x, h and w. In practice the spectral representation is more often used since it leads to efficient implementations of restoration filters in the (discrete) Fourier domain. If deblurring is obtained by dividing the Fourier transform by the blurring filter PSF, this will lead to the explosion of noise.

Prior knowledge of natural images is used to regularize the IR problem. The Bayesian approach [2] provides the means to incorporate prior knowledge in data analysis. Bayesian analysis revolves around the posterior probability, which summarizes the degree of one's certainty concerning a given situation. Bayes's law states that the posterior probability is proportional to the product of the likelihood and the prior probability. The likelihood encompasses the information contained in the new data. The prior expresses the degree of certainty concerning a parameter *x* the present state of certainty is characterized by the probability density function p(x). Perform an experiment and take some data *d*. Bayes's law becomes:

$$p(x \mid d) = \frac{p(d \mid x)p(x)}{p(d)}$$
(3)

p(x|d) is the posterior probability density function, or simply the posterior, because it effectively follows the experiment. It is the conditional probability of *x* given the new data *d*. The probability p(x) is called the prior because it represents the state of knowledge before the experiment. The quantity p(d|x) is the likelihood, which expresses the probability of the data *d* given any particular *x*. The likelihood is usually derived from a model for predicting the data, given *x*, as well as a probabilistic model for the noise. The term in the denominator p(d) may be considered necessary only for normalization purposes. As the normalization can be evaluated from the other terms, Bayes's law is often written as a proportionality, leaving out the denominator.

Although the posterior probability completely describes the state of certainty about any possible image, it is often necessary to select a single image as the 'result' or reconstruction. A typical choice is that image that maximizes the posterior probability, which is called the MAP (maximum a posteriori) estimate. Given the data  $\mathbf{y}$ , the posterior probability of any image  $\mathbf{x}$  is given by Bayes's law (3) in terms of the proportionality

$$p(x \mid y)\alpha p(y \mid x)p(x) \tag{4}$$

where  $p(\mathbf{y}|\mathbf{x})$ , the probability of the observed data given  $\mathbf{x}$ , is the likelihood and  $p(\mathbf{x})$  is the prior probability of  $\mathbf{x}$ . The negative logarithm of the posterior probability density function is given by

$$\log[p(x \mid y)] = \phi(x) = \Lambda(x) + \Pi(x)$$
(5)

where the first term comes from the likelihood and the second term from the prior probability.

The likelihood is specified by the assumed probability density function of the fluctuations in the measurements about their predicted values (in the absence of noise). For the additive, uncorrelated Gaussian noise assumed, the negative log (likelihood) is just half of chi-squared

$$-\log[p(\mathbf{y} | \mathbf{x})] = \Lambda(\mathbf{x}) = \frac{1}{2}\chi^{2} = \frac{1}{2\sigma_{\mathbf{w}}^{2}} \|\mathbf{y} - \mathbf{h}\mathbf{x}\|^{2}$$
(6)

which is quadratic in the residuals. The choice for the likelihood function should be based on the actual statistical characteristics of the measurement noise. The restored image can be obtained from Eq. (7).

$$\hat{\mathbf{x}} = \arg\min_{x} \frac{1}{2} \|\mathbf{y} - \mathbf{h}\mathbf{x}\|_{2}^{2} + \lambda \Pi(\mathbf{x})$$
(7)

where  $\hat{\mathbf{x}}$  is the restored image or image estimate and  $\hat{\lambda}$  is the regularization parameter.

### A. Blur Model

In this paper, only Gaussian and Uniform blurs are considered.

#### a) Uniform Blur

The uniform rectangular blur is described by the following function:

$$h(n_1, n_2) = \begin{cases} \frac{1}{l^2}, & |n_1| < \frac{l}{2}, |n_2| < \frac{l}{2}, \\ 0, & elsewhere \end{cases}$$
(8)

where parameter l defines the size of smoothing area. The frequency characteristics of Eq. (8) is shown in Fig. 1.

