

Adaptive MMSE Equalizer through LMS Algorithm Based CMA Channel Equalization

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Abstract. The adaptive algorithm has been widely used in the digital signal processing like channel estimation, channel equalization, echo cancellation, and so on. One of the most important adaptive algorithms is the LMS algorithm. We present in this paper an multiple objective optimization approach to fast blind channel equalization. By investigating first the performance (mean-square error) of the standard fractionally spaced CMA (constant modulus algorithm) equalizer in the presence of noise, we show that CMA local minima exist near the minimum mean-square error (MMSE) equalizers. Consequently, CMA may converge to a local minimum corresponding to a poorly designed MMSE receiver with considerably large mean-square error. The step size in the LMS algorithm decides both the convergence speed and the residual error level, the highest speed of convergence and residual error level.

Keywords: CMA, LMS Algorithm, MMSE.

1 Introduction

Blind equalization has the potential to improve the efficiency of communication systems by eliminating training signals. Difficulties of its application in wireless communications, however, are due largely to the characteristics of the propagation media - multipath delays and fast fading. The challenge is achieving blind equalization using only a limited amount of data. A widely tested algorithm is the constant modulus algorithm (CMA). In the absence of noise, under the condition of the channel invertibility, the CMA converges globally for symbol-rate IIR equalizers and fractionally spaced FIR equalizers . It is shown in [9] that CMA is less affected by the ill-conditioning of the channel. However, Z. Ding *et. al*, showed that CMA may converge to some local minimum symbol rate for FIR equalizer. In the presence of noise, the analysis of convergence of CMA is difficult and little conclusive results are available. Another drawback of CMA is that its convergence rate may not be sufficient for fast fading channels. Another approach to the blind equalization is based on the blind channel estimation. Some of the recent eigen structure-based channel estimations require a relatively smaller data size comparing with higher-order statistical methods.

However the asymptotic performance of these eigen structure-based schemes is limited by the condition of the channel [12, 13]. Specifically, the asymptotic normalized mean square error (ANMSE) is lower bounded by the condition number of the channel matrix. Unfortunately, frequency selective fading channels with long multipath delays often result in ill-conditioned channel matrices. The key idea of this paper is to combine the approach based on minimizing the constant modulus cost and that based on matching the second-order cyclostationary statistics. The main feature of the proposed approach is the improved convergence property over the standard CMA equalization and the improved robustness for ill-conditioned channels.

2 Blind Channel Equalization and Types

The field of blind channel equalization has been existence for a little over twenty years. Research during this time has centered on developing new algorithms and formulating a theoretical justification for these algorithms. Blind channel equalization is also known as a self recovering equalization.. The objective of blind equalization is to recover the unknown input sequence to the unknown channel based solely on the probabilistic and statistical properties of the input sequence. The receiver synchronizes with the received signal and adjusts to equalizer without any training sequences. The term blind is used in this equalizer because it performs the equalization on the data without a reference signal. Instead, the blind equalizer relies on knowledge of the signal structure and its statistic to perform the equalization.

1. Blind signal is the unknown signal which would be identified in output signal with accommodated noise signal at receiver. 2. Channel equalization uses the idea & knowledge of training sequences for channel estimation where as Blind channel equalization doesn't utilizes the characteristics of training sequences for frequency and impulse response analysis of channel. 3. Blind Channel Equalization differs from channel equalization and without knowing the channel characteristics like transfer function & SNR, it efficiently estimate the channel and reduces the Inter Symbol Interferences (ISI) by blind signal separation and also eliminating noise at the receiver.

3 CMA-(Constant Modulus Algorithm)

In digital communication, equalizer was designed to compensate the channel distortions, through a process known as equalization. Equalization are categories in to

1) Trained equalization, 2) Blind (self-recovering) Equalization

Blind equalization finds important application in data communication system. In data communications, digital signals are generated and transmitted by the sender through an analog channel to the receiver. Linear channel distortion as a result of limited channel bandwidth, multipath and fading is often the most serious distortion in digital communication system. Blind equalization improves system bandwidth efficient by avoiding the use of training sequence. The linear channel distortion, known as the Inter-symbol interference (ISI), can severely corrupt the transmitted signal and make it difficult for the receiver to directly recover the transmitted data. Channel equalization and identification has proven to be an effective means to compensate the linear distortion by removing much of the ISI.

