Estimation of *f*-Similarity in *f*-Triangles Using FIS

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Abstract. Today, some high profile crimes grab our attention and headlines of the world. But, the core problem underlies is to identify the criminals. However, we acquire the features of miscreants narrated by spectators, the fuzzy patterns of finger prints, shoe prints and sometimes the handwriting found, are the crucial clues to apprehend the criminals. Identifying similarity of fuzzy information with the criminal database is not an easy task; this is what is being investigated in our work. We begin our work with a novel approach of estimating fuzzy similarity call it as f-similarity in fuzzy triangles using the membership values. Undoubtedly, the degrees of similarities persist in figures and hand drawn sketches, but, estimating them is performed here. In this sequel, we have discussed about f-geometry, which are the basics of f-similarity defined in terms of membership values. The membership values generated using three popularly known postulates of similar triangles like AAA, SSS and SAS are applied as inputs to the Fuzzy Inference System (FIS). We have found good results of FIS, which can be applied for any inexact geometric shape evaluation.

Keywords: *f*-geometry, *f*-principle, *f*-similarity, FIS.

1 Introduction

In [1], *f*-geometry intensifies about the high validity index in similarity that can be very well justified by human cognition as triangle like shape, which is not possible by computer systems. This high validity index in similarity called *f*-similarity. However, there is no such algorithm or novel approach developed to find the similarity measure between two fuzzy triangles or f-triangles. *f*-triangles are those triangles drawn in free hand without using rulers or compass. In [4], Zadeh emphasizes that an Impossibility principle is an *f*-principle, means it may have fuzzy validity or f-validity, which can neither be proved nor be disproved. So, for such *f*-valid triangles, we have assigned membership values to the different parameters of the postulates of triangle and the values are subjected to FIS to compute the similarity called as *f*-similarity [1]. This methodology of calculating *f*-similarity in *f*-triangles or *f*-patters can be a base work for irregular pattern recognition, face recognition, inexact modeling and many fields of intelligent image retrieval under uncertainty. We have discussed the basic definitions in our previous paper [3]. Therefore, it can be referred for better understanding of our work.

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2 Related Work

Many researchers have contributed their efforts on fuzzy geometry. Nevertheless, the work carried out belongs to the category of Precisiated Fuzzy Logic (FLp). We have discussed about some fuzzy geometry types and previous work related to it in [3]. In [1, 3] discusses about Extended Fuzzy Logic FLe, that is, represented as FLe = FLp + FLu, where FLp is the Fuzzy Logic [11] and FLu is the Unprecisiated FL [1]. Major difference between FLu and FLp is the absence of preciseness in FLu, where the objects of discourse and analysis are imperfect information; in short FLu is an addendum to the existing FLp [5]. FLu facilitates fuzzy validity or f-validity, which emphasizes on high validity index among the collection of f-valid results; this is not allowed in FLp. Whereas FLe permits *f*-valid reasoning for solutions based on perceptions, when there are no provable solutions based on measurements. On similar lines, *f*-geometric shapes of FLu like *f*-line, *f*-point, *f*-circle, *f*-triangle, *f*-proof and *f*-theorem in terms of membership functions were discussed in [3]. To the best of our knowledge, we have not found any intense work on *f*-similarity in *f*-triangles using FIS.

This paper is organized as follows, in Section 1, we have discussed about the introduction of this paper. In section 2, related work which was carried out by various researchers in this domain. In Section 3, computing *f*-similarity in triangles using the postulates is discussed. Section 4 elaborates the Fuzzy Inference System and our method of application with practical results to validate f-similarity between f-triangles. Section 5 sheds light on the future direction in this area and concludes.

3 Computing f-Similarity in Triangles

A key idea in geometry for estimating similarity is that whenever two corresponding angles in two triangles are congruent, then two triangles are called as similar triangles. However, there are three well known postulates AAA, SSS and SAS that proves triangles to be similar. In our work we fuzzify these postulates in terms of membership function, further, computation of f-similar triangles are made using the Fuzzy Inference System. We have taken the data set from hand drawn f-triangles.

