# **Fuzzy Identification of Geometric Shapes**

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**Abstract.** The identification of criminals with sketches can no longer sustain using conventional image processing techniques. Since, it behaves mechanistically, that is, a system which behaves as per given set of rules. We propose a humanistic system for identification of sketches of criminals. Certainly, one must be looking forward for a novel approach, which identifies similarity between a photographic image and a transformed fuzzy image i.e., a sketched image. The transformation on images could be anyone among rotation, reflection, translation, scaling or shearing. In this regard, our approach identifies fuzzy geometric shapes, like humans identify any imprecise shape with their cognition. Such fuzzy shapes cannot be left unidentified under crucial conditions. We begin with estimation of f-validity and then the f-similarity for f-geometric objects, which are considered as basics for developing a humanistic identification system. We implement OWA operators for computing f-similarity in fuzzy geometric shapes. Moreover, the results are found to be justified with the extent of fuzziness.

**Keywords:** *f*-geometry, *f*-similarity, fuzzy identification, *f*-patterns, fuzzy geometry.

## 1 Introduction

Of Late, the world faces merciless crimes and terrorism leading to a massive loss to the mankind by unidentified wrongdoers. Although the wrongdoers escapes intelligently and does not caught on the surveillance camera either. But, the identification of such persons can only be possible using sketching, which is based on the narrations by spectators in Natural Language. Nevertheless, those sketches do not match with the criminal database using the conventional methods of image processing. Since, there is always a vast difference between a sketched image and the image of database. Therefore, the conventional methodologies can no longer sustain in identification of sketches. The sketches may be in any of the transformation forms such as translation, reflection, rotation, scaling or shearing. Despite all these transformation that remains in sketches, it has to be identified, which is proposed in our approach. Therefore, this approaches making inroads to fuzzy face identification, fuzzy feature identification and other fuzzy pattern recognition fields. This paper is organized as follows. In section 1.1, we look back on previous related work. In section 2, we discuss on the introduction of f-geometry. In section 3, we compute the experimental results of f-triangle, f-rectangle and f-square using f-theorem, followed by estimation using OWA operators. Section 4, we conclude along with future directions.

### 1.1 Related Work

Many researchers have performed lot of research in face identification through soft computing techniques, including fuzzy face recognition using c-means clustering, neural network based face recognition.

However, the fuzzy identification of geometric shapes has not been discussed in terms of membership functions except in [2]. However, their effort of fuzzy definitions in exponential membership function is resolved using triangular membership function in [9]. Moreover, estimation of f-similarity in f-triangles is performed by Imran.et.al in [11]. Undoubtedly, their work is implemented in perception based image retrieval in [10], which gives rise to many intelligent image retrieval. Nevertheless, there has been a vast literature about fuzzy geometry, which is discussed in [2]. Our work is stepping towards identifying fuzzy geometric shapes. To the best of our knowledge we have not found any such work on identification of fuzzy geometric shapes.

# 2 *f*- GEOMETRY

In [1], Zadeh has distinguished geometry in two worlds: the world of fuzzy geometry usually referred as *f*-geometry, Wfg, and world of Euclidean geometry, Weg. In Weg, drawing instruments such as ruler, ball point pen and compass exist. On the other hand, in Wfg, no such instruments except an unprecisiated spray pen, so the figures in Wfg looks fuzzy in appearances. The transform of Euclidean or crisp geometry, Weg, results in f-point, f-line, f-circle, f-parallel, f-triangle, f-rectangle and f-square are formalized from point, line, circle, parallel, triangle, rectangle and square respectively. The counterpart of crisp concept *C* in Weg is the fuzzy concept represented as *f*-concept or *f*-*C* or sometimes \**C* in Wfg. The *f*-concept is referred as f-transform of crisp concept. But, there are no formal definitions for *f*-transformation. Nevertheless, it can be applied to the theorems, proofs and axioms. An *f*-theorem is formalized by Euclidean geometric object to form corresponding fuzzy geometric object (see section 2.1). However, the *f*-geometry in this sequel is different in both spirit and substance, which is discussed in detail in [1, 2].

