Algebra of Geometric Filters Defined over Three Dimenisonal Rectangular Lattice-PartI

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Abstract. This paper gives a novel concept of what we call as Geometric Filters defined over a 3-D rectangular grid of pixels. These G-Filters have potential applications to processing of volumetric images. In addition, the paper describes in brief the algebra of G-Filters by formulating a lattice of convex polyhedrons constructed in a 3X3X3 grid of pixels. The results are visualized as a distributive lattice. The lattice also contains its proper envelopes as its successors.

Keywords: Geometric Filters, 3-D image processing, Image Algebra, Extended Topology, Topological Filters.

1 Introduction

The processing of digital images, based on well defined mathematical techniques has remained a subject of interest for many years. In particular mathematical theories associated with processing, enhancement, analysis , and recognition[8],[10] of sensed imageries have received significance amount of attention and effort. At present there are methodologies for image processing using rigorous mathematical framework, for example, that of mathematical morphology [1], [7], [8]. In spite of these efforts, the wide variety of existing methodologies associated with image processing operations are yet to be consolidated under one rigorous unifying mathematical structure.

The term mathematical structure refers to 3-tuple <X, O, R>, [2] where X is a set of mathematical objects, O is the set of operations and R is a set of relations. A mathematical structure with binary operations alone would fall under the algebraic structures of monoids, groups, rings, integral domain and fields. Alternatively, a mathematical structure with binary operations alone fall under the category of formal relational structures like that of lattices.

Based on the above said mathematical structures we construct an image algebra either for 2D or 3D images. [2] [4] [5] [9] [11]. The image algebra is defined as a mathematical theory concerned with the transformation and analysis of images. The central idea of image algebra is based, is that of treating an image

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N. Meghanathan et al. (Eds.): CCSIT 2012, Part III, LNICST 86, pp. 101–111, 2012.

as a construct made up of certain convex polygons or polyhedrons according as the image being 2-D or 3-D, and treating an image processing operation as filtering by means of what we call geometric filters (G-filters) .The notion of G-filters has been derived from extended topology, which is a generalization of the traditional mathematical topology [3],[8],[10].There exists a mathematical framework of G-Filters for 2-D images .

The algebra of 2-D filters is discussed in section 2. In this paper we present a mathematical framework of G-Filters and its algebra for 3-D images in section 3, and the corresponding lattice generated from a 256 convex polyhedrons is shown in section 4. The algorithms related to the same work is discussed in [12].

2 2-D Geometric Filters (2-D G-Filters)

The theory of geometric filters has been formulated purely based on the extended topological filters [2],[8],[10].Consider a 3x3 array of cells as shown in Fig.1a. The smallest convex polygon that would be found inside this cell array is also shown in Fig. 1b.

1	2	3
4	5	6
7	8	9

	2	
4	5	6
	8	

Fig. 1. (a) 3x3 array of cell (b) Smallest Convex Polygon

Note that sixteen such different convex polygons can be formed by dropping indirect neighbour (corner) cells, which are pixels 1, 3, 7 and 9. Fig. 2 shows all 16 convex polygons. The suffixes denote the cells that are dropped.

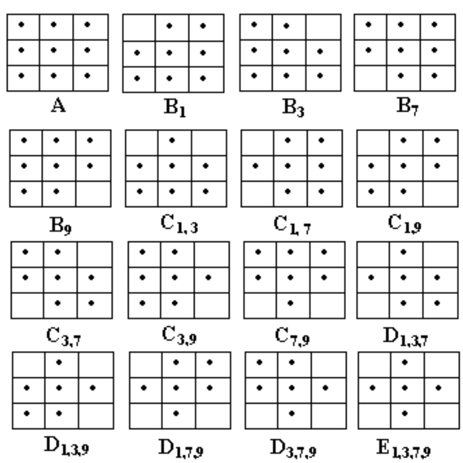

Fig. 2. 16 Convex Polygons

2.1 Algebra Of 2 Dimensional G-Filters

The relational mathematical structure in which the G-filters are studied is a lattice $\phi =< X, \preceq >$ where X is a set of G-filters and $\preceq$ is the partial order relation "finer than". For example, the filter F1 = { A, B_1, B_7, B_9, $C_{1,7}$ } is finer than that of F2 = { A ,B_1, B_7,$C_{1,7}$ }. [2],[4],[5],[6]. All the 2D convex polygons form a lattice with A as their supremum and E as their infimum. Note that there are 5 levels in the lattice Fig. 3. All the elements in a level constitute a group which is viewed as an immediate sub group of the group of elements in the just higher level. We can see from lattice that there are 24 linear chains consisting of proper envelopes starting from E .Each chain exhibits a linear hierarchy of generating A from E.