Atmospheric turbulence is a severe limitation in remote sensing. Although the blur introduced by atmospheric turbulence depends on a variety of factors (such as temperature, wind speed, exposure time), for long-term exposures the point-spread function can be described reasonably well by a Gaussian function:

$$h(n_1, n_2) = \frac{1}{2\pi\tau^2} \exp\left(-\frac{n_1^2 + n_2^2}{\tau^2}\right)$$
(9)

b) Atmospheric Turbulence Blur

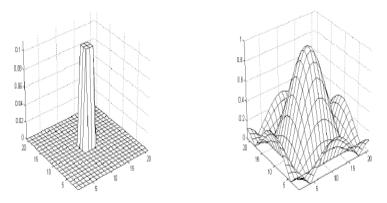


Fig. 1. Uniform blur of size 3 x 3 and its frequency characteristics

where  $\tau^2$  is a parameter of the PSF (the variance of the Gaussian function) (Fig. 2). Its Fourier transform is also a Gaussian function and its absolute values are shown in Fig. 2.

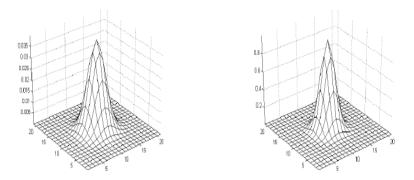
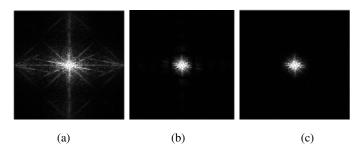


Fig. 2. Gaussian PSF with  $\tau = 2$  and its frequency characteristics

### B. Estimation of Blurring Filter PSF

A prior knowledge about the distorting operator and its parameters is of crucial importance in blurred image restoration. In this paper, a novel method is proposed for the identification of the type of blur and of its parameters. Only Gaussian and uniform blur can be identified with this method. The Fourier spectrum of an image is influenced by the blur. This is shown in Fig. 3. So here a method to identify the blur from this Fourier amplitude spectrum is proposed. After identifying the type of blur, its parameters are also extracted from the Fourier spectrum thresholded image.



**Fig. 3.** Influence of blur on Fourier spectrum amplitude: (a) spectrum amplitude of image that is not corrupted (b) spectrum amplitude of the same image corrupted by the uniform blur (c) spectrum amplitude of the same image corrupted by the Gaussian blur

#### 1) Blur Template

The blur template is constructed from the thresholded Fourier spectrum image of several blurred images. First, the Fourier spectrum amplitude of blurred images are thresholded to reduce the noise effect. After thresholding, a morphological operation [18] is performed on the resultant image. Morphological operations apply a structuring element to an input image, creating an output image of the same size. In morphological operations, the value of each pixel in the output image is based on a comparison of the corresponding pixel in the input image with its neighbors. By choosing the size and shape of the neighborhood, a morphological operation that is sensitive to specific shapes in the input image can be constructed. Here dilation is performed with the disk structuring element. A structuring element is a matrix consisting of only 0's and 1's that can have any arbitrary shape and size. The pixels with values of 1 define the neighborhood. Dilation adds pixels to the boundaries of objects in an image. The number of pixels added or removed from the objects in an image depends on the size and shape of the structuring element used to process the image. The value of the output pixel is the maximum value of all the pixels in the input pixel's neighborhood. In a binary image, if any of the pixels is set to the value 1, the output pixel is set to 1. The dilation is performed inorder to set the boundary pixels to connect all the white regions.

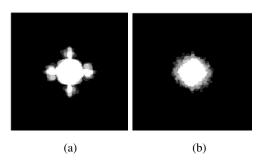


Fig. 4. Blur templates: (a) Uniform blur template (b) Gaussian blur template

Several blurred images are created for both Gaussian and Uniform blur. From each of these blurred images, the dilated images are constructed. The mean image of the dilated images of Gaussian blurred images is created and this is taken as the template for identifying Gaussian blur. Similarly the template for uniform blur identification is also constructed. Thus the templates for identifying both the blurs are available (Fig. 4), so by using these templates it can decide to which blur type, a new blurred image belongs.