3.1 Angle Angle Angle (AAA)

In f-geometry, two triangles are said to be as f-similar if its membership function has high validity index to the property of similar triangles (AAA) and the membership values decreases in difference of the corresponding angles. Mathematically represented as

$$\mu_{AAA}(f - Similar) = \mu_{A1} * \mu_{A2} * \mu_{A3}$$

$$\mu_{AAA}(f - Similar) = e^{-|\theta_1 - \theta_4|} * e^{-|\theta_2 - \theta_5|} * e^{-|\theta_3 - \theta_6|} \quad ...(1)$$

where $\mu_{A1}, \mu_{A2}, \mu_{A3}$ - membership functions of angle1, angle2, angle3 are based on the difference in the corresponding angles; Fig 1 shows $\theta_1, \theta_2, \theta_3$ and $\theta_4, \theta_5, \theta_6$ are the f-angles of f-triangle1 and f-triangle2.

3.2 Side Angle Side (SAS)

In f-geometry, two triangles are said to be as f-similar if its membership function has high validity index to the property of similar triangles (SAS) and the membership values decreases in difference in corresponding angle and difference in proportion of two corresponding sides. Mathematically represented as

$$\mu_{SAS}(f - Similar) = \mu_{S1} * \mu_A * \mu_{S2}$$

$$\mu_{SAS}(f - Similar) = e^{-|k - k_1|} * e^{-|\theta_1 - \theta_2|} * e^{-|k - k_2|} \quad \text{... (2)}$$

where $\mu_{S1}, \mu_A, \mu_{S2}$ - membership functions of side1, angle, side2 respectively; Fig 2 shows f- proportions of an f-triangle. In case of SAS, we assume that A/A' *=B/B' *= k (A constant) i.e., corresponding sides of the two triangles are in the same ratio as in geometry. Where A/A', B/B' takes the fuzzy proportion values $k1, k2$ respectively, θ1 - θ2 is the difference between angles θ1 and θ2. Point to be noticed is a *=b means a is approximately equals to b, in the sense the fuzzy proportions are approximately equal [1, 2].

3.3 Side Side Side (SSS)

In f-geometry, two triangles are said to be as f-similar if its membership function has high validity index to the property of similar triangles (SSS), with all the three corresponding sides are equal in proportion and the membership values decreases even in slight difference in proportion of the sides. Mathematically represented as

$$\mu_{SSS}(f - Similar) = \mu_{S1} * \mu_{S2} * \mu_{S3}$$

$$\mu_{SSS}(f - Similar) = e^{-|k - k_1|} * e^{-|k - k_2|} * e^{-|k - k_3|}. \quad \text{..(3)}$$

where $\mu_{S1}, \mu_{S2}, \mu_{S3}$ - membership functions of side1, side2, side3 respectively; Fig 2 shows an f-triangle. A/A' *=B/B' *= C/C' *= k (A constant), where, A/A', B/B', C/C' takes the fuzzy proportion values k1, k2, k3 respectively, and * denotes that the fuzzy proportions are approximately equal.

4 Fuzzy Inference System

Fuzzy Inference System (FIS) are systems that create the mapping from a given input to an output using Fuzzy Logic. With the mapping in hand, decisions can be done or patterns distinguished. Processes in fuzzy inference are carried by membership functions, fuzzy operators and if-then rules. Inference system has two types of fuzzy toolboxes in general, Mamdani type and Sugeno type was proposed by Ebrahim Mamdani [6] and Takagi Sugeno [7] respectively. FIS is widely used in applications such as expert systems, data classification, control systems and computer vision, but not restricted with the above said applications. Mamdani was amongst the first to propose control system using fuzzy set theory. Much of his work was based on Zadeh's fuzzy algorithms for complex systems and decision processes by [8]. Mamdani's method is extensively used methodology, modelled in conditions where the outputs in membership functions are non-linear. In short Sugeno's method is modelled where the output in membership functions are linear or constant. Even though both these types enhance efficiency of the defuzzification process, the output cannot be expressed in terms of linguistics. We have applied Mamdani's type of fuzzy inference and following steps in our work includes (i) Fuzzification of input variables as angles or sides, (ii) Fuzzy operators are applied to estimate degree of similarity, (iii) Fuzzy outputs are mapped based on similarity, (iv) Aggregation of rules for fuzzy outputs and (v) Defuzzification of aggregated fuzzy output.

Defuzzification is carried on the aggregate output fuzzy set, which consists of range of output values; therefore it has to be defuzzified in order to determine a single output value from the aggregate output. Most widely used defuzzification method is centroid, which returns the centre of area under the curve and we have applied *middle of maximum* method as defuzzification method to estimate the f-similarity between any two f-triangles.