### 2.1 The Concept of f-Theorem

The *f*-theorem emphasizes on the fuzzification of generic rules of a Euclidean triangle. With reference to the discussion in [2, 9], an f-triangle should have three f-lines and the sum of interior angles of about  $180^{\circ}$ . Any form of increase in fuzziness

of the previously said aspects result in the decrement of f-validity. So, we need to estimate the fuzziness in three lines and the interior angles in an f-triangle. At first, we apply f-algorithm to estimate the fuzziness in the lines through the f-transformation distance (d) [2, 9]. For f-rectangles, we consider the following basic aspects, such as: i) four straight lines and ii) right angles at the four corners. Similarly, in case of f-squares, we take an additional parameter of four equal sides as pre-requisite along with four lines and four right angles at the corner.

#### **Definition 1:**

μ

In *f-geometry*, any polygon is called as *f-triangle*, if its membership value is closer to the membership value of a crisp triangle. Moreover, the membership value decreases with increase in fuzziness. Represented as:

$$\mu(f - triangle) = \mu_D * \mu_{SIA} \quad \text{(or)} \quad \mu_{d1} * \mu_{d2} * \mu_{d3} * \mu_{SIA} \tag{1}$$

where  $\mu_D$  denotes the membership value of D, with  $D = d_1 + d_2 + d_3$  and  $d_1, d_2, d_3$  are distance of f-line1, f-line2 and f-line3 from the reference straight line.  $\mu_{SIA}$  denotes the membership value of sum of internal angles with  $\theta = \angle a + \angle b + \angle c$ , and  $\angle a, \angle b, \angle c$  are the internal angles of a triangle.

The computation of membership function for f-line is same as in Equation (1a) and sum of interior angles in Equation (1b) are represented as

$$\mu(f - line) = \begin{cases} \frac{c - D}{c - b} & \text{if } b \le D \le c \\ 0 & \text{if } c \le D \end{cases}$$
(1a)  
$$(f - Sum \text{ of Internal } angle ) = \begin{cases} 0 & \text{if } \theta \le a \\ \frac{\theta - a}{b - a} & \text{if } a \le \theta \le b \\ \frac{c - \theta}{c - b} & \text{if } b \le \theta \le c \\ 0 & \text{if } c \le \theta \end{cases}$$
(1b)

where D,b and c are the distances in real numbers in Equation 1(a), and  $\theta$ , a, b and c are the angles in real numbers in Equation 1(b). The Example 2 given below shows the computation of f-validity for a crisp triangle.

**Example 1:** Any polygon formed with three f-lines of D = 0, makes  $\mu_D = 1$  and sum of interior angles  $\theta = 180^{\circ}$  makes  $\mu_{SIA} = 1$ . By definition.1,  $\mu(f - triangle) = 1$ , estimates the polygon to be a crisp triangle.

#### **Definition 2:**

In *f-geometry*, any quadrilateral is called an *f-rectangle*, if its membership value is closer to the membership value of a crisp rectangle and it decreases with increase in fuzziness. Represented as:

$$\mu(f - rec \tan gle) = \mu_D * \mu_{IA} \text{ or } (\mu_{d1} * \mu_{d2} * \mu_{d3} * \mu_{d4}) * (\mu_{IA1} * \mu_{IA2} * \mu_{IA3} * \mu_{IA4})$$
(2)

where  $\mu_{d1}$ ,  $\mu_{d2}$ ,  $\mu_{d3}$ ,  $\mu_{d4}$  are the individual membership values of f-line1, f-line2, f-line3 and f-line4 respectively, as depicted in Equation 1(a).  $\mu_{IA1}$ ,  $\mu_{IA2}$ ,  $\mu_{IA3}$ ,  $\mu_{IA4}$  are the individual membership values of interior angle1, interior angle2, interior angle3 and interior angle4 respectively. The membership function of each f-line is given as depicted by Equation 1(a), while that of the interior angle are represented as