### 2) Blur Identification

For deblurring, first the Fourier spectrum amplitude of the blurred image is taken. This image is thresholded and dilated. The resultant image is matched with both of the blur templates shown in Fig. 4. For this, the difference image is computed. i.e., the resultant image is subtracted from each of the blur templates. Now there are two difference images corresponding to the Gaussian and Uniform blur templates. The number of non-zero elements in these difference image is computed. The type of blur template which generates the difference image whose number of non-zero elements is minimum, is identified as the blur type. If the blurred image belongs to some other blur model then the number of non-zero elements in the difference image will be very high. But still there exists a minimum value with the two blur templates. This leads to incorrect blur estimate. So to avoid this, a constraint is made that this minimum value should be less than a constant.

### 3) Blur Parameter Estimation

For Gaussian blur parameter estimation, the radius of the center white portion of the thresholded and dilated image of the Fourier spectrum amplitude of the blurred image is used. In the case of Gaussian blur, variance is the blur parameter. Variance has influence over the radius of the center white portion of the dilated image. i.e., the radius corresponding to a range of variance values will be different. Once the radius is computed and if it is found less than a specific value, then the variance value corresponding to that will be computed as follows:

$$val = 1 - ((radius - lower_radius)/(upper_radius - lower_radius))$$
  
variance = (0.5 \* val) + lower\_variance (10)

The radius and variance range is given in Table 1. For example, if radius > 50, the variance is computed as:

$$val = 1 - ((radius - 50)/(61 - 50))$$
  
variance = (0.5 \* val) + 1

For uniform blur, the breadth and height of the center white portion is calculated and minimum of it is taken as length. If length is in the range or less than a specific value, then the kernel size corresponding to that is selected, shown in Table 2. With this estimated blur type and blur parameter, the blurring filter PSF can be constructed as specified in section II.A. This estimated PSF is used for deblurring the image. Image deblurring in Bayesian framework using different priors are explained in the following section.

### 3 Priors

The choice of prior is very important. Five priors (Tikhonov (L2), Sobolev, Total variational (TV), Sparse, Sparse and redundant prior) are taken here.

### A. Tikhonov and Sobolev Priors

The Tikhonov prior is also known as the L2-norm. This prior is based on the fact that the energy present in images is bounded, but this is not the case with noise. The image estimate (deblurred image) can be obtained from the Eq. (11).

$$\hat{x} = \arg\min_{x} \|y - h^* x\|_2^2 + \lambda \|x\|_2^2$$
(11)

where  $||x||_{2}^{2} = \left(\sum_{i} |x_{i}|^{2}\right)^{\frac{1}{2}}$ 

Since the filtering is diagonalized over Fourier, the solution is simply computed over the Fourier domain as:

$$\hat{X}(\omega) = \frac{Y(\omega)\hat{H}(\omega)}{\left\|\hat{H}(\omega)\right\|^2 + \lambda}$$
(12)

where  $\hat{X}(\omega)$ ,  $Y(\omega)$ , and  $H(\omega)$  denotes the Fourier transform of restored image, blurred image and blurring filter PSF.  $\lambda$  is the regularization parameter.

L2 regularization does not perform any denoising. So to remove noise, high frequencies in the blurred image can be penalized using the Sobolev prior. The prior is as follows:

$$\Pi(x) = \sum_{i} \left\| \nabla x(i) \right\|^{2} = \sum_{\omega} S(\omega) \left\| X(\omega) \right\|^{2}.$$
(13)

where  $S(\omega) = \|\omega\|^2$ .