Theorem 1. *One can construct a total of 95 hierarchical G-filters using 3x3 cell structure.*

Proof. Refer [2]

we obtain the total number of heirarchical G-filters as

$$2 + 2\sum_{i=1}^{4} {}^{4}C_i + \sum_{i=1}^{6} {}^{6}C_i = 95 \tag{1}$$

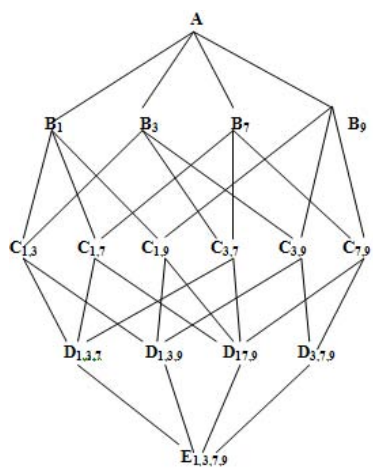

Fig. 3. Lattice of 16 Convex Polygons

3 3-D GEOMETRIC FILTERS (3 D G-Filters)

Consider a 3X3X3 array of 27 cells as shown in Fig. 4. The smallest convex polyhedron that could be formed inside this cell array is also shown in Fig. 5

(K-1) Plane			K Plane			(K-1) Plane		
1	2	3	10	11	12	19	20	21
4	5	6	13	14	15	22	23	24
7	8	9	16	17	18	25	26	27

Fig. 4. Cubic array of size 3x3x3

(K-1) Plane			K Plane			(K-1) Plane		
	2		10	11	12		20	
4	5	6	13	14	15	22	23	24
	8		16	17	18		26	

Fig. 5. Smallest 3-D Convex Polyhedron. $I_{1,3,7,9,19,21,25,27}$

The term convex polyhedron refers to a 3-D wire frame contour that could be drawn using certain neighbourhood pixels including a minimum of all 19 neighbourhood pixels shown in figure 5, that is, cells 2, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18 20, 22, 24, 26. The cells 1, 3, 7, 9, 19, 21, 25 and 27 are the corner cells with respect to the central cell 14. For example the pixels (coner pixels) 1, 3, 7, 9,19, 21, 25 and 27 in Fig. 5 form a 3D contour (convex polyhedron) with respect to the central pixel 14. There are 8 corner cells in a 3x3x3 rectangular grid and the main idea is to construct various unique possible convex polyhedrons. 256 convex polyhedrons are obtained using the formula

$$\sum_{i=0}^{8} {}^8C_i = {}^8C_0 + {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8 = 256 \quad (2)$$

Starting from, say A which has corner pixels 1, 3, 7, 9,19, 21, 25 and 27 as in Fig .5, the process starts first by eliminating one corner pixel at a time and listing the different possibilities under group B, for set A we are able to find 8, B subsets (B1,B3,B7,B9,B19,B21,B25,B27) which is ${}^8C_1$.The process continues, explained in THEOREM 2. The following are the combinations for different sets

$$A = {}^8C_0 = 1, B = {}^8C_1 = 8, C = {}^8C_2 = 28, D = {}^8C_3 = 56, E = {}^8C_4 = 70,$$
$$F = {}^8C_5 = 56, G = {}^8C_6 = 28, H = {}^8C_7 = 8, I = {}^8C_8 = 1.$$

256 convex polyhedrons are classified under nine groups as A, B, C, D, E, F, G, H, and I as shown in the Table 1.

