Hybrid Synchronization of Arneodo and Rössler Chaotic Systems by Active Nonlinear Control

Sundarapandian Vaidyanathan and Suresh Rasappan

Research and Development Centre, Vel Tech Dr. RR & Dr. SR Technical University
Avadi-Alamathi Road, Avadi, Chennai-600 062, India
sundarvtu@gmail.com
http://www.vel-tech.org/

Abstract. This paper investigates the hybrid chaos synchronization of identical Arneodo systems (1981), identical Rössler systems (1976) and non-identical Arneodo and Rössler systems. In hybrid synchronization of chaotic systems, one part of the systems is synchronized and the other part is anti-synchronized so that complete synchronization (CS) and anti-synchronization (AS) co-exist in the systems. The co-existence of CS and AS is very useful in secure communication and chaotic encryption schemes. Active nonlinear control is the method used for the hybrid synchronization of the chaotic systems addressed in this paper. Since the Lyapunov exponents are not required for these calculations, the active control method is effective and convenient to achieve hybrid synchronization of the two chaotic systems. Numerical simulations are shown to verify the results.

Keywords: Hybrid synchronization, chaos, Arneodo system, Rössler system, active nonlinear control.

1 Introduction

Chaos is very interesting nonlinear phenomenon, exhibiting sensitive dependence on initial conditions. Synchronization of chaos is an important research problem, which has been attracting considerable interest in the chaos literature. Chaos synchronization has been widely explored in a variety of fields including physical systems [1], chemical systems [2], ecological systems [3], secure communications ([4]-[5]), etc.

Since Pecora and Carroll published a seminal paper ([6], 1990) for synchronizing two identical chaotic systems with different conditions, many chaos synchronization methods have been developed extensively over the past few decades ([6]-[20]). Some important methods for the chaos synchronization are the PC method [6], sampled-data feedback synchronization method [7], OGY method [8], time-delay feedback method [9], backstepping method [10], adaptive design method [11], sliding control method [12], etc.

So far, many types of synchronization phenomenon have been presented such as complete synchronization [6], phase synchronization ([3],[13]), generalized synchronization ([5], [14]), anti-synchronization ([15], [16]), projective synchronization [17], generalized projective synchronization ([18], [19]) etc.

Complete synchronization (CS) is characterized by the equality of state variables evolving in time, while anti-synchronization (AS) is characterized by the disappearance

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of the sum of relevant state variables evolving in time. Projective synchronization (PS) is characterized by the fact the master and slave systems could be synchronized up to a scaling factor, whereas in generalized projective synchronization (GPS), the responses of the synchronized dynamical states synchronize up to a constant scaling matrix α. It is easy to see that the complete synchronization and anti-synchronization are the special cases of the generalized projective synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

In hybrid synchronization of chaotic systems [19], one part of the systems is synchronized and the other part is anti-synchronized so that complete synchronization (CS) and anti-synchronization (AS) co-exist in the systems. The co-existence of CS and AS is very useful in secure communication and chaotic encryption schemes.

This paper is organized as follows. In Section 2, we discuss the hybrid synchronization between identical Arneodo systems ([21], 1981). In Section 3, we discuss the hybrid synchronization between identical Rössler systems ([22], 1976).In Section 4, we discuss the hybrid synchronization between non-identical Arneodo and Rössler systems. In Section 5, we conclude with the main results obtained in this paper.

2 Hybrid Synchronization of Identical Arneodo Systems

In this section, we consider the hybrid synchronization of identical Arneodo systems ([21], 1981).

Thus, we consider the *master* system as the Arneodo dynamics described by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= mx_1 - sx_2 - x_3 - x_1^2 \end{aligned} \tag{1}$$

where $x_i (i = 1, 2, 3)$ are the *state* variables and s, m are positive constants.

The Arneodo system (1) is chaotic when $s = 3.8$ and $m = 7.5$.

The state orbits of the chaotic Arneodo system are shown in Figure 1.

We consider the controlled Arneodo system as the *slave* system, which is described by the dynamics

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1 \\ \dot{y}_2 &= y_3 + u_2 \\ \dot{y}_3 &= my_1 - sy_2 - y_3 - y_1^2 + u_3 \end{aligned} \tag{2}$$

where $y_i (i = 1, 2, 3)$ are the *state* variables and $u_i (i = 1, 2, 3)$ are the active controls.