$$\mu(f - Interior \quad angle \ ) = \begin{cases} 0 & if \quad ia \le a \\ \frac{ia - a}{b - a} & if \quad a \le ia \le b \\ \frac{c - ia}{c - b} & if \quad b \le ia \le c \\ 0 & if \quad c \le ia \end{cases}$$
(a)

where ia, a, b and c are interior angles in Equation 2(a) in real numbers.

**Example 2.** Any quadrilateral formed with four f-lines and four f-interior angles having the distances  $d_1 = d_2 = d_3 = d_4 = 0$ , makes the membership values  $\mu_D = 1$ . With interior angles at 90° each makes  $\mu_{IA} = 1$ , then from Equation (2),  $\mu(f - rec \tan gle) = 1$ , which estimates the quadrilateral to be a crisp rectangle.

#### **Definition 3:**

In *f-geometry*, any quadrilateral is called an *f-square*, if its membership value is closer to the membership value of a crisp square and it decreases with increase in fuzziness. Represented as:

$$\mu(f - square) = \mu_D * \mu_{IA} * \mu_{AS}$$
(3)  
or  
$$= (\mu_{d1} * \mu_{d2} * \mu_{d3} * \mu_{d4}) * (\mu_{IA1} * \mu_{IA2} * \mu_{IA3} * \mu_{IA4}) * (\mu_{S1/S2} * \mu_{S3/S4})$$

where  $\mu_{d1}, \mu_{d2}, \mu_{d3}, \mu_{d4}$  denotes individual membership values of f-line1, f-line2, f-line3 and f-line4 respectively as depicted by Equation 1(a). While interior angles are depicted by Equation 2(a) and  $s = \mu_{S1/S2}, \mu_{S3/S4}$  denotes the difference in length of adjacent sides. Although, the membership function of an f-square can be computed by taking the product using Equation (3). However, we refer computing

 $\mu(f - square)$  using the individual membership function as inputs to Ordered Weighted Averaging operators. The equal sides of an f-square can be represented as:

$$\mu(f - length) = \begin{vmatrix} \frac{c-s}{c-b} & \text{if } b \le s \le c \\ 0 & \text{if } c \le s \end{vmatrix}$$
 3(a)

where s,b,c are the difference in length of adjacent sides in Equation 3(a).

**Example 3:** Any quadrilateral formed with four f-lines, four f-interior angles, difference in length of two adjacent sides be zero. Therefore, the distances  $d_1 = d_2 = d_3 = d_4 = 0$ , makes the membership value  $\mu_D = 1$ . If all the interior angles at 90° each makes  $\mu_{IA} = 1$  and with equal length of all four sides, such that s = (s1/s2) - (s3/s4) = 0 makes  $\mu_{AS} = 1$ . Thus, from Equation (5),  $\mu(f - square) = 1$  estimates the quadrilateral to be a crisp square.

### 3 Computing *f*-Validity and *f*-Similarity in *f*-Geometric Shapes

The computation of f-validity is performed by applying the f-algorithm followed by ftheorem for f-triangles as discussed in [2, 9]. We have inferred a triangle to be crisp triangle in the Example 1. But, the estimation of f-validity in case of fuzzy triangles is shown in the forth coming Example. We have taken three sets of f-triangles for fvalidity in this section and f-similarity later in this paper.

**Example 4(a):** In the first f-triangle shown in Figure 1(a), the f-transformation distance *d* for three f-lines are (0.4, 0.6, 0.6), which results  $\mu_d$  as {0.98, 0.96, 0.96}. The sum of interior angles as 179.25° results in  $\mu_{\delta}$  as {0.925}. The *f*-validity is calculated by taking the product of the above membership function {0.98, 0.96, 0.96 0.925} as 0.8354.