Table 1. 256 Convex Polyhedrons

A -No Pixel is eliminated we get 1 combination A={1,3,7,9,19,21,25,27}		
Group B - Eliminating one voxel we obtain 8 combinations		
B_1={3,7,9,19,21,25,27}	B_3={1,7,9,19,21,25,27}	B_7={1,3,9,19,21,25,27}
B_9={1,3,7,19,21,25,27}	$B_{1,9}$={1,3,7,9,21,25,27}	B_{21}={1,3,7,9,19,25,27}
B_{25}={1,3,7,9,19,21,27}	B_{27}={1,3,7,9,19,21,25}	
Group C - By Eliminating two voxels we obtain 28 combinations		
$C_{1,3}$={7,9,19,21,25,27}	$C_{1,7}$={3,9,19,21,25,27}	$C_{1,9}$={3,7,19,21,25,27}
$C_{1,19}$={3,7,9,21,25,27}	$C_{1,21}$={3,7,9,19,25,27}	$C_{1,25}$={3,7,9,19,21,27}
$C_{1,27}$={3,7,9,19,21,25}	$C_{3,7}$={1,9,19,21,25,27}	$C_{3,9}$={1,7,19,21,25,27}
$C_{3,19}$={1,7,9,21,25,27}	$C_{3,21}$={1,7,9,19,25,27}	$C_{3,25}$={1,7,9,19,21,27}
$C_{3,27}$={1,7,9,19,21,25}	$C_{7,9}$={1,3,19,21,25,27}	$C_{7,19}$={1,3,9,21,25,27}
$C_{7,21}$={1,3,9,19,25,27}	$C_{7,25}$={1,3,9,19,21,27}	$C_{7,27}$={1,3,9,19,21,25}
$C_{9,19}$={1,3,7,21,25,27}	$C_{9,21}$={1,3,7,19,25,27}	$C_{9,25}$={1,3,7,19,21,27}
$C_{9,27}$={1,3,7,19,21,25}	$C_{19,21}$={1,3,7,9,25,27}	$C_{19,25}$={1,3,7,9,21,27}
$C_{19,27}$={1,3,7,9,21,25}	$C_{21,25}$={1,3,7,9,19,27}	$C_{21,27}$={1,3,7,9,19,25}
$C_{25,27}$={1,3,7,9,19,21}		
Group D - By Eliminating three voxels we obtain 56 combinations		
$D_{1,3,7}$={9,19,21,25,27}	$D_{1,3,9}$={7,19,21,25,27}	$D_{1,3,19}$={7,9,21,25,27}
$D_{1,3,21}$={7,9,19,25,27}	$D_{1,3,25}$={7,9,19,21,27}	$D_{1,3,27}$={7,9,19,21,25}
$D_{1,7,9}$={3,19,21,25,27}	$D_{1,7,19}$={3,9,21,25,27}	$D_{1,7,21}$={3,9,19,25,27}
$D_{1,7,25}$={3,9,19,21,27}	$D_{1,7,27}$={3,9,19,21,25}	$D_{1,9,19}$={3,7,21,25,27}
$D_{1,9,21}$={3,7,19,25,27}	$D_{1,9,25}$={3,7,19,21,27}	$D_{1,9,27}$={3,7,19,21,25}
$D_{1,19,21}$={3,7,9,25,27}	$D_{1,19,25}$={3,7,9,21,27}	$D_{1,19,27}$={3,7,9,21,25}
$D_{1,21,25}$={3,7,9,19,27}	$D_{1,21,27}$={3,7,9,19,25}	$D_{1,25,27}$={3,7,9,19,21}
$D_{3,7,9}$={1,19,21,25,27}	$D_{3,7,19}$={1,9,21,25,27}	$D_{3,7,21}$={1,9,19,25,27}
$D_{3,7,25}$={1,9,19,21,27}	$D_{3,7,27}$={1,9,19,21,25}	$D_{3,9,19}$={1,7,21,25,27}
$D_{3,9,21}$={1,7,19,25,27}	$D_{3,9,25}$={1,7,19,21,27}	$D_{3,9,27}$={1,7,19,21,25}
$D_{3,19,21}$={1,7,9,25,27}	$D_{3,19,25}$={1,7,9,21,27}	$D_{3,19,27}$={1,7,9,21,25}
$D_{3,21,25}$={1,7,9,19,27}	$D_{3,21,27}$={1,7,9,19,25}	$D_{3,25,27}$={1,7,9,19,21}
$D_{7,9,19}$={1,3,21,25,27}	$D_{7,9,21}$={1,3,19,25,27}	$D_{7,9,25}$={1,3,19,21,27}
$D_{7,9,27}$={1,3,19,21,25}	$D_{7,19,21}$={1,3,9,25,27}	$D_{7,19,25}$={1,3,9,21,27}
$D_{7,19,27}$={1,3,9,21,25}	$D_{7,21,25}$={1,3,9,19,27}	$D_{7,21,27}$={1,3,9,19,25}
$D_{7,25,27}$={1,3,9,19,21}	$D_{9,19,21}$={1,3,7,25,27}	$D_{9,19,25}$={1,3,7,21,27}
$D_{9,19,27}$={1,3,7,21,25}	$D_{9,21,25}$={1,3,7,19,27}	$D_{9,21,27}$={1,3,7,19,25}