Since this prior can be written over the Fourier domain, the solution to the deblurring with Sobolev prior can be simply computed with the Fourier coefficients:

$$\hat{X}(\omega) = \frac{Y(\omega)\hat{H}(\omega)}{\left\|\hat{H}(\omega)\right\|^2 + \lambda S(\omega)}$$
(14)

Radius	Variance
	range
>= 61	1
>= 50	<= 1.5
>= 41	<= 2
>= 36	<= 2.5
>= 32	<= 3
>= 29	<= 3.5
>= 25	<= 4
>= 23	<= 4.5
>= 21	<= 5
otherwise	>= 5.5

Table1.Gaussianparameterselection

Table 2. Uniform para	meter selection
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Length	Kernel size
	taken
> 60	7
Between 49 and 55	9
Otherwise	11

The lena image is blurred using Gaussian blur of variance 2.3. The blurred lena image is deblurred using the L2 and Sobolev priors and the resultant image is shown in Fig. 5. The blur estimated is Gaussian with variance 2.27, the blurring filter constructed using this is used to deblur the image.

#### B. Total Variation (TV) Prior

Sobolev regularization perform a denoising but also tends to blur the edges. The TV prior [3] is able to better reconstruct sharp edges. It reads:

$$\Pi(x) = \sum_{i} \left\| \nabla x(i) \right\| \tag{15}$$

With respect to the Sobolev energy, it simply measure the L1-norm instead of the L2-norm, thus dropping the square in the functional. L1-norm is the summation of absolute values of all the image intensity elements. Unfortunately, the TV functional  $\Pi(x)$  is not a smooth function of the image x. It thus requires the use of advanced convex optimization method to be minimized for regularization. An alternative is to replace the absolute value by a smooth absolute value. The smoothed TV norm reads:

$$\Pi(x) = \sum_{i} \sqrt{\left\|\nabla x(i)\right\|^2 + \varepsilon^2}$$
(16)

When  $\mathcal{E}$  gets close to zero, the smoothed energy becomes closer to the original total variation, but the optimization becomes more difficult. When  $\mathcal{E}$  becomes large, the smoothed energy becomes closer to the Sobolev energy, thus blurring the edges. Unfortunately, this prior is non-quadratic, and cannot be expressed over the Fourier domain. So an iterative scheme such as a gradient descent is used to approximate the solution.

For deblurring, initially, the deblurred image denoted by x is initialized with the value of blurred image. By using the gradient descent optimization method, the value of x is updated in each iteration. Gradient descent is implemented in TV with the following equation.

$$x_{(k+1)} = x_{(k)} - \gamma \left( h * (h * x_{(k)} - y) + \lambda Grad \Pi (x_{(k)}) \right)$$
(17)

where  $x_{(k+1)}$  and  $x_{(k)}$  denotes the value of x in the current and previous iterations respectively. This process continues until the stopping criteria are met. Here, the number of iterations is fixed which is considered as the stopping criteria. The deblurred image obtained with TV prior of blurred lena image is shown in Fig. 5(e).

### C. Sparsity Prior

Sparsity prior[4] considers a synthesis-based regularization, that compute a sparse set of coefficients  $(a_m^*)_m$  in a frame  $\Psi = (\Psi_m)_m$ , i.e., wavelet transform [19] is applied over the blurred image. After applying the wavelet transform, the energy will be present in lower number of coefficients (low-low band coefficients are taken). The L1-norm of the sparse set of coefficients is taken as the prior. The deblurred image *x* can be obtained from

$$x^{*} = \arg\min_{x} \frac{1}{2} \|y - h^{*}x\|^{2} + \lambda \|\langle x, \psi_{m} \rangle \|_{1}$$
(18)

where  $\left\|\left\langle x, \psi_m \right\rangle\right\|_1 = \sum_m \left|\left\langle x, \psi_m \right\rangle\right|$ , y - the blurred image and h - blurring filter PSF.