Fig. 1. (a) f-triangle with f-validity as 0.8354 and 1.0

Similarly, we have performed the same method to find the f-validity for the second f-triangle shown in Figure 1a. However, the value of d in f-lines is zero and the sum of interior angles is crisply  $180^{\circ}$ . Therefore, it is apparent that the membership

function for three f-lines and the sum of interior angles are 1.0. Hence, the f-validity is inferred by their product which is computed as1.0. Let us look into some more examples of f-triangle in Figure 1(b) and Figure 1(c).







Fig. 1. (c) f- validity as 0.166 and 0.6409

**Example 4(b):** In the first f-triangle shown in Figure 1(b), the value of *d* for three flines are (0.6, 0.2, 0.8), which results  $\mu_d$  as {0.94, 0.98, 0.93}. The sum of interior angles as 178.0° results  $\mu_{\delta}$  as 0.8. The *f*-validity is calculated by taking the product of the above membership function as 0.6853.

In the second f-triangle shown in Figure 1(b), the value of *d* for three f-lines are (0.6, 0.7, 0.6), which results  $\mu_d$  as {0.94, 0.94, 0.94}. The sum of interior angles as 179.34° results  $\mu_{\delta}$  as 0.934. The *f*-validity is calculated by taking the product of the above membership function as 0.7757.

In the same way, the f-validity of f-triangles shown in Figure 1(c) is also computed. Their f-validity is 0.166 and 0.6409. The f-validity obtained in this section is a pre-requisite for computing the f-similarity between them, which is discussed in the next section. Since, we have only four parameters involved in the computation of f-validity in f-triangles, we have directly multiplied them. However, we implement Ordered Weighted Averaging for aggregation of multiple criteria as inputs for f-rectangles, f- squares and also for f-similarity in f-triangles. Therefore, we have a brief discussion of the same in this section.

#### 3.1 Ordered Weighted Averaging

Ordered Weighted Averaging (OWA) is the central concept of information aggregation, was originally introduced by Yager [4]. The calculation of weights with example is discussed in detail in [7].

#### 3.2 Computing f-Similarity in f-Triangles

The well known approaches for finding similarity are the AAA, SSS and SAS postulates. However, to estimate the f-similarity among a set of f-triangles, we consider AAA postulate. The parameters are the f-validities of two f-triangles of the

(4)

set and the difference between the three f-interior angles. In view of the fact that, anyone of the f-triangle can undergo reflection, rotation, translation, scaling or shearing. So, we consider the difference among the closest of any two f- interior angles. Later, the differences of next closer f-interior angles are computed. Finally, the difference of remaining last f-interior angles is computed. Therefore, the mathematical expression for finding f-similarity can be represented as given below.

$$f - similarity = f - val_{1}^{*} f - val_{2}^{*} \mu_{diff_{1}}^{*} \mu_{diff_{2}}^{*} \mu_{diff_{3}}^{}$$
<sup>(4)</sup>

where *f*-val1, *f*-val2 are the f-validities of triangle1 and triangle2 respectively and  $\mu_{diff_1}, \mu_{diff_2}, \mu_{diff_3}$  are the differences of the f-interior angles as discussed before in this section.

For the first set of triangles shown in Figure 1(a), we have got the following values for f-validity as 0.8354 and 1.0. From Figure 1(a), the difference among the closest f-interior angles are found as [1.09, 7.99, 9.83] generates the membership function {0.91, 0.207, 0.01}. Using Equation 4, f-similarity is computed as:

Undoubtedly, the process of aggregation is found better with OWA operator. So, we implement the same here in the forth coming example.