4.1 Compution of f-similarity Using FIS

We have computed f-similarity between two f-similar triangles drawn by free hand. Data sets fed as input for the first postulate AAA, SAS and SSS are given from the metric values given in the table 1, 2 and 3 respectively, are calculated using an *f*-algorithm. We have the following as input to generate the membership values for FIS.

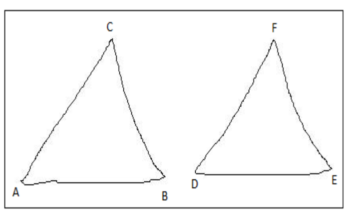


Fig. 1. f-Similarity in triangles based on AAA

Table 1. Values of f-angles for the postulate AAA from Fig 1

Geometric shape I	**Angle A**	**Angle B**	**Angle C**
f-triangle 1	64.65	51.31	66.87
Geometric shape II	**Angle D**	**Angle E**	**Angle F**
f-triangle 2	66.27	53.37	60.87

From (1), Membership values are calculated as,

$$\mu_{AAA}(f-Similar) = \mu_{A1} * \mu_{A2} * \mu_{A3}$$

$$\mu_{AAA}(f-Similar) = e^{-|\theta_1-\theta_4|} * e^{-|\theta_2-\theta_5|} * e^{-|\theta_3-\theta_6|}$$

$$\mu_{AAA}(f-Similar) = e^{-|1.62|} * e^{-|2.06|} * e^{-|6.0|}$$

$$= 0.19 * 0.13 * 0.002$$

These are the membership values given as inputs to the FIS for AAA postulate.

Table 2. Values to prove SAS postulate from Fig 2 for f-similarity

Geometric shape I	**Side A**	**Angle**	**Side B**
f-triangle 1	16.11	64.18	17.08
Geometric shape II	**Side A'**	**Angle**	**Side B'**
f-triangle 2	28.73	64.18	31.94

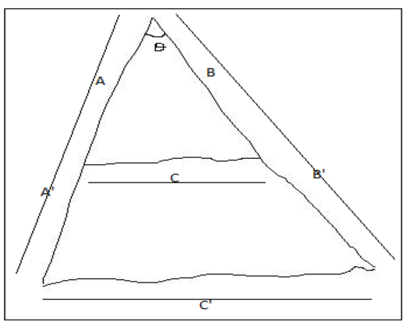


Fig. 2. f-Similarity using SSS and SAS

From (2), Membership values for SAS are calculated as,

$$\mu_{SAS}(f-Similar) = \mu_{S1} * \mu_{A} * \mu_{S2}$$

$$\mu_{SAS}(f-Similar) = e^{-|k - k_1|} * e^{-|\theta_1-\theta_2|} * e^{-|k - k_2|}$$

$$\mu_{SAS}(f-Similar) = e^{-|0.5 - 0.56|} * e^{-|0|} * e^{-|0.5 - 0.534|}$$

$$= 0.94 * 1.0 * 0.97$$

These are the membership values given as inputs to the FIS for SAS postulate.

Table 3. Values obtained from Fig 2, to prove SSS postulate for f-similarity

Geometric shape I	**Side A**	**Side B**	**Side C**
f-triangle 1	16.11	17.08	13.85
Geometric shape II	**Side A'**	**Side B'**	**Side C'**
f-triangle 2	28.73	31.94	27.04

Membership function of SSS From (3) with k=0.5 as a constant, k1= 0.56, K2=0.534, k3=0.5122 are the fuzzy proportions A/A', B/B' and C/C' respectively. From Equation (3),

$$\mu_{SSS}(f\text{-}Similar) = e^{-|k - k_1|} * e^{-|k - k_2|} * e^{-|k - k_3|}$$

$$\mu_{SSS}(f\text{-}Similar) = e^{-|0.5 - 0.56|} * e^{-|0.5 - 0.534|} * e^{-|0.5 - 0.5122|}$$

$$\mu_{SSS}(f\text{-}Similar) = e^{-|0.06|} * e^{-|0.034|} * e^{-|0.0122|}$$

$$= 0.94 * 0.97 * 0.98$$

These are the membership values given as inputs to the FIS for SSS postulate.