To solve this non-smooth optimization problem, iterative soft thresholding is used. It computes a series of images  $x^{(l)}$  defined as

$$x^{(l+1)} = S_{\tau\lambda}^{\psi} \left( x^{(l)} - \tau h * \left( h * x^{(l)} - y \right) \right)$$
(19)

where  $\lambda$  is a constant which should be adapted to noise. For  $x^{(l)}$  to converge to a solution of the problem, the gradient step size should be chosen as

$$\tau < \frac{2}{\|h * h\|}$$

Total number of iterations is set to 1000. Soft thresholding is performed for denoising. The deblurred image of blurred lena image using this sparsity prior is shown in Fig. 5(f).

#### D. Sparse and Redundant Prior

This prior considers that the contents can vary significantly across different images or different patches in a single image. The adaptive sparse domain selection (ASDS) method [5] uses this prior. This method learns various sets of bases from a precollected dataset of example image patches, and then for a given patch to be processed, one set of bases are adaptively selected to characterize the local sparse domain. Consider the Eq (20).

$$x = \Phi \alpha \tag{20}$$

where  $\Phi = [\phi_1, \phi_2, ..., \phi_m]$  is a given dictionary of atoms (i.e., code set),  $\alpha$  – the set of coefficients where most of coefficients are close to zero and x, the deblurred image. With the sparsity prior, the representation of x over  $\Phi$  can be estimated from its observation y by solving the following L0-minimization problem:

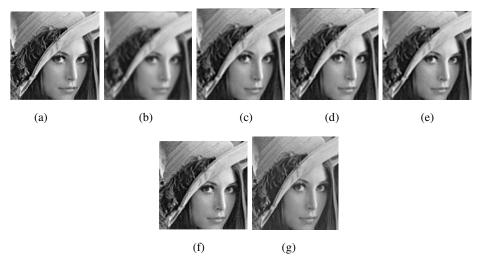
$$\hat{\alpha} = \arg\min_{\alpha} \left\{ \left\| y - H\Phi\alpha \right\|_{2}^{2} + \lambda \left\| \alpha \right\|_{0} \right\}$$
(21)

where the L0-norm counts the number of nonzero coefficients in vector  $\alpha$ . Once  $\hat{\alpha}$  is obtained, x can then be estimated as  $\hat{x} = \Phi \hat{\alpha}$ . The L0-minimization is an NP-hard combinatorial search problem, and is usually solved by greedy algorithms. Set of compact sub-dictionaries from high quality example image patches are learned using the principal component analysis (PCA) technique. For an image patch to be coded, the best sub-dictionary that is most relevant to the given patch is selected.

Suppose that  $\{\phi_k\}$ , k = 1, 2, ..., K, is a set of *K* orthonormal sub-dictionaries. Let  $\mathbf{x}$  be an image vector, and  $\mathbf{x}_i = \mathbf{R}_i \mathbf{x}$ , i=1,2,...,N, be the *i*th patch (size:  $7 \times 7$ ) vector of  $\mathbf{x}$ , where  $\mathbf{R}_i$  is a matrix extracting patch  $\mathbf{x}_i$  from  $\mathbf{x}$ . For patch  $\mathbf{x}_i$ , suppose that a sub-dictionary  $\phi_{k_i}$  is selected for it. Then,  $\mathbf{x}_i$  can be approximated as  $\hat{x}_i = \phi_{k_i} \alpha_i$ ,  $\|\alpha_i\|_0 \leq T$  via sparse coding. The whole image  $\mathbf{x}$  can be reconstructed by averaging all the reconstructed patches  $\hat{x}_i$ , which can be mathematically written as:

$$\hat{x} = \left(\sum_{i=1}^{N} R_i^T R_i\right)^{-1} \sum_{i=1}^{N} \left(R_i^T \phi_{k_i} \alpha_i\right)$$
(22)

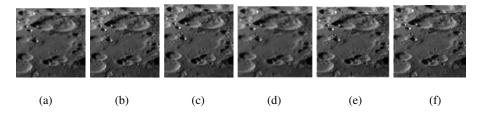
The deblurred image of blurred lena image using the ASDS is shown in Fig. 5(g).