For the hybrid synchronization of the identical Arneodo systems (1) and (2), the *errors* are defined as

$$e_1 = y_1 - x_1, \;\; e_2 = y_2 + x_2 \;\text{ and }\; e_3 = y_3 - x_3 \tag{3}$$

A simple calculation yields the error dynamics as

$$\begin{aligned} \dot{e}_1 &= e_2 - 2x_2 + u_1 \\ \dot{e}_2 &= e_3 + 2x_3 + u_2 \\ \dot{e}_3 &= me_1 - se_2 - e_3 + 2sx_2 - y_1^2 + x_1^2 + u_3 \end{aligned} \tag{4}$$

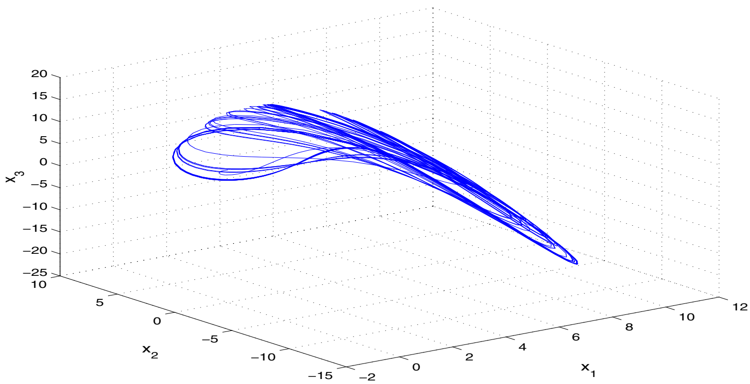

Fig. 1. State Orbits of the Arneodo System (1)

We consider the active nonlinear controller defined by

$$\begin{aligned} u_1 &= -e_2 + 2x_2 - k_1 e_1 \\ u_2 &= -e_3 - 2x_3 - k_2 e_2 \\ u_3 &= -m e_1 + s e_2 - 2s x_2 + y_1^2 - x_1^2 \end{aligned} \tag{5}$$

where k_1 and k_2 are positive constants.

Substitution of (5) into (4) yields the linear error dynamics

$$\dot{e}_1 = -k_1 e_1, \;\; \dot{e}_2 = -k_2 e_2, \;\; \dot{e}_3 = -e_3 \tag{6}$$

We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2}\, e^T e = \frac{1}{2}\, \left(e_1^2 + e_2^2 + e_3^2\right) \tag{7}$$

Differentiating (7) along the trajectories of the system (6), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - e_3^2 \tag{8}$$

which is a negative definite function on $\mathbb{R}^3$, since k_1, k_2 are positive constants.

Thus, by Lyapunov stability theory [23], the error dynamics (6) is globally exponentially stable. Hence, we obtain the following result.

Theorem 1. *The identical Arneodo systems (1) and (2) are globally and exponentially hybrid synchronized with the active nonlinear controller (5).* ■

Numerical Simulations

For the numerical simulations, the fourth order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the two systems of differential equations (1) and (2) with the active controller (5).

The parameters of the identical Arneodo systems (1) and (2) are selected as $s = 3.8$ and $m = 7.5$ so that the systems (1) and (2) exhibit chaotic behaviour. Also, we take $k_1 = 2, k_2 = 2$.

The initial values for the master system (1) are taken as

$$x_1(0) = 6, \; x_2(0) = 8, \; x_3(0) = 2$$

and the initial values for the slave system (2) are taken as

$$y_1(0) = 9, \; y_2(0) = 4, \; y_3(0) = 3$$

Figure 2 shows the hybrid synchronization of the Arneodo systems (1) and (2).

