Adaptive Controller and Synchronizer Design for the Qi-Chen Chaotic System

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Abstract. This paper investigates the design problem of adaptive controller and synchronizer for the Qi-Chen chaotic system (2005) with unknown parameters. First, adaptive control laws are derived to stabilize the Qi-Chen chaotic system to its unstable equilibrium at the origin. Then adaptive control laws are also derived to achieve global chaos synchronization of identical Qi-Chen chaotic systems with unknown parameters. The results derived for adaptive stabilization and synchronization for the Qi-Chen chaotic system are established using Lyapunov stability theory. Numerical simulations are presented to demonstrate the effectiveness of the adaptive control and synchronization schemes derived in this paper.

Keywords: Adaptive control, chaos, synchronization, Qi-Chen system.

1 Introduction

Chaotic systems are dynamical systems that possess some special features, such as being extremely sensitive to small variations of initial conditions, having bounded trajectories in the phase space, and so on. The chaos phenomenon was first observed in weather models by Lorenz ([1], 1963). This was followed by a discovery of a large number of chaos phenomena and chaos behaviour in physical, social, economical, biological and electrical systems [2].

The problem of controlling a chaotic system was introduced by Ott *et al.* ([3], 1990). The control of chaotic systems is basically to design state feedback control laws that stabilizes the chaotic systems around the unstable equilibrium points. Active control method is used when the system parameters are known and adaptive control method is used when some or all of the system parameters are unknown ([3]-[5]).

Chaos synchronization is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of chaos synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

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Since the pioneering work by Pecora and Carroll ([6], 1990), several approaches have been proposed for chaos synchronization such as the active control method [8], adaptive control method [9], sampled-data control method [10], backstepping method [11], sliding mode control method ([12]-[13]), etc.

This paper investigates the design of adaptive controller and synchronizer for the Qi-Chen chaotic system (Qi, Chen *et al.* [14], 2005). First, we devise adaptive control scheme using state feedback control for the Qi-Chen chaotic system about its unstable equilibrium at the origin. Then we devise adaptive synchronization scheme for identical Qi-Chen chaotic systems. The adaptive control and synchronization results derived in this paper are established using Lyapunov stability theory.

This paper has been organized as follows. In Section 2, we give a system description of the Qi-Chen chaotic system (2005). In Section 3, we derive results for the adaptive control of the Qi-Chen chaotic system with unknown parameters. In Section 4, we derive results for the adaptive synchronization of identical Qi-Chen chaotic systems with unknown parameters. In Section 5, we summarize the main results obtained in this paper.

2 System Description

The Qi-Chen chaotic system ([14], 2005) is described by the dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_2 x_3\\ \dot{x}_2 &= c x_1 - x_2 - x_1 x_3\\ \dot{x}_3 &= x_1 x_2 - b x_3\end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the state variables of the system and a, b, c are constant, positive parameters of the system.

The system (1) is *chaotic* when

$$a = 35, \ \ b = 8/3 \ \text{ and } \ c = 80. \tag{2}$$

Figure 1 describes the state orbits of the Qi-Chen system (1).

When the parameter values are taken as in (2), the Qi-Chen system (1) and the system linearization matrix at the equilibrium point $E_0 = (0, 0, 0, 0)$ is given by

$$A = \begin{bmatrix} -35 & 35 & 0 \\ 80 & -1 & 0 \\ 0 & 0 & -8/3 \end{bmatrix}$$

which has the eigenvalues

$$\lambda_1 = -73.5788, \ \ \lambda_2 = -2.6667 \ \text{ and } \ \lambda_3 = 37.5788$$

Since λ_3 is a positive eigenvalue of A, it is immediate from Lyapunov stability theory [15] that the system (1)is unstable at the equilibrium point $E_0 = (0, 0, 0)$.