The Fuzzy Inference System for finding the f-similarity between two f-triangles requires three parameters as input, as discussed in these postulates of AAA, SAS and SSS. We have fuzzified each input variable into three linguistic labels, each of which is represented in triangular membership function (trimf), with a range (1-10) angles / centimetres, Exact [-3 0 3], Similar [2 5 8] and Dissimilar [7 10 13]. All the inputs are in the form of angles or sides. Inputs values are substituted for the rules generated in the FIS, input to the three antecedents will determine the fuzzy output for each rule followed by the fuzzy operators. Fuzzy implication operator is applied to determine the area under the membership value curve in output, maximum is applied in aggregation, followed by defuzzification by middle of maximum. Output is categorized based on 27 rules, categorizes into three levels as exact triangle, f-similar triangle and dissimilar triangle as shown in the output snapshots. However, output will be in crisp form after defuzzification.

We have applied 27 rules formed using the three parameters in each of the postulates as discussed in later part of section 3, includes different weights ranging between [0.1, 1]. However, the rules are given in the Table:4. Quiver view in the snapshots depicts the exactness, f- similarity and dissimilarity by small arrows, dots and bigger arrows respectively. Surface view provides a clear vision of similarity in 3D, but it is not possible to represent in snapshots, therefore we have represented in different views as in Fig 4 (b), (c) and (d). Snapshots obtained are the similar in all the three postulates, because, we have applied similar rules all the cases.

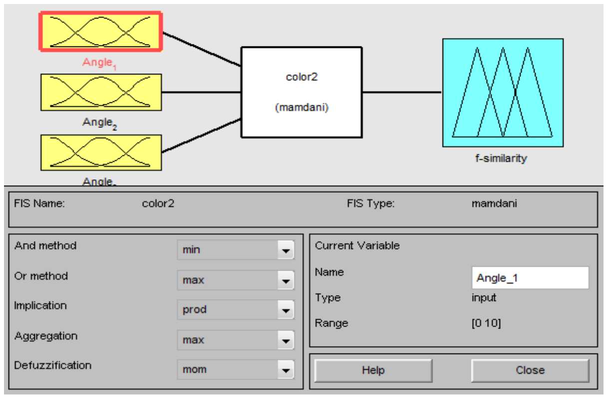


Fig. 3. a) Snapshot of FIS editor for finding f-similarity

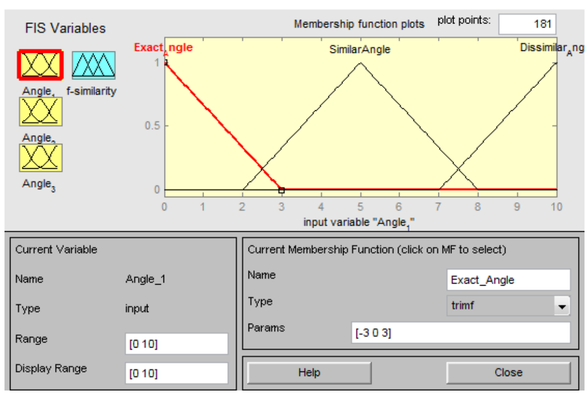
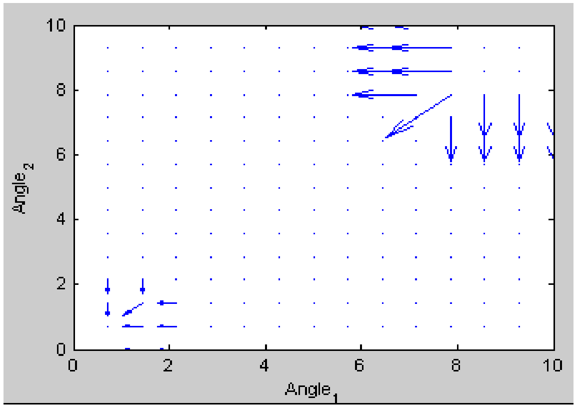