**Example 5(a):** With the input value m = 5, the fuzzy quantifier 'most' generates the weight vector W [0, 0.2, 0.4, 0.4, 0] (refer [7]). The f-similarity is computed for the ordered inputs X [1.0, 0.91, 0.8354, 0.207, 0.01].

$$f\text{-similarity} = [1.0 \ 0.91 \ 0.8354 \ 0.207 \ 0.01] * [0 \ 0.2 \ 0.4 \ 0.4 \ 0]$$
$$= [(1.0 * 0) + (0.91 * 0.2) + (0.8354 * 0.4) + (0.207 * 0.4) + (0.01 * 0)]$$
$$= [0 + 0.182 + 0.3341 + 0.0828 + 0]$$
$$= 0.5989$$

By the same method, we compute the f- similarity for other set of f- triangles given below in the following examples.

Similarly, the f-similarity for the f-triangles in Figure 1(b) and 1(c) is computed as 0.461 and 0.656 respectively.

#### 3.3 Computing f-Validity and f-Rectangles

Firstly, we compute the level of f-validity in f-rectangles, here we consider 8 important parameters as inputs, (see Equation 2) i.e., with m = 8, the fuzziness in an f-rectangle will be computed using the OWA operator R [4].



Fig. 2. showing the *f*-validity of rectangles in terms of membership values

**Example 6(a):** The Multiple parameters that necessitate for estimating an f-rectangle are the following 8 inputs in the form of membership values ( $\mu_{d1}$ ,  $\mu_{d2}$ ,  $\mu_{d3}$ ,  $\mu_{d4}$ ,  $\mu_{IA1}$ ,  $\mu_{IA2}$ ,  $\mu_{IA3}$ ,  $\mu_{IA4}$ ). Using the *f*-algorithm the following values are computed from the Figure 2(a) as (0.9, 0.88, 0.88, 0.86, 0.852, 0.662, 0.442, 0.232). With the weights being *W* (0, 0, 0, 0.25, 0.25, 0.25, 0.25, 0), the OWA operator estimates the *f*-validity as 0.704.

In Figure 2(b), the membership values (0.985, 0.98, 0.96, 0.957, 0.95, 0.95, 0.883, 0.879) obtained using *f*-algorithm [2, 9]. With the above weights, the OWA operator estimates the *f*-validity as 0.935.

#### 3.4 Computing f-Similarity in f-Rectangles

To find the f-similarity between two fuzzy rectangles require some of the basic parameters that are described in this section. For a set of two f-rectangles, the f-validity of two rectangles whose f-similarity is need to be found are required. In addition, the difference of any two closest f-interior angles, then the difference of next closer f-interior angles and so on. The method of estimating the angle difference from the minimum is due to reflection, rotation, shearing and translation of anyone of the f-rectangles. Let us consider *A*, *B*, *C* and *D* are the lengths of sides of an f-rectangle. Then, we assume A/A' =B/B' \*C/C' \*=D/D' \*= k (fuzzy proportion), where A/A' = k1, B/B' = k2 C/C' = k3 and D/D' = k4. Moreover, the differences in the length of fuzzy proportions (*k*-*k1*, *k*-*k2*, *k*-*k3*, and *k*-*k4*) of the four sides are calculated. The fuzzy proportions k-k1 denoted as fp1 in the equation given below. Therefore, a total of ten parameters constitute as inputs for OWA operators for computing f-similarity as done before.

$$f - similarity = f - val^{*} f - val^{2} * \mu_{diff^{1}} * \mu_{diff^{2}} * \mu_{diff^{3}} * \mu_{diff^{4}} *_{fp^{4}} * \mu_{fp^{1}} * \mu_{fp^{2}} * \mu_{fp^{3}} * \mu_{fp^{4}}$$
(5)