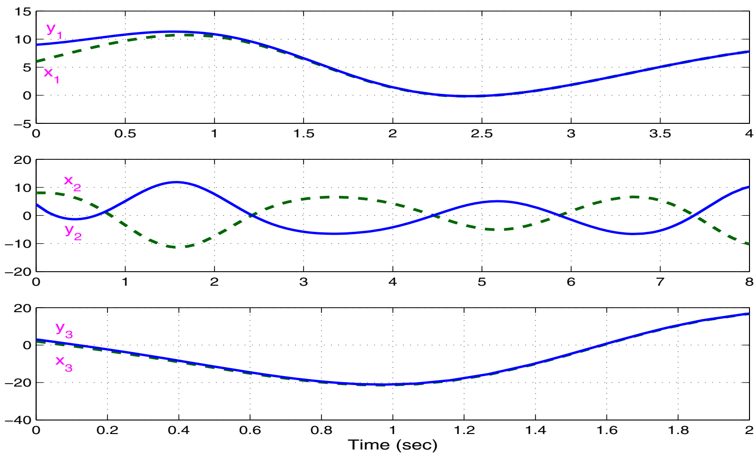

Fig. 2. Hybrid Synchronization of Identical Arneodo Systems

3 Hybrid Synchronization of Identical Rössler Systems

In this section, we discuss the hybrid synchronization of identical Rössler systems. Thus, we consider the Rössler system [22] as the *master* system, which is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3 \\ \dot{x}_2 &= x_1 + ax_2 \\ \dot{x}_3 &= b + (x_1 - c)x_3 \end{aligned} \tag{9}$$

where $x_i (i = 1, 2, 3)$ are the *state* variables and a, b, c are positive constants.

When $a = 0.2, b = 0.2$ and $c = 5.7$, the Rössler system (9) is chaotic. The state orbits of the Rössler system are shown in Figure 3.

Next, we consider the controlled Rössler dynamics as the *slave* system, which is described by

$$\begin{aligned} \dot{y}_1 &= -y_2 - y_3 + u_1 \\ \dot{y}_2 &= y_1 + ay_2 + u_2 \\ \dot{y}_3 &= b + (y_1 - c)y_3 + u_3 \end{aligned} \tag{10}$$

where $y_i (i = 1, 2, 3)$ are the *state* variables and $u_i (i = 1, 2, 3)$ are the active controls.

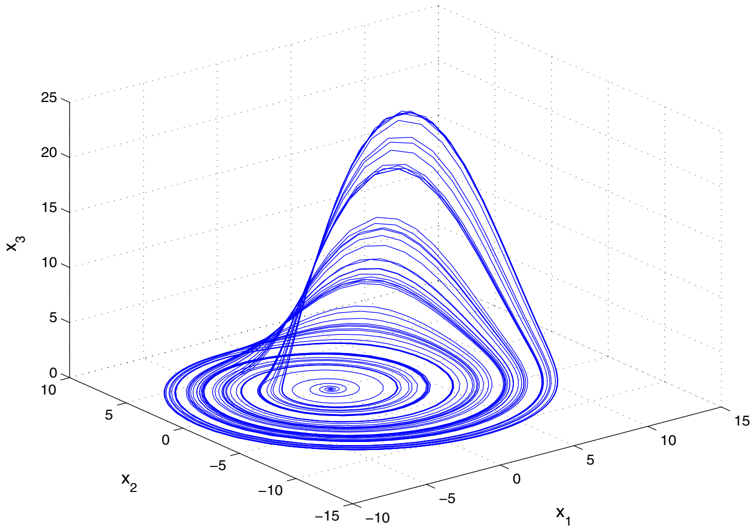

Fig. 3. State Orbits of the Rössler System (9)

For the hybrid synchronization of the identical Rössler systems (9) and (10), the *errors* are defined as

$$e_1 = y_1 - x_1, \;\; e_2 = y_2 + x_2 \;\text{ and }\; e_3 = y_3 - x_3 \tag{11}$$

A simple calculation yields the error dynamics as

$$\begin{aligned} \dot{e}_1 &= -e_2 - e_3 + 2x_2 + u_1 \\ \dot{e}_2 &= e_1 + ae_2 + 2x_1 + u_2 \\ \dot{e}_3 &= -ce_3 + y_1 y_3 - x_1 x_3 + u_3 \end{aligned} \tag{12}$$

We consider the active nonlinear controller defined by

$$\begin{aligned} u_1 &= e_2 + e_3 - 2x_2 - k_1 e_1 \\ u_2 &= -e_1 - ae_2 - 2x_1 - k_2 e_2 \\ u_3 &= -y_1 y_3 + x_1 x_3 \end{aligned} \tag{13}$$

where k_1 and k_2 are positive constants.