**Fig. 5.** L2 & Sobolev Restored images (a) Original Lena image (b) Gaussian blurred image with variance 2.5 (c) L2 (d) Sobolev (e) TV (f) Sparsity (g) ASDS

### 4 Results and Discussion

Experiments were conducted on gray scale images [20], color images [20] and satellite images. The performance of the restoration process is quantified using Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index (SSIM). The PSNR and SSIM values obtained for 6 gray scale images blurred with Uniform blur of length 9are shown in tables 3 - 4. The blur is estimated as Uniform with blur parameter 9. The PSNR and SSIM values obtained for 6 gray scale images blurred with Gaussian blur of variance 2.5 are shown in tables 5 - 6. The blur is estimated as Gaussian. The blur parameter obtained are shown in Table. 5. Naturally blurred satellite image due to atmospheric turbulence is deblurred where the blur type estimated as Gaussian and parameters estimated are 1.3. The deblurred satellite image using various priors is shown in Fig. 6.



**Fig. 6.** Restoration performed on blurred raw\_moon1 image (a) original blurred raw\_moon1 image (b) L2 restored (c) Sobolev restored (d) TV restored (e) Sparsity (f) ASDS

Name	Blurred	L2	Sobolev	TV	Sparsity	ASDS
boat	20.37	23.91	24.45	24.60	26.02	29.95
cameraman	19.34	22.72	23.04	23.45	25.48	29.26
lena	21.32	24.12	24.54	24.67	25.79	29.77
cactus	20.33	23.51	23.89	23.69	25.08	26.06
building	18.56	21.27	21.49	21.36	23.67	25.32
animal	21.76	24.63	24.96	24.72	25.84	27.09

 Table 3. Table showing PSNR values in dB of the blurred and restored images blurred with

 Uniform kernel of size 9x9

 Table 4. Table showing SSIM values of the restored images blurred with Uniform kernel of size 9x9

Name	L2	Sobolev	TV	Sparsity	ASDS
boat	0.575	0.578	0.578	0.582	0.855
cameraman	0.604	0.606	0.608	0.612	0.922
lena	0.588	0.591	0.591	0.593	0.822
cactus	0.572	0.574	0.574	0.577	0.782
building	0.574	0.577	0.577	0.583	0.819
animal	0.609	0.612	0.612	0.614	0.770

**Table 5.** Table showing **PSNR** values in dB of the blurred and restored images blurred with
 Gaussian blur of standard deviation 2.5

		Estimated					
Name	Blurred	Variance	L2	Sobolev	TV	Sparsity	ASDS
boat	21.43	2.36	23.53	24.13	24.10	25.78	26.88
cameraman	20.39	2.34	22.42	22.92	22.85	25.25	25.58
lena	22.24	2.3	24.35	24.69	24.85	25.84	28.48
cactus	21.06	2.4	23.39	23.92	23.70	25.29	25.66
building	19.33	2.5	20.90	21.34	21.11	23.07	23.45
animal	22.54	2.4	24.32	24.85	24.68	26.06	26.45

 Table 6. Table showing SSIM values of the restored images blurred with Gaussian blur of standard deviation 2.5

Name	L2	Sobolev	TV	Sparsity	ASDS
boat	0.574	0.577	0.577	0.579	0.779
cameraman	0.603	0.606	0.606	0.608	0.855
lena	0.588	0.591	0.591	0.593	0.797
cactus	0.572	0.574	0.574	0.576	0.717
building	0.574	0.577	0.577	0.581	0.751
animal	0.609	0.612	0.612	0.613	0.727

# 5 Conclusion

In this paper, we proposed a new method for estimating the blur type and its parameter. The blurring filter PSF constructed from this estimation is used for deblurring the images. Image restoration is performed in the Bayesian framework where L2, Sobolev, Total variation, sparse, sparse and redundant priors are used. The performance measures used are Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity Index (SSIM). From the deblurred images obtained and performance measured used, it can infer that the sparse and redundant prior gives better result.

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