where *f-val1* and *f-val2* are the f-validity of two f-rectangles,  $\mu_{diff1}, \mu_{diff2}, \mu_{diff3}, \mu_{diff4}$  are the membership function of the difference in f-interior angles and  $\mu_{fp1}, \mu_{fp2}, \mu_{fp3}, \mu_{fp4}$  are the membership function of fuzzy proportional difference in four sides of two f-rectangles. Finally, the OWA operator does the computation by aggregating those input values. Let us find f-similarity among two frectangles shown in Figure 2(a) and 2(b) in the forth coming example. **Example 7(a):** With m = 10, the weight vector W [0 0 0 0.2 0.2 0.2 0.2 0.2 0 0] as computed in [7]. The input vector X [0.99 0.99 0.99 0.99 0.943 0.935 0.757 0.704 0.307 0] is in ordinal position as per the Equation 5. The f-similarity between two f-rectangles shown in Figure 2(a) and 2(b) is found as 0.864. Similarly, the f-similarity in Figure 2(b) and 2(c) is found as 0.945

#### 3.5 Computing f-Validity in f-Squares

In this part, we compute f-validity in f-squares, which reveals the validity index of an f-square. For that, we require membership values of the 10 important parameters of an f-square as inputs  $X(x_1, x_2, x_3, ..., x_m)$ , therefore m = 10, along with the weights W(0,0,0,0.2,0.2,0.2,0.2,0.2,0.0,0).

In Figure 3(b), the membership values are (0.85, 0.673, 0.51, 0.4, 0.4, 0.2, 0.15, 0, 0, 0). With the above given weight estimates the OWA that signifies the validity index =0.23



Fig. 3. Showing the *f*-validity of squares in terms of membership values

#### 3.6 Computing f-Similarity in f- Squares Using OWA Operators

To find the f-similarity among a set of f-squares, the following parameters are considered as basics criteria. The parameters required are the f-validity of two f-squares, the difference of the f-interior angles and the difference in the fuzzy proportion of length of the sides. Moreover, the difference among the closest is estimated as performed for the f-rectangles. The mathematical expression for finding f-similarity is the same. Unlike f- rectangles which have two fuzzy proportions in f-similarity of two f-rectangles. But, there is only one common fuzzy proportion k for all the four sides.

Therefore, the difference in fuzzy proportion of lengths k-k1, k-k2, k-k3 and k-k4 are calculated. Let us look into some of the examples in this section, in which we compute f-similarity in f-squares.

**Example 9(a):** With m = 10, the weight vector W [0 0 0 0.2 0.2 0.2 0.2 0.0 0]. The input vector X [0.99 0.99 0.99 0.99 0.85 0.715 0.591 0.23 0 0] is as per the Equation 3 and in the ordinal position. The f-similarity between two f-squares shown in Figure 3(a) and 3(b) is found as 0.6752. Similarly, the f-similarity between two f-rectangles shown in Figure 3(b) and 3(c) is found as 0.692. From the above results, the estimated *f*-similarity among f-triangles, f-rectangles and f-squares are evidently true with respect to our perceptions. Moreover, the measure of fuzziness in *f*-geometric objects, such as *f*-circle, *f*-rectangle, *f*-square are estimated as *f*-validity. The *f*-validity in *f*-point, *f*-line, *f*-circle, *f*-parallel is estimated using the *f*-algorithm [2, 9].

## 4 Conclusions and Future Directions

In this paper, we have estimated the *f*-similarity in *f*-geometric objects of the same class such as rectangle, square and triangle using *f*-theorem. Indeed, we have considered the transformation changes that occur with any geometric shape. Therefore, we find the difference between two closest angles, assuming that the closest angle as the transformed angle from the crisp angle. Additionally, OWA operators are employed to aggregate the membership values of individual features. Undoubtedly, this discussion does not come to an end, instead, it leads to estimate the fuzzy similarity between an image and images of criminal database. Specifically, the features of face like eye, ear, cheek, chin, nose and others in terms of membership values. Therefore, the sketches are estimated by individual membership values of features of a face. However, the application of *f*-similarity is not restricted to Computational Forensics and fuzzy pattern recognition. But, has wide applications in inexact shape modeling, diabetic retinopathy and other fields of intelligent image retrieval using Computing with Words.

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