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$D_{9,25,27}=\{1,3,7,19,21\}$	$D_{19,21,25}=\{1,3,7,9,27\}$	$D_{19,21,27}=\{1,3,7,9,25\}$
$D_{19,25,27}=\{1,3,7,9,21\}$	$D_{21,25,27}=\{1,3,7,9,19\}$	
Group E - Eliminating four voxels we obtain 70 combinations		
$E_{1,3,7,9}=\{19,21,25,27\}$	$E_{1,3,7,19}=\{9,21,25,27\}$	$E_{1,3,7,21}=\{9,19,25,27\}$
$E_{1,3,7,25}=\{9,19,21,27\}$	$E_{1,3,7,27}=\{9,19,21,25\}$	$E_{1,3,9,19}=\{7,21,25,27\}$
$E_{1,3,9,21}=\{7,19,25,27\}$	$E_{1,3,9,25}=\{7,19,21,27\}$	$E_{1,3,9,27}=\{7,19,21,25\}$
$E_{1,3,19,21}=\{7,9,25,27\}$	$E_{1,3,19,25}=\{7,9,21,27\}$	$E_{1,3,19,27}=\{7,9,21,25\}$
$E_{1,3,21,25}=\{7,9,19,27\}$	$E_{1,3,21,27}=\{7,9,19,25\}$	$E_{1,3,25,27}=\{7,9,19,21\}$
$E_{1,7,9,19}=\{3,21,25,27\}$	$E_{1,7,9,21}=\{3,19,25,27\}$	$E_{1,7,9,25}=\{3,19,21,27\}$
$E_{1,7,9,27}=\{3,19,21,25\}$	$E_{1,7,19,21}=\{3,9,25,27\}$	$E_{1,7,19,25}=\{3,9,21,27\}$
$E_{1,7,19,27}=\{3,9,21,25\}$	$E_{1,7,21,25}=\{3,9,19,27\}$	$E_{1,7,21,27}=\{3,9,19,25\}$
$E_{1,7,25,27}=\{3,9,19,21\}$	$E_{1,9,19,21}=\{3,7,25,27\}$	$E_{1,9,19,25}=\{3,7,21,27\}$
$E_{1,9,19,27}=\{3,7,21,25\}$	$E_{1,9,21,25}=\{3,7,19,27\}$	$E_{1,9,21,27}=\{3,7,19,25\}$
$E_{1,9,25,27}=\{3,7,19,21\}$	$E_{1,19,21,25}=\{3,7,9,27\}$	$E_{1,19,21,27}=\{3,7,9,25\}$
$E_{1,19,25,27}=\{3,7,9,21\}$	$E_{1,21,25,27}=\{3,7,9,19\}$	$E_{3,7,9,19}=\{1,21,25,27\}$
$E_{3,7,9,21}=\{1,19,25,27\}$	$E_{3,7,9,25}=\{1,19,21,27\}$	$E_{3,7,9,27}=\{1,19,21,25\}$
$E_{3,7,19,21}=\{1,9,25,27\}$	$E_{3,7,19,25}=\{1,9,21,27\}$	$E_{3,7,19,27}=\{1,9,21,25\}$
$E_{3,7,21,25}=\{1,9,19,27\}$	$E_{3,7,21,27}=\{1,9,19,25\}$	$E_{3,7,25,27}=\{1,9,19,21\}$
$E_{3,9,19,21}=\{1,7,25,27\}$	$E_{3,9,19,25}=\{1,7,21,27\}$	$E_{3,9,19,27}=\{1,7,21,25\}$
$E_{3,9,21,25}=\{1,7,19,27\}$	$E_{3,9,21,27}=\{1,7,19,25\}$	$E_{3,9,25,27}=\{1,7,19,21\}$
