Hybrid Synchronization of Hyperchaotic Chen Systems via Sliding Mode Control

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Abstract. This paper investigates the hybrid synchronization of hyperchaotic Chen systems (Jia, Dai and Hui, 2010) via sliding mode control. The stability results for the sliding mode control based synchronization schemes derived in this paper are established using Lyapunov stability theory. The sliding mode control method is very effective and convenient to achieve global chaos synchronization of the identical hyperchaotic Chen systems because the Lyapunov exponents are not required for these calculations. Numerical simulations are presented to demonstrate the sliding mode control results derived in this paper for the hybrid synchronization of identical hyperchaotic Chen systems.

Keywords: Sliding mode control, hybrid synchronization, hyperchaos, hyperchaotic Chen system.

1 Introduction

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly known as the *butterfly effect* [1].

In most of the synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the idea of synchronization is to use the output of the master system to control the slave system so that the output of the response system tracks the output of the master system asymptotically.

Since the pioneering work by Pecora and Carroll ([2], 1990), chaos synchronization problem has been studied extensively in the literature. Chaos theory has been applied to a variety of fields including physical systems [3], chemical systems [4], ecological systems [5], secure communications ([6]-[8]) etc.

In the last two decades, various control schemes have been developed and successfully applied for the chaos synchronization such as PC method [2], OGY method [9], active control ([10]-[12]), adaptive control ([13]-[15]), time-delay feedback method [16], backstepping design method ([17]-[18]), sampled-data feedback synchronization method ([19]-[20]) etc.

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So far, many types of synchronization phenomenon have been presented such as complete synchronization [2], generalized synchronization [21], anti-synchronization [22], projective synchronization [23], generalized projective synchronization [24], etc.

Complete synchronization (CS) is characterized by the equality of state variables evolving in time, while anti-synchronization (AS) is characterized by the disappearance of the sum of relevant state variables evolving in time. Projective synchronization (PS) is characterized by the fast that the master and slave systems could be synchronized up to a scaling factor. In generalized projective synchronization (GPS), the responses of the synchronized dynamical states synchronize up to a constant scaling matrix. It is easy to see that the complete synchronization and anti-synchronization are special cases of the generalized projective synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively. In hybrid synchronization of two chaotic systems [25], one part of the systems is completely synchronized and the other part is anti-synchronized so that the complete synchronization (CS) and anti-synchronization (AS) co-exist in the systems.

In this paper, we derive new results based on the sliding mode control ([26]-[28]) for the global chaos synchronization of identical hyperchaotic Chen systems ([29], Jia, Dai and Hui, 2010).

The stability results for the sliding mode control based synchronization schemes derived in this paper are established using Lyapunov stability theory [30]. In robust control systems, sliding mode control is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and external disturbances.

This paper has been organized as follows. In Section 2, we describe the problem statement and our methodology using sliding mode control. In Section 3, we discuss the global chaos synchronization of identical hyperchaotic Chen systems ([29], 2010). In Section 4, we summarize the main results obtained in this paper.

2 Problem Statement and Our Methodology Using Sliding Mode Control

In this section, we detail the problem statement for global chaos synchronization of identical chaos systems and our methodology using sliding mode control (SMC) and Lyapunov stability theory.

Consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : \mathbb{R}^n \to \mathbb{R}^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in \mathbb{R}^n$ is the state of the system and $u \in \mathbb{R}^m$ is the controller of the slave system.

In hybrid synchronization, we define the synchronization error so that the odd states of the systems (1) and (2) are completely synchronized and the even states of the systems (1) and (2) are anti-synchronized.

Thus, we define the *hybrid synchronization error* as

$$e_i = \begin{cases} y_i - x_i, \text{ if } i \text{ is odd} \\ y_i + x_i, \text{ if } i \text{ is even} \end{cases} \tag{3}$$

Then the error dynamics can be expressed in the form

$$\dot{e} = Ae + \eta(x, y) + u \tag{4}$$

The objective of the hybrid synchronization problem is to find a controller u such that

$$\lim_{t \to \infty} \|e(t)\| = 0 \quad \text{for all } \ e(0) \in \mathbb{R}^n \tag{5}$$

To solve this problem, we first define the control u as

$$u(t) = -\eta(x, y) + Bv(t) \tag{6}$$

where B is a constant gain vector selected such that (A, B) is controllable.