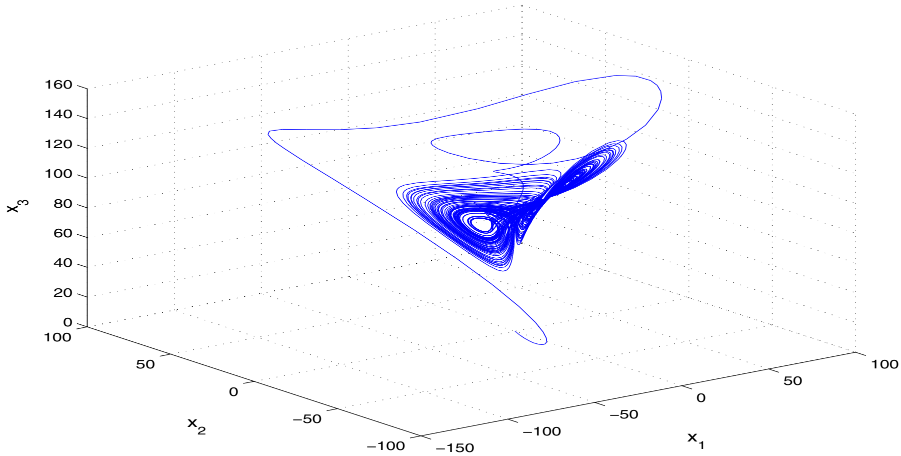

Fig. 1. State Orbits of the Qi-Chen System

3 Adaptive Controller Design of the Qi-Chen Chaotic System

3.1 Main Results

In this section, we discuss the design of adaptive controller for globally stabilizing the Qi-Chen system (2005), when the parameter values are unknown.

Thus, we consider the controlled Qi-Chen system described by the dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 + u_1 \\ \dot{x}_2 &= c x_1 - x_2 - x_1 x_3 + u_2 \\ \dot{x}_3 &= x_1 x_2 - b x_3 + u_3 \end{aligned} \tag{3}$$

where u_1, u_2, u_3 are feedback controllers to be designed using the states x_1, x_2, x_3 and estimates $\hat{a}, \hat{b}, \hat{c}$ of the unknown system parameters a, b, c of the system.

Next, we consider the following adaptive control functions

$$\begin{aligned} u_1 &= -\hat{a}(x_2 - x_1) - x_2 x_3 - k_1 x_1 \\ u_2 &= -\hat{c} x_1 + x_2 + x_1 x_3 - k_2 x_2 \\ u_3 &= -x_1 x_2 + b x_3 - k_3 x_3 \end{aligned} \tag{4}$$

where $\hat{a}, \hat{b}$ and $\hat{c}$ are estimates of the system parameters a, b and c, respectively, and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting the control law (4) into the plant equation (3), we obtain

$$\begin{aligned} \dot{x}_1 &= (a - \hat{a})(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= (c - \hat{c}) x_1 - k_2 x_2 \\ \dot{x}_3 &= -(b - \hat{b}) x_3 - k_3 x_3 \end{aligned} \tag{5}$$

We define the parameter estimation error as

$$e_a = a - \hat{a}, \;\; e_b = b - \hat{b} \;\text{ and }\; e_c = c - \hat{c} \tag{6}$$

Using (6), the state dynamics (5) can be simplified as

$$\begin{aligned} \dot{x}_1 &= e_a(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= e_c x_1 - k_2 x_2 \\ \dot{x}_3 &= -e_b x_3 - k_3 x_3 \end{aligned} \tag{7}$$

We use Lyapunov approach for the derivation of the update law for adjusting the parameter estimates $\hat{a}, \hat{b}$ and $\hat{c}$.

Consider the quadratic Lyapunov function defined by

$$V(x_1, x_2, x_3, e_a, e_b, e_c) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2\right) \tag{8}$$

which is a positive definite function on $\mathbb{R}^6$.

Note that

$$\dot{e}_a = -\dot{\hat{a}}, \;\; \dot{e}_b = -\dot{\hat{b}}, \;\; \dot{e}_c = -\dot{\hat{c}}. \tag{9}$$

Differentiating V along the trajectories of (5) and using (9), we obtain

$$\begin{aligned} \dot{V} = &-k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] \\ &+ e_b\left[-x_3^2 - \dot{\hat{b}}\right] + e_c\left[x_1 x_2 - \dot{\hat{c}}\right] \end{aligned} \tag{10}$$

In view of Eq. (10, the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= x_1(x_2 - x_1) + k_4 x_1 \\ \dot{\hat{b}} &= -x_3^2 + k_5 x_2 \\ \dot{\hat{c}} &= x_1 x_2 + k_6 x_3 \end{aligned} \tag{11}$$

where k_4, k_5 and k_6 are positive constants.

Substituting (11) into (10), we obtain

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 \tag{12}$$

which is a negative definite function on $\mathbb{R}^6$.

Thus, by Lyapunov stability theory [15], it follows that the plant dynamics (7) is globally exponentially stable and also that the parameter estimate errors e_a, e_b, e_c converge to zero exponentially with time.

Hence, we obtain the following result.

Theorem 1. *The Qi-Chen chaotic system (3) with unknown parameters is globally and exponentially stabilized by the adaptive control law (4), where the update law for the parameters is given by (11) and* $k_i, (i = 1, 2, \ldots, 6)$ *are positive constants.* □

3.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method with step-size $h = 10^{-6}$ is used to solve the Qi-Chen system (3) with the adaptive control law (4) and the parameter update law (11).