$E_{3,19,21,25}=\{1,7,9,27\}$	$E_{3,19,21,27}=\{1,7,9,25\}$	$E_{3,19,25,27}=\{1,7,9,21\}$
$E_{3,21,25,27}=\{1,7,9,19\}$	$E_{7,9,19,21}=\{1,3,25,27\}$	$E_{7,9,19,25}=\{1,3,21,27\}$
$E_{7,9,19,27}=\{1,3,21,25\}$	$E_{7,9,21,25}=\{1,3,19,27\}$	$E_{7,9,21,27}=\{1,3,19,25\}$
$E_{7,9,25,27}=\{1,3,19,21\}$	$E_{7,19,21,25}=\{1,3,9,27\}$	$E_{7,19,21,27}=\{1,3,9,25\}$
$E_{7,19,25,27}=\{1,3,9,21\}$	$E_{7,21,25,27}=\{1,3,9,19\}$	$E_{9,19,21,25}=\{1,3,7,27\}$
$E_{9,19,21,27}=\{1,3,7,25\}$	$E_{9,19,25,27}=\{1,3,7,21\}$	$E_{9,21,25,27}=\{1,3,7,19\}$
$E_{19,21,25,27}=\{1,3,7,9\}$		
Group F - By Eliminating five voxels we obtain 56 combinations		
$F_{1,3,7,9,19}=\{21,25,27\}$	$F_{1,3,7,9,21}=\{19,25,27\}$	$F_{1,3,7,9,25}=\{19,21,27\}$
$F_{1,3,7,9,27}=\{19,21,25\}$	$F_{1,3,7,19,21}=\{9,25,27\}$	$F_{1,3,7,19,25}=\{9,21,27\}$
$F_{1,3,7,19,27}=\{9,21,25\}$	$F_{1,3,7,21,25}=\{9,19,27\}$	$F_{1,3,7,21,27}=\{9,19,25\}$
$F_{1,3,7,25,27}=\{9,19,21\}$	$F_{1,3,9,19,21}=\{7,25,27\}$	$F_{1,3,9,19,25}=\{7,21,27\}$
$F_{1,3,9,19,27}=\{7,21,25\}$	$F_{1,3,9,21,25}=\{7,19,27\}$	$F_{1,3,9,21,27}=\{7,19,25\}$
$F_{1,3,9,25,27}=\{7,19,21\}$	$F_{1,3,19,21,25}=\{7,9,27\}$	$F_{1,3,19,21,27}=\{7,9,25\}$
$F_{1,3,19,25,27}=\{7,9,21\}$	$F_{1,3,21,25,27}=\{7,9,19\}$	$F_{1,7,9,19,21}=\{3,25,27\}$
$F_{1,7,9,19,25}=\{3,21,27\}$	$F_{1,7,9,19,27}=\{3,21,25\}$	$F_{1,7,9,21,25}=\{3,19,27\}$
$F_{1,7,9,21,27}=\{3,19,25\}$	$F_{1,7,9,25,27}=\{3,19,21\}$	$F_{1,7,19,21,25}=\{3,9,27\}$
$F_{1,7,19,21,27}=\{3,9,25\}$	$F_{1,7,19,25,27}=\{3,9,21\}$	$F_{1,7,21,25,27}=\{3,9,19\}$
$F_{1,9,19,21,25}=\{3,7,27\}$	$F_{1,9,19,21,27}=\{3,7,25\}$	$F_{1,9,19,25,27}=\{3,7,21\}$
$F_{1,9,21,25,27}=\{3,7,19\}$	$F_{1,19,21,25,27}=\{3,7,9\}$	$F_{3,7,9,19,21}=\{1,25,27\}$