Substituting (6) into (4), the error dynamics simplifies to

$$\dot{e} = Ae + Bv \tag{7}$$

which is a linear time-invariant control system with single input v.

Thus, the original global chaos synchronization problem can be replaced by an equivalent problem of stabilizing the zero solution $e = 0$ of the linear system (7) be a suitable choice of the sliding mode control.

In the sliding mode control, we define the variable

$$s(e) = Ce = c_1 e_1 + c_2 e_2 + \cdots + c_n e_n \tag{8}$$

where $C = [\, c_1 \quad c_2 \quad \cdots \quad c_n \,]$ is a constant vector to be determined.

In the sliding mode control, we constrain the motion of the system (7) to the sliding manifold defined by

$$S = \{x \in \mathbb{R}^n \mid s(e) = 0\} = \{x \in \mathbb{R}^n \mid c_1 e_1 + c_2 e_2 + \cdots + c_n e_n = 0\}$$

which is required to be invariant under the flow of the error dynamics (7).

When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \tag{9}$$

which is the defining equation for the manifold S and

$$\dot{s}(e) = 0 \tag{10}$$

which is the necessary condition for the state trajectory $e(t)$ of the system (7) to stay on the sliding manifold S.

Using (7) and (8), the equation (10) can be rewritten as

$$\dot{s}(e) = C\,[Ae + Bv] = 0 \tag{11}$$

Solving (11), we obtain the equivalent control law given by

$$v_{\text{eq}}(t) = -(CB)^{-1}CAe(t) \tag{12}$$

where C is chosen such that $CB \neq 0$.

Substituting (12) into the error dynamics (7), we get the closed-loop dynamics as

$$\dot{e} = [I - B(CB)^{-1}C]Ae \tag{13}$$

where C is chosen such that the system matrix $[I - B(CB)^{-1}C]A$ is Hurwitz.

Then the controlled system (13) is globally asymptotically stable.

To design the sliding mode controller for the linear time-invariant system (7), we use the constant plus proportional rate reaching law

$$\dot{s} = -q\text{sgn}(s) - ks \tag{14}$$

where sgn$(\cdot)$ denotes the sign function and the gains $q > 0, k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.

From equations (11) and (14), we obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1}[C(kI + A)e + q\text{sgn}(s)] \tag{15}$$

Theorem 1. *The master system (1) and the slave system (2) are globally and asymptotically synchronized for all initial conditions $x(0), y(0) \in \mathbb{R}^n$ by the feedback control law*

$$u(t) = -\eta(x, y) + Bv(t) \tag{16}$$

where $v(t)$ is defined by (15) and B is a column vector such that (A, B) is controllable. Also, the sliding mode gains k, q are positive.

Proof. First, we note that substituting (16) and (15) into the error dynamics (7), we obtain the closed-loop dynamics as

$$\dot{e} = Ae - B(CB)^{-1}[C(kI + A)e + q\text{sgn}(s)] \tag{17}$$

To prove that the error dynamics (17) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(e) = \frac{1}{2}\,s^2(e) \tag{18}$$

which is a positive definite function on $\mathbb{R}^n$.

Differentiating V along the trajectories of (17) or the equivalent dynamics (14), we obtain

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\text{sgn}(s) \tag{19}$$

which is a negative definite function on $\mathbb{R}^n$.

Thus, by Lyapunov stability theory [30], it is immediate that the error dynamics (17) is globally asymptotically stable for all initial conditions $e(0) \in \mathbb{R}^n$.