The parameters of the system (3) are selected as $a = 35, b = 8/3$ and $c = 80$.

We also take $k_i = 4$ for $i = 1, 2, \dots, 6$.

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 5, \quad \hat{b}(0) = 18, \quad \hat{c}(0) = 10$$

Suppose that we take the initial values of the states of (3) as

$$x_1(0) = 20, \quad x_2(0) = 34, \quad x_3(0) = 18$$

When the adaptive control law (4) and the parameter update law (11) are applied, the controlled Qi-Chen system (3) converges to the equilibrium $E_0 = (0, 0, 0)$ exponentially as shown in Figure 2 and the parameter estimates $\hat{a}, \hat{b}, \hat{c}$ converge to the system parameters a, b, c exponentially as shown in Figure 3.

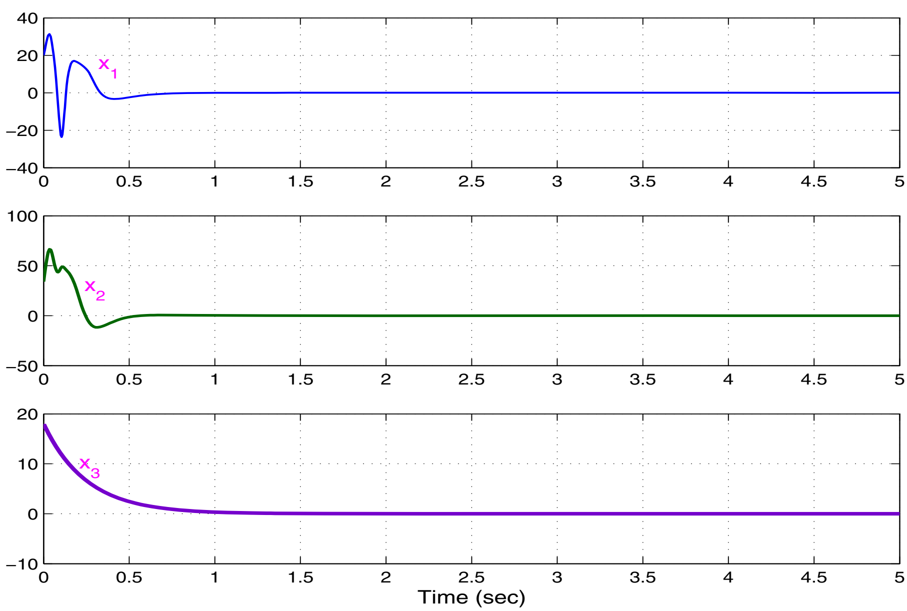

Fig. 2. Time Responses of the Controlled Qi-Chen System

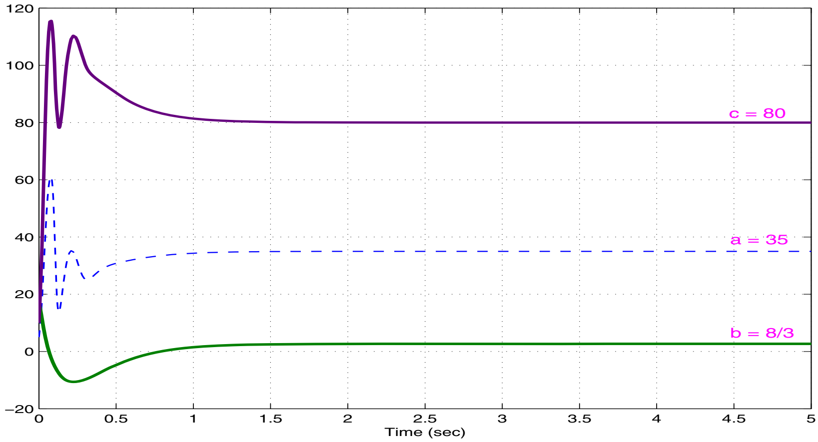

Fig. 3. Parameter Estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t)$

4 Adaptive Synchronizer Design of the Qi-Chen Chaotic System

4.1 Main Results

In this section, we discuss the design of adaptive synchronization of identical Qi-Chen systems (2005) with unknown parameters.

As the master system, we consider the Qi-Chen dynamics described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= c x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 &= x_1 x_2 - b x_3 \end{aligned} \tag{13}$$

where x_1, x_2, x_3 are the state variables and a, b, c are unknown system parameters.