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$F_{3,7,9,19,25}$={1,21,27}	$F_{3,7,9,19,27}$={1,21,25}	$F_{3,7,9,21,25}$={1,19,27}
$F_{3,7,9,21,27}$={1,19,25}	$F_{3,7,9,25,27}$={1,19,21}	$F_{3,7,19,21,25}$={1,9,27}
$F_{3,7,19,21,27}$={1,9,25}	$F_{3,7,19,25,27}$={1,9,21}	$F_{3,7,21,25,27}$={1,9,19}
$F_{3,9,19,21,25}$={1,7,27}	$F_{3,9,19,21,27}$={1,7,25}	$F_{3,9,19,25,27}$={1,7,21}
$F_{3,9,21,25,27}$={1,7,19}	$F_{3,19,21,25,27}$={1,7,9}	$F_{7,9,19,21,25}$={1,3,27}
$F_{7,9,19,21,27}$={1,3,25}	$F_{7,9,19,25,27}$={1,3,21}	$F_{7,9,21,25,27}$={1,3,19}
$F_{7,19,21,25,27}$={1,3,9}	$F_{9,19,21,25,27}$={1,3,7}	
Group G - By Eliminating six voxels we obtain 28 combinations		
$G_{1,3,7,9,19,21}$={25,27}	$G_{1,3,7,9,19,25}$={21,27}	$G_{1,3,7,9,19,27}$={21,25}
$G_{1,3,7,9,21,25}$={19,27}	$G_{1,3,7,9,21,27}$={19,25}	$G_{1,3,7,9,25,27}$={19,21}
$G_{1,3,7,19,21,25}$={9,27}	$G_{1,3,7,19,21,27}$={9,25}	$G_{1,3,7,19,25,27}$={9,21}
$G_{1,3,7,21,25,27}$={9,19}	$G_{1,3,9,19,21,25}$={7,27}	$G_{1,3,9,19,21,27}$={7,25}
$G_{1,3,9,19,25,27}$={7,21}	$G_{1,3,9,21,25,27}$={7,19}	$G_{1,3,19,21,25,27}$={7,9}
$G_{1,7,9,19,21,25}$={3,27}	$G_{1,7,9,19,21,27}$={3,25}	$G_{1,7,9,19,25,27}$={3,21}
$G_{1,7,9,21,25,27}$={3,19}	$G_{1,7,19,21,25,27}$={3,9}	$G_{1,9,19,21,25,27}$={3,7}
$G_{3,7,9,19,21,25}$={1,27}	$G_{3,7,9,19,21,27}$={1,25}	$G_{3,7,9,19,25,27}$={1,21}
$G_{3,7,9,21,25,27}$={1,19}	$G_{3,7,19,21,25,27}$={1,9}	$G_{3,9,19,21,25,27}$={1,7}
$G_{7,9,19,21,25,27}$={1,3}		
Group H - By Eliminating seven voxels we obtain 8 combinations		
$H_{1,3,7,9,19,21,25}$={27}	$H_{1,3,7,9,19,21,27}$={25}	$H_{1,3,7,9,19,25,27}$={21}
$H_{1,3,7,9,21,25,27}$={19}	$H_{1,3,7,19,21,25,27}$={9}	$H_{1,3,9,19,21,25,27}$={7}
$H_{1,7,9,19,21,25,27}$={3}	$H_{3,7,9,19,21,25,27}$={1}	
I Eliminating eight voxels we obtain 1 combination $I_{1,3,7,9,19,21,25,27}$**={}**		