This completes the proof. ■

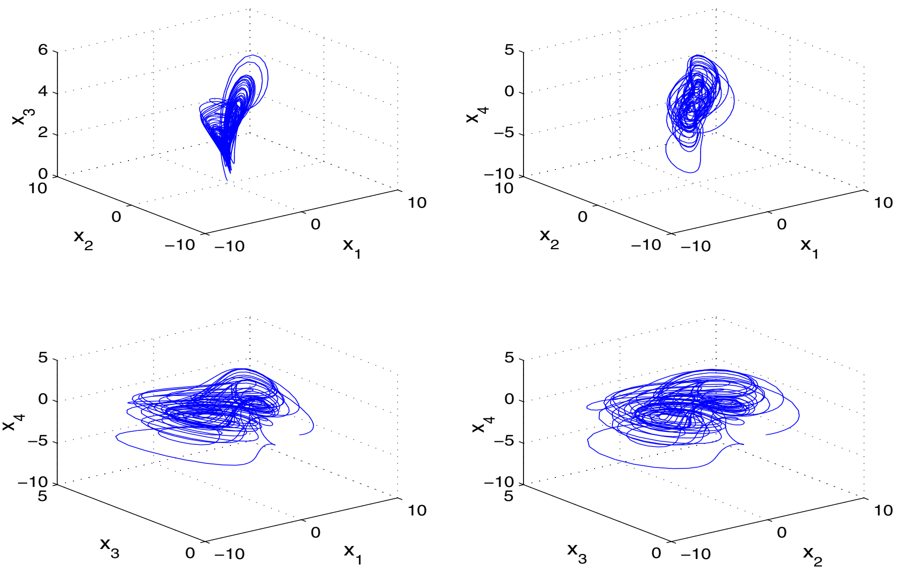

Fig. 1. State Portrait of the Hyperchaotic Chen System

3 Global Chaos Synchronization of Identical Hyperchaotic Chen Systems

3.1 Main Results

In this section, we apply the sliding mode control results obtained in Section 2 for the global chaos synchronization of identical hyperchaotic Chen systems ([29], 2010).

Thus, the master system is described by the hyperchaotic Chen dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= 4x_1 - 10x_1x_3 + cx_2 + 4x_4 \\ \dot{x}_3 &= x_2^2 - bx_3 \\ \dot{x}_4 &= -dx_1 \end{aligned} \tag{20}$$

where x_1, x_2, x_3, x_4 are the states of the system and a, b, c, d are constant, positive parameters of the system.

The slave system is also described by the controlled hyperchaotic Chen dynamics

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= 4y_1 - 10y_1y_3 + cy_2 + 4y_4 + u_2 \\ \dot{y}_3 &= y_2^2 - by_3 + u_3 \\ \dot{y}_4 &= -dy_1 + u_4 \end{aligned} \tag{21}$$

where y_1, y_2, y_3, y_4 are the states of the system and u_1, u_2, u_3, u_4 are the active controllers to be designed.

The systems (20) and (21) are hyperchaotic when

$$a = 35, \quad b = 3, \quad c = 21 \quad \text{and} \quad d = 2$$

The state portrait of the hyperchaotic Chen system (20) is illustrated in Figure 1.

The hybrid synchronization error e is defined by

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 - x_3 \\ e_4 &= y_4 + x_4 \end{aligned} \tag{22}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) - 2ax_2 + u_1 \\ \dot{e}_2 &= 4e_1 + ce_2 + 4e_4 + 8x_1 - 10(y_1y_3 + x_1x_3) + u_2 \\ \dot{e}_3 &= -be_3 + y_2^2 - x_2^2 + u_3 \\ \dot{e}_4 &= -de_1 - 2dx_1 + u_4 \end{aligned} \tag{23}$$

We can write the error dynamics (23) in the matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{24}$$

where the associated matrices are

$$A = \begin{bmatrix} -a & a & 0 & 0 \\ 4 & c & 0 & 4 \\ 0 & 0 & -b & 0 \\ -d & 0 & 0 & 0 \end{bmatrix}, \quad \eta(x, y) = \begin{bmatrix} -2ax_2 \\ 8x_1 - 10(y_1y_3 + x_1x_3) \\ y_2^2 - x_2^2 \\ -2dx_1 \end{bmatrix} \quad \text{and} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{25}$$

The sliding mode controller design is carried out as detailed in Section 2.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{26}$$

where B is chosen such that (A, B) is controllable. We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{27}$$

In the hyperchaotic case, the parameter values are

$$a = 35, \quad b = 3, \quad c = 21 \quad \text{and} \quad d = 2$$

The sliding mode variable is selected as

$$s = Ce = [\,-1 \quad -2 \quad\quad 0 \quad 1\,]\, e \tag{28}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as

$$k = 5 \text{ and } q = 0.1$$

We note that a large value of k can cause chattering and an appropriate value of q is chosen to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering.

From equation (15), we can obtain $v(t)$ as

$$v(t) = 10e_1 - 43.5e_2 - 1.5e_4 + 0.05\,\mathrm{sgn}(s) \tag{29}$$

Thus, the required sliding mode controller is obtained as

$$u(t) = -\eta(x, y) + Bv(t) \tag{30}$$

where $\eta(x, y)$, B and $v(t)$ are defined in equations (25), (27) and (29).