As the slave system, we consider the controlled Qi-Chen dynamics described by

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + u_1 \\ \dot{y}_2 &= c y_1 - x_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= y_1 y_2 - b y_3 + u_3 \end{aligned} \tag{14}$$

where y_1, y_2, y_3 are the state variables and u_1, u_2, u_3 are the nonlinear controllers to be designed.

The synchronization error e is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3) \tag{15}$$

Then the error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + y_2 y_3 - x_2 x_3 + u_1 \\ \dot{e}_2 &= c e_1 - e_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 &= -b e_3 + y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{16}$$

We define the adaptive synchronizing law

$$\begin{aligned} u_1 &= -\hat{a}(e_2 - e_1) - y_2 y_3 + x_2 x_3 - k_1 e_1 \\ u_2 &= -\hat{c} e_1 + e_3 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 &= \hat{b} e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3 \end{aligned} \tag{17}$$

where $\hat{a}, \hat{b}$ and $\hat{c}$ are estimates of the system parameters a, b and c, respectively, and $k_i, (i = 1, 2, 3)$ are positive constants.

Substituting (17) into (16), we obtain the error dynamics as

$$\begin{aligned} \dot{e}_1 &= (a - \hat{a})(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= (c - \hat{c}) e_1 - k_2 e_2 \\ \dot{e}_3 &= -(b - \hat{b}) e_3 - k_3 e_3 \end{aligned} \tag{18}$$

We define the parameter estimation error as

$$e_a = a - \hat{a}, \;\; e_b = b - \hat{b} \;\text{ and }\; e_c = c - \hat{c} \tag{19}$$

Substituting (19) into (18), the error dynamics (18) can be simplified as

$$\begin{aligned} \dot{e}_1 &= e_a(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= e_c e_1 - k_2 e_2 \\ \dot{e}_3 &= -e_b e_3 - k_3 e_3 \end{aligned} \tag{20}$$

We use Lyapunov approach for the derivation of the update law for adjusting the parameter estimates $\hat{a}, \hat{b}$ and $\hat{c}$.

Consider the quadratic Lyapunov function defined by

$$V(e_1, e_2, e_3, e_a, e_b, e_c) = \frac{1}{2} \left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 \right) \tag{21}$$

which is a positive definite function on $\mathbb{R}^6$.

Note that

$$\dot{e}_a = -\dot{\hat{a}}, \;\; \dot{e}_b = -\dot{\hat{b}}, \;\; \dot{e}_c = -\dot{\hat{c}}. \tag{22}$$

Differentiating V along the trajectories of (20) and using (22), we obtain

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a \left[e_1(e_2 - e_1) - \dot{\hat{a}} \right] \\ &+ e_b \left[-e_3^2 - \dot{\hat{b}} \right] + e_c \left[e_1 e_2 - \dot{\hat{c}} \right] \end{aligned} \tag{23}$$

In view of Eq. (23), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= e_1(e_2 - e_1) + k_4 e_1 \\ \dot{\hat{b}} &= -e_3^2 + k_5 e_2 \\ \dot{\hat{c}} &= e_1 e_2 + k_6 e_3 \end{aligned} \tag{24}$$

where k_4, k_5 and k_6 are positive constants.

Substituting (24) into (23), we obtain

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 e_a^2 - k_5 e_b^2 - k_6 e_c^2 \tag{25}$$

which is a negative definite function on $\mathbb{R}^6$.

Thus, by Lyapunov stability theory [15], it follows that the error dynamics (20) is globally exponentially stable and also that the parameter estimate errors e_a, e_b, e_c converge to zero exponentially with time.

Hence, we obtain the following result.

Theorem 2. *The identical Qi-Chen chaotic systems (13) and (14) with unknown parameters are globally and exponentially synchronized by the adaptive control law (17), where the update law for the parameters is given by (24) and* $k_i, (i = 1, 2, \ldots, 6)$ *are positive constants.* □

4.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method with step-size $h = 10^{-6}$ is used to solve the Qi-Chen systems (13) and (14) with the adaptive control law (17) and the parameter update law (24).

The parameters of the system (3) are selected as $a = 35, b = 8/3$ and $c = 80$.

We also take $k_i = 4$ for $i = 1, 2, \ldots, 6$.