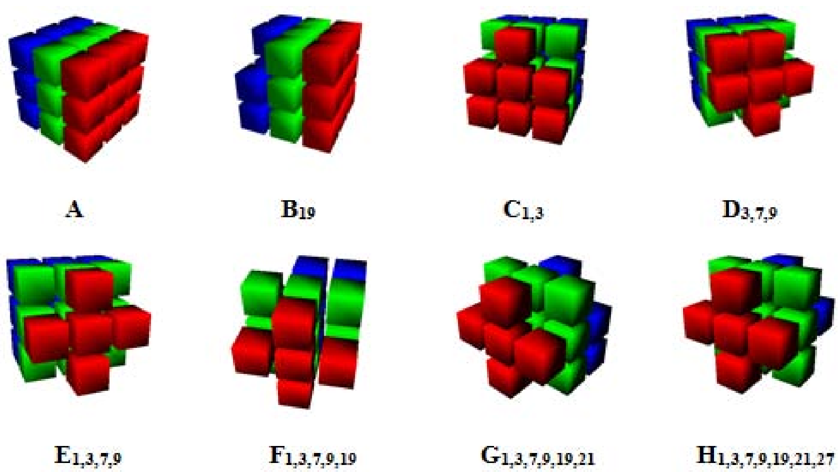

Fig. 6. Convex Polyhedrons

Few examples of Convex polyhedrons, constructed in a 3X3X3 grid of pixels as listed in table 1, is shown in Figure 6. An algorithm is developed for constructing the above 256 Convex polyhedrons.The algorithm is not discussed here. Red represents front plane (k-1), Green represents central plane (k), Blue represents rear plane (k+1).

Theorem 2. *One can construct a total of*

$$2 + 2\sum_{i=1}^{8} {}^{8}C_i + 2\sum_{i=1}^{28} {}^{28}C_i + 2\sum_{i=1}^{56} {}^{56}C_i + \sum_{i=1}^{70} {}^{70}C_i \tag{3}$$

hierarchical G-filters[2] in a 3 x3 x3 cell structure.

Proof. Note that there are eight corners cells in a 3x3x3 cell structure and we construct$\sum_{i=1}^{8} {}^{8}C_i$ convex patterns in the following manner. The first group A contains ${}^{8}C_0$ that is , one polyhedron which is 3x3x3 cell structure itself. The group B contains ${}^{8}C_1$ that is, 8 polyhedrons B1 (without cell 1),B2 (without cell 2), B3 , B7 , B9 ,B19, B21, B25 and B27. Similarly the the group C contains ${}^{8}C_2$ that is, 28 polyhedrons C1,3 (without cells 1 and 3), C1,7, C1,9, C3,7, C3,9 and C7,9 C25,27 .In this manner ,the elements of group D, E,F,G,H and I are identified. Now the first G-filter F1 contains the polyhedron I in the set, and as per the definition of G-filter,F1 should contain all of its 256 proper envelopes. It is easy to see the cardinality of the set to be 256. Now, the absence of I in the set would yeild a coarse G-filter F2 with 255 remaining elements. Then by dropping one element at a time from the group H we can generate eight heirarchical G-filters with cardinality 254. Next, in order to get a filter with a cardinality 253,we leave out G and any two elements in group H. We see here ${}^{8}C_2$ such possibilities. subsequently we construct ${}^{8}C_3$ filters with cardinality 252 by dropping I and any three other elements of group D. Lastly we generate one filter with cardinality 251 by dropping I and any four other elements of H.This procedure is repeated for the remaining groups also. Ultimately we obtain the total number of heirarchical G-filters.

$$\begin{aligned} 2 + 2\sum_{i=1}^{8} {}^{8}C_i + 2\sum_{i=1}^{28} {}^{28}C_i + 2\sum_{i=1}^{56} {}^{56}C_i + \sum_{i=1}^{70} {}^{70}C_i \\ = 1,180,735,735,908,220,000,000 \end{aligned}$$

4 Results

The G-Filters for 3 Dimensional rectangular grid is identified and their corresponding convex polyhedrons are listed in the Table 1. All the 256 convex polyhedrons are arranged in 9 levels with A set as root node, I set as leaf node and sets B, C, D, E, F, G, H as intermediate nodes to form a complete distributive lattice, which is shown in the Fig 7. Only connections between A to B and H to I are shown in the Fig 7.,due to the complexity involved in forming