Fig. 3. b) Snapshot of Inputs in triangular membership function

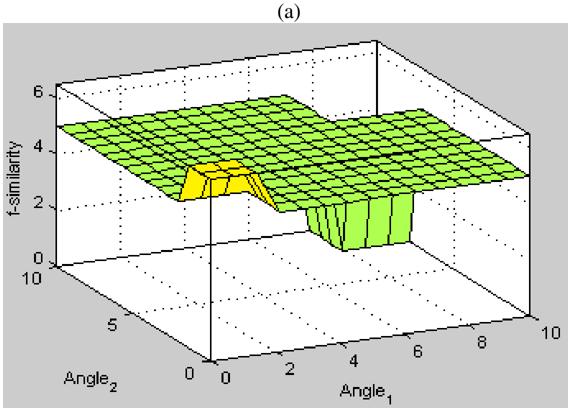
Table 4. Rule base for FIS

S.No	Angle / Side	Angle / Side	Angle / Side	Similarity in triangle
1.	Exact	Exact	Exact	Exact
2.	Exact	Exact	Similar	f-similar
3.	Exact	Dissimilar	Exact	f-similar
4.	Similar	Exact	Exact	f-similar
5.	Exact	Similar	Similar	f-similar
6.	Similar	Exact	Similar	f-similar
7.	Similar	Similar	Exact	f-similar
8.	Similar	Similar	Similar	f-similar
9.	Similar	Similar	Dissimilar	f-similar
10.	Dissimilar	Similar	Similar	f-similar
11.	Similar	Dissimilar	Similar	f-similar
12.	Exact	Similar	Dissimilar	f-similar
13.	Exact	Dissimilar	Similar	f-similar
14.	Dissimilar	Exact	Similar	f-similar
15.	Dissimilar	Similar	Exact	f-similar
16.	Similar	Dissimilar	Exact	f-similar
17.	Similar	Exact	Dissimilar	f-similar
18.	Similar	Dissimilar	Dissimilar	f-similar
19.	Dissimilar	Similar	Dissimilar	f-similar
20.	Dissimilar	Dissimilar	Similar	f-similar
21.	Exact	Exact	Dissimilar	f-similar
22.	Dissimilar	Exact	Exact	f-similar
23.	Exact	Dissimilar	Exact	f-similar
24.	Exact	Dissimilar	Dissimilar	f-similar
25.	Dissimilar	Exact	Dissimilar	f-similar
26.	Dissimilar	Dissimilar	Similar	Dissimilar
27.	Dissimilar	Dissimilar	Dissimilar	Dissimilar

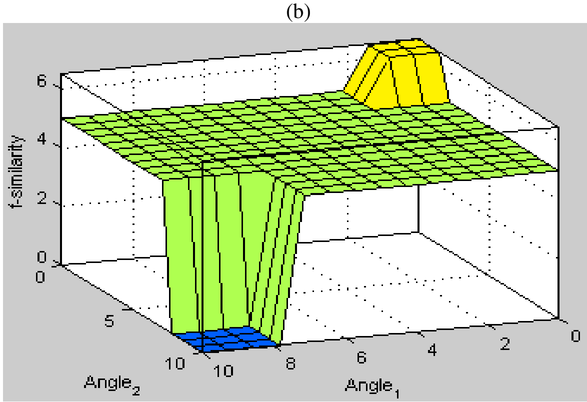
4.2 Snapshots



(a)

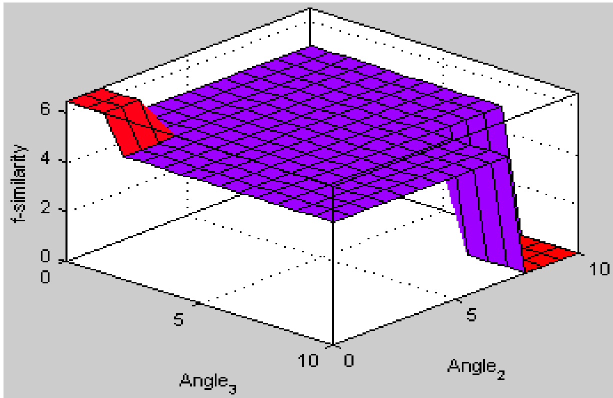


(b)



(c)

Fig. 4. (a) Quiver view with small arrows depicting exact triangle, dotted for f-triangle and bigger arrows for dissimilar. (a) Surface view shows yellow colour for exactness, green for f-similarity with first 2 parameters. (b) Surface view shows blue colour for dissimilarity at zero for first 2 parameters.(c) Surface view shows exactness and dissimilarity in red, f-similarity in blue colours of last 2 parameters.



(d)

Fig. 4. *(continued)*

5 Conclusion

In this paper, we have discussed about f-similarity in f-triangles in terms of membership functions. Nevertheless, the number of input parameters can be extended based on the requirements. Our work is not just restricted to fuzzy geometric shapes, but can be implemented for face recognition, inexact modeling, biometrics, many fields where there is possibilities of uncertainty and other areas of intelligent image processing. Further, FIS designed generates good results, without being erroneous. On similar lines we look forward for further work, which can be extended in OWA with multi criteria decision making.

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