Substitution of (13) into (12) yields the linear error dynamics

$$\dot{e}_1 = -k_1 e_1, \;\; \dot{e}_2 = -k_2 e_2, \;\; \dot{e}_3 = -ce_3 \tag{14}$$

We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2}\, e^T e = \frac{1}{2}\, \left(e_1^2 + e_2^2 + e_3^2\right) \tag{15}$$

Differentiating (15) along the trajectories of the system (14), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - c e_3^2 \tag{16}$$

which is a negative definite function on $\mathbb{R}^3$, since k_1, k_2, c are positive constants.

Thus, by Lyapunov stability theory [23], the error dynamics (14) is globally exponentially stable. Hence, we obtain the following result.

Theorem 2. *The identical Rössler systems (9) and (10) are globally and exponentially hybrid synchronized with the active nonlinear controller (13).* ■

Numerical Simulations

For the numerical simulations, the fourth order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the two systems of differential equations (9) and (10) with the active controller (13).

The parameters of the identical Rössler systems (9) and (10) are selected as $a = 0.2, b = 0.2$ and $c = 5.7$ so that the systems (9) and (10) exhibit chaotic behaviour. Also, we take $k_1 = 2, k_2 = 2$.

The initial values for the master system (9) are taken as

$$x_1(0) = 6,\ x_2(0) = 17,\ x_3(0) = 12$$

and the initial values for the slave system (10) are taken as

$$y_1(0) = 1,\ y_2(0) = 10,\ y_3(0) = 2$$

Figure 4 shows the hybrid synchronization of the Rössler systems (9) and (10).

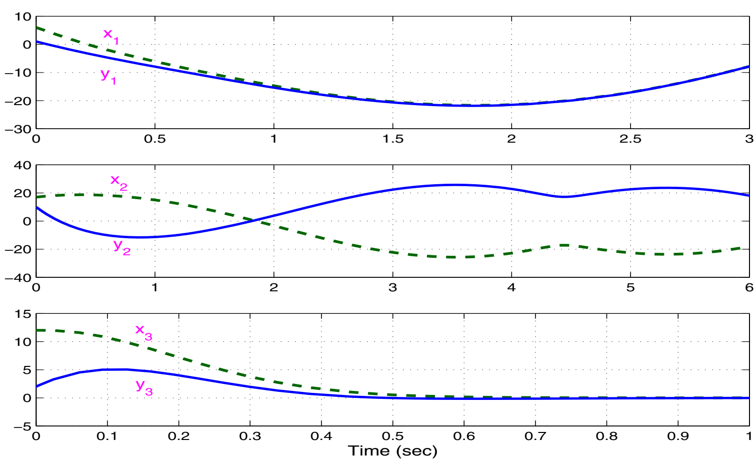

Fig. 4. Hybrid Synchronization of Identical Rössler Systems

4 Hybrid Synchronization of Arneodo and Rössler Systems

In this section, we discuss the hybrid synchronization of non-identical chaotic systems, *viz.* Arneodo and Rössler chaotic systems. Thus, we consider the Arneodo system [21] as the *master* system, which is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= mx_1 - sx_2 - x_3 - x_1^2 \end{aligned} \tag{17}$$

where x_1, x_2, x_3 are the *state* variables and s, m are positive constants.

Next, we consider the controlled Rössler dynamics [22] as the *slave* system, which is described by

$$\begin{aligned} \dot{y}_1 &= -y_2 - y_3 + u_1 \\ \dot{y}_2 &= y_1 + ay_2 + u_2 \\ \dot{y}_3 &= b + (y_1 - c)y_3 + u_3 \end{aligned} \tag{18}$$

where y_1, y_2, y_3 are the *state* variables, a, b, c are positive constants and u_1, u_2, u_3 are the active controls.

For the hybrid synchronization of the non-identical systems (17) and (18), the *errors* are defined as

$$e_1 = y_1 - x_1, \;\; e_2 = y_2 + x_2 \;\text{ and }\; e_3 = y_3 - x_3 \tag{19}$$

A simple calculation yields the error dynamics as

$$\begin{aligned} \dot{e}_1 &= -e_2 - y_3 + u_1 \\ \dot{e}_2 &= e_1 + x_1 + x_3 + ay_2 + u_2 \\ \dot{e}_3 &= b - ce_3 - mx_1 + sx_2 + (1 - c)x_3 + x_1^2 + y_1 y_3 + u_3 \end{aligned} \tag{20}$$

We consider the active nonlinear controller defined by

$$\begin{aligned} u_1 &= e_2 + y_3 - k_1 e_1 \\ u_2 &= -e_1 - x_1 - x_3 - ay_2 - k_2 e_2 \\ u_3 &= -b + mx_1 - sx_2 - (1 - c)x_3 - x_1^2 - y_1 y_3 \end{aligned} \tag{21}$$

where k_1 and k_2 are positive constants.