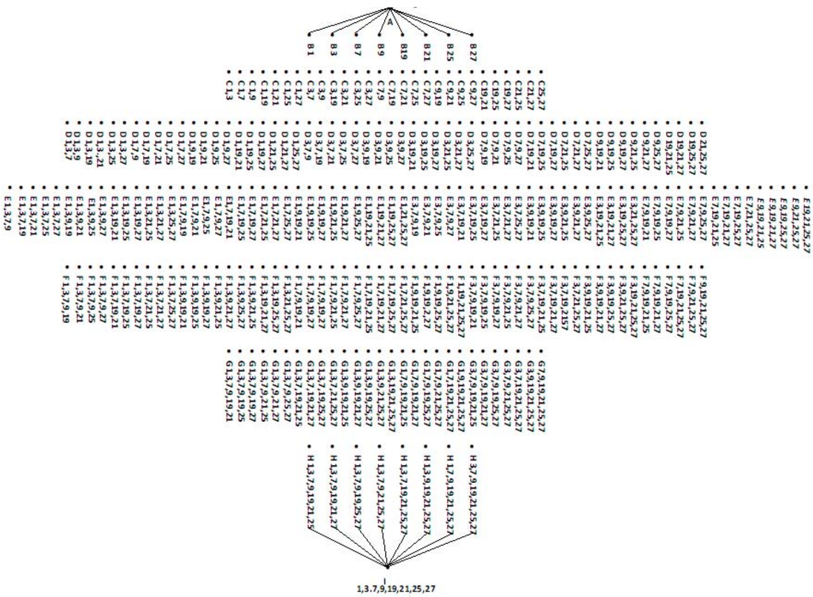

Fig. 7. Lattice Formed By 256 Convex Polyhedrons

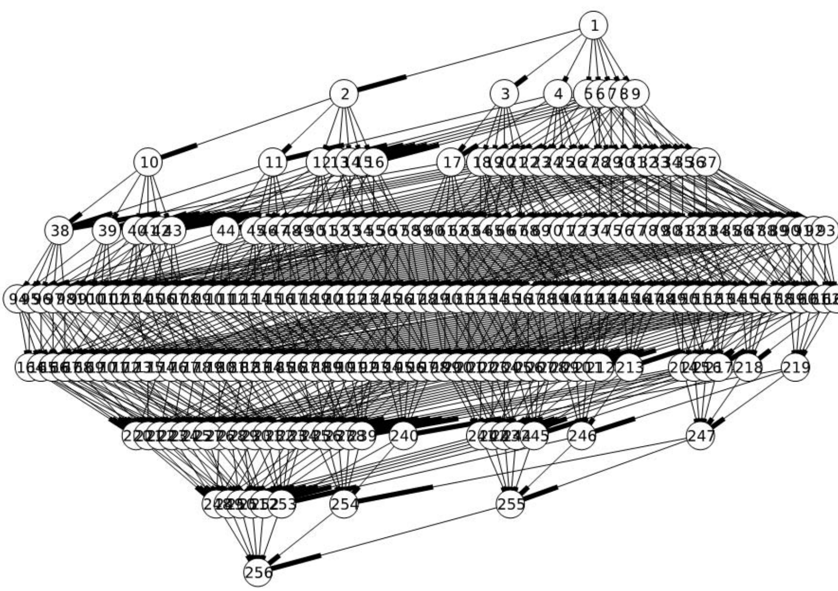

Fig. 8. Lattice Formed By 256 Convex Polyhedrons

the connections of 3D lattice. In the constructed distributive lattice root node A is supremum and leaf node I is infimum. The number of subsets formed from their corresponding sets at each level is identified as 1, 8, 28, 56, 70, 56, 28, 8, 1 at level 0, level 1, level 2, level 3, level 4, level 5, level 6, level 7, level 8, level 9 respectively in the lattice. The hierarchical relationships among the sets and their corresponding sub sets is identified. and visualized by using the hierarchy construction algorithm . Due to the complexity in visualisation the nodes are mentioned in numbers instead of set name. For example, node 1 in Fig 7 is A. Similarly, node 256 corresponds to I1, 3, 7, 9, 19, 21, 25, 27. The number of unique linear chains in the 2-D and 3-D lattices are identified and proved in the Theorem1 and Theorem 2 respectively.

5 Conclusion

In this paper, we have considered three dimensional polyhedrons developed in a 3 X 3 X 3 lattice. These 256 polyhedrons form a lattice with A as the supremum and $I_{1,3,7,9,19,21,25,27}$ as infimum. We developed the notion of 3-D Geometric Filters and showed their potential applications to 3 D image processing.

Acknowledgments. We sincerely thank Mrs. G.Chitra, Managing Director, Pentagram Research Centre Pvt. Ltd.for her support to carry out research work.

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