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 12, \ \ \hat{b}(0) = 6, \ \ \hat{c}(0) = 20$$

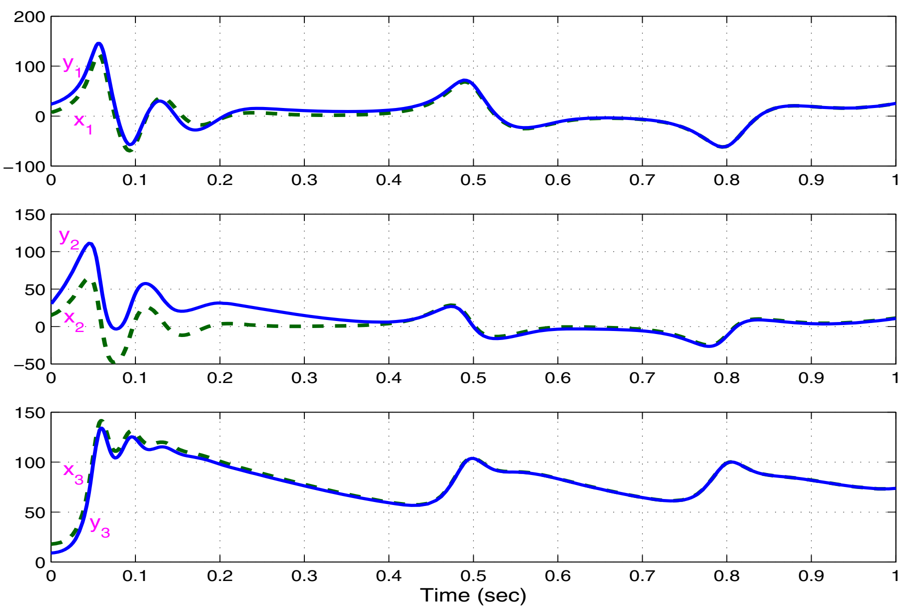

Fig. 4. Time Responses of the Controlled Qi-Chen System

Suppose that the initial values of the master system (13) are taken as

$$x_1(0) = 7, \;\; x_2(0) = 15, \;\; x_3(0) = 18$$

Suppose that the initial values of the slave system (14) are taken as

$$y_1(0) = 24, \;\; y_2(0) = 30, \;\; y_3(0) = 9$$

The identical Qi-Chen systems (13) and (14) are synchronized exponentially as shown in Figure 4 and the parameter estimates $\hat{a}, \hat{b}, \hat{c}$ converge to the system parameters a, b, c exponentially as shown in Figure 5.

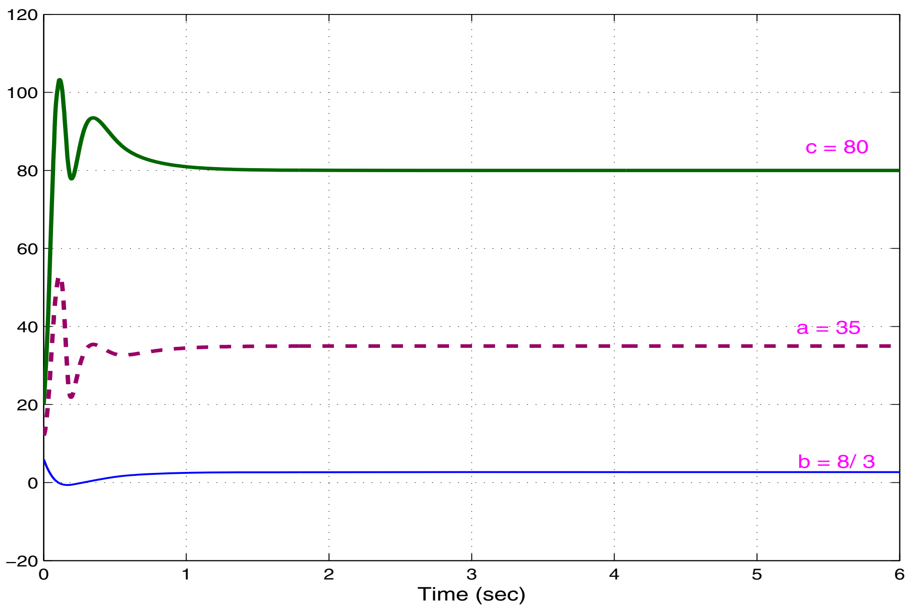

Fig. 5. Parameter Estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t)$

5 Conclusions

In this paper, we derived results for the adaptive controller and synchronizer for the Qi-chen chaotic system (Qi, Chen *et al.* 2005) with unknown parameters. First, we designed adaptive control law to stabilize the Qi-Chen system to its unstable equilibrium point at the origin based on the Lyapunov stability theory. Then we designed adaptive synchronizer for the global chaos synchronization of identical Qi-Chen systems with unknown parameters. Our synchronization results were established using Lyapunov stability theory. Numerical simulations are presented to illustrate the effectiveness of the proposed adaptive controller and synchronizer schemes for the Qi-Chen chaotic system (2005).

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