Substitution of (21) into (20) yields the linear error dynamics

$$\dot{e}_1 = -k_1 e_1, \;\; \dot{e}_2 = -k_2 e_2, \;\; \dot{e}_3 = -ce_3 \tag{22}$$

We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2}\, e^T e = \frac{1}{2}\, \left(e_1^2 + e_2^2 + e_3^2\right) \tag{23}$$

Differentiating (23) along the trajectories of the system (22), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - ce_3^2 \tag{24}$$

which is a negative definite function on $\mathbb{R}^3$, since k_1, k_2, c are positive constants.

Thus, by Lyapunov stability theory [23], the error dynamics (14) is globally exponentially stable. Hence, we obtain the following result.

Theorem 3. *The non-identical Arneodo system (17) and Rössler system (18) are globally and exponentially hybrid synchronized with the active nonlinear controller (21).* ■

Numerical Simulations

For the numerical simulations, the fourth order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the two systems of differential equations (17) and (18) with the active controller (21).

The parameters of the Arneodo and Rössler systems are selected so that they are chaotic, *viz.*

$$s = 3.8, \;\; m = 7.5, \;\; a = 0.2, \;\; b = 0.2, \;\; c = 5.7$$

The initial values for the master system (17) are taken as

$$x_1(0) = 2, \; x_2(0) = 8, \; x_3(0) = 5$$

and the initial values for the slave system (18) are taken as

$$y_1(0) = 16, \; y_2(0) = 3, \; y_3(0) = 12$$

Figure 5 shows the hybrid synchronization of the non-identical Arneodo system (17) and Rössler system(18).

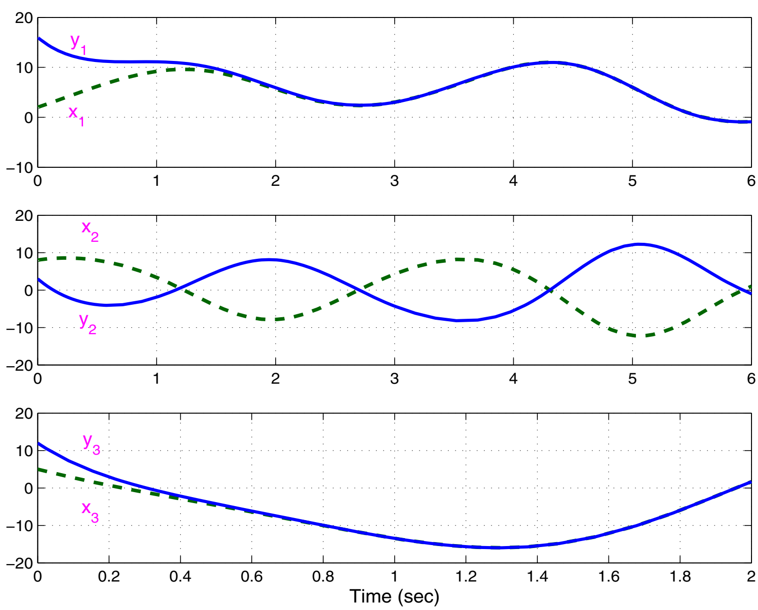

Fig. 5. Hybrid Synchronization of Arneodo and Rössler Systems

5 Conclusions

In this paper, we have used active nonlinear control method so as to achieve hybrid chaos synchronization of the following chaotic systems:

(A) Identical Arneodo chaotic systems (1981)
(B) Identical Rössler chaotic systems (1976)
(C) Non-identical Arneodo and Rössler chaotic systems

Numerical simulations are also shown to verify the proposed active nonlinear controllers to achieve hybrid synchronization of the chaotic systems addressed in this paper. Since Lyapunov exponents are not required for the calculations, the proposed nonlinear control method is effective and convenient to achieve hybrid synchronization of the identical and non-identical Arneodo and Rössler chaotic systems. Numerical simulations are given to illustrate the synchronization results.

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