By Theorem 1, we obtain the following result.

Theorem 2. *The identical hyperchaotic Chen systems (20) and (21) are globally and asymptotically hybrid-synchronized for all initial conditions with the sliding mode controller u defined by (30).* ■

3.2 Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic Chen systems (20) and (21) with the sliding mode controller u given by (30) using MATLAB.

For the hyperchaotic Chen systems, the parameter values are taken as

$$a = 35, \;\; b = 3, \;\; c = 21 \text{ and } d = 2$$

The sliding mode gains are chosen as

$$k = 5 \text{ and } q = 0.1$$

The initial values of the master system (20) are taken as

$$x_1(0) = 16, \;\; x_2(0) = 19, \;\; x_3(0) = 21, \;\; x_4(0) = 11$$

and the initial values of the slave system (21) are taken as

$$y_1(0) = 8, \;\; y_2(0) = 26, \;\; y_3(0) = 36, \;\; y_4(0) = 24$$

Figure 2 depicts the hybrid synchronization of the hyperchaotic Chen systems (20) and (21).

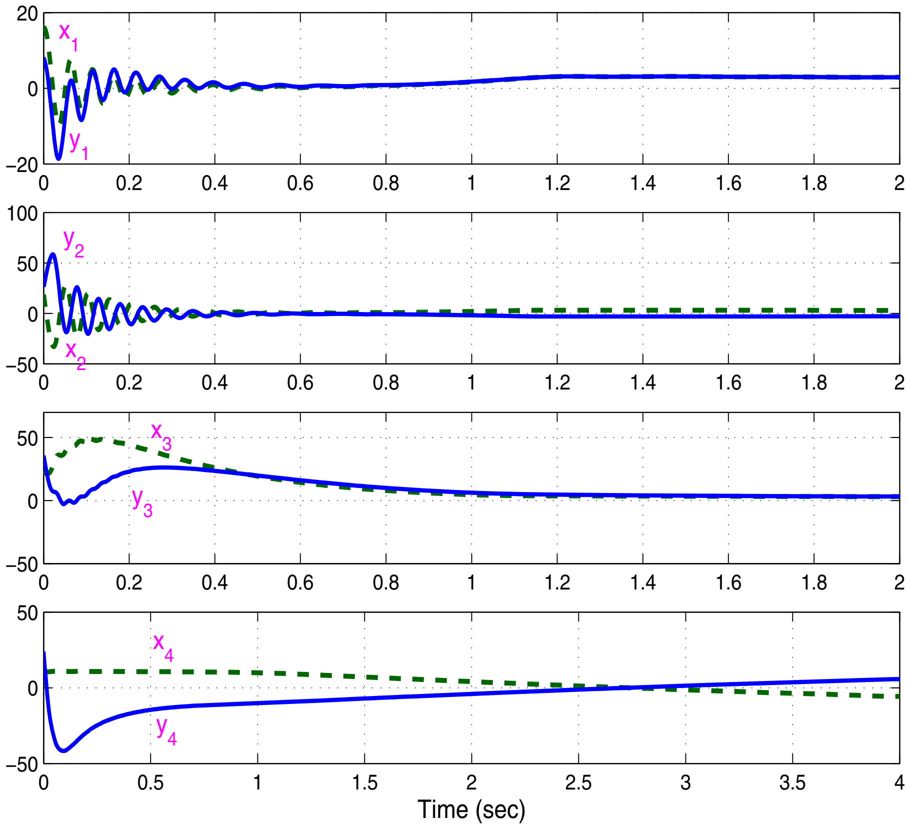

Fig. 2. Hybrid Synchronization of the Identical Hyperchaotic Chen Systems

4 Conclusions

Sliding control method (SMC) is an effective method in control engineering. In robust control systems, sliding mode control is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and external disturbances.In this paper, we have used sliding mode control (SMC) to achieve hybrid chaos synchronization for the identical hyperchaotic Chen systems (Jia, Dai and Hui, 2010). Our synchronization results for the identical hyperchaotic Chen systems have been established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the sliding mode control method is very effective and convenient to achieve hybrid chaos synchronization for identical hyperchaotic Chen systems. Numerical simulations are also presented to demonstrate the effectiveness of the synchronization results derived in this paper using sliding mode control.

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