Channel Equalization

A typical communication system design involves first passing the signal to be transmitted through a whitening filter to reduce redundancy or correlation and then transmitting the resultant whitened signal. At the receiver, the recorded signal is passed through the inverse whitening filter and the original signal is thus restored. However, the channel will affect the transmitted signal because of a) Channel noise b) Channel dispersion leading to inter symbol interference. It is necessary to pass the received signal through a so called equalizing filter to undo the dispersion effect as shown in figure 2 below. Equalization compensates for Inter symbol Interference (ISI) created by multi path within time dispersive Channel message signal whitening signal receiver.

Blind Channel Equalization

The field of blind channel equalization has been existence for a little over twenty years. Research during this time has centered on developing new algorithms and formulating a theoretical justification for these algorithms. Blind channel equalization is also known as a self-recovering equalization. The objective of blind equalization is to recover the unknown input sequence to the unknown channel based solely on the probabilistic and statistical properties of the input sequence. The receivers synchronize to the received signal and adjust the equalizer without the training sequence. The term blind is used in this equalizer because it performs the equalization on the data without a reference signal. Instead, the blind equalizer relies on knowledge of the signal structure and its statistic to perform the equalization. A natural question from for direct adaptive equalization with training is, "How can we adapt our filter F , without the use of a training signal?". Figure 2 shows such a system. There has been extensive research on this subject for single user applications as well as multi-user applications. The Constant Modulus Algorithm is one such algorithm employed for the blind adaptation problem.

4 LMS Algorithm

Usually, the adaptive algorithm consists of a transfer filter for processing the input single and an algorithm unit for update the transfer filter's coefficients. $x(n)$ is the input signal; $w(n) = [w_0, w_1, w_2, \dots, w_l]$ is the vector of the transfer filter's coefficients; $d(n)$ is the desired output of the transfer filter; $y(n)$ is the output of the transfer filter; $e(n)$ is the error value, and it can be written as:

$$e(n) = d(n) - y(n) \quad (1)$$

The Adaptive algorithm unit represents some algorithm to update the coefficients of the transfer filter. For LMS algorithm, the method to update the coefficients of the transfer filter is given as follows:

$$w(n) = w(n+1) + \mu^* x(n) * e(n) \quad (2)$$

μ , is the step of LMS algorithm.

5 Adaptive MMSE Equalizer

The Sampled signal after MMSE Equalizer can be expressed in matrix form as

$$\hat{s}(i) = w^H y(i) \tag{3}$$

Where $y(i) = H^T(i)s(i) + n(i)$, 4

is the length of the MMSE Equalizer: $w = [w_1, w_2, w_3, w_4, w_5, \dots, w_M]^T$ is the equalizer coefficients vector; Then the error signal $e(i)$ is given

$$e(i) = d(i) - \hat{s}(i) \tag{5}$$

where $d(i)$ is the desired response. For MMSE equalizer, $d(i) = s(i + D)$, D is a time delay parameter which is $L + 1$ usually. The MMSE criterion is used to derive the optimal equalizer coefficients vector w :

$$w = \underset{w}{\text{minimize}} E\{|e|^2\} \tag{6}$$

We make the assumption that signal $s(i)$ and noise $n(i)$ are independent identity distribution stochastic

Variable and uncorrelated each other, then the equalizer coefficients vector w can be expressed as[2]:

$$w = (H^H H + \frac{1}{SNR} I)^{-1} H^H \delta_D \tag{7}$$

Where $\delta_D = [0, \dots, 1_D; 0, \dots, 0]_{1 \times (L+M-1)}^T$, $SNR = \frac{\sigma_s^2}{\sigma_n^2}$ denotes the signal noise ratio I is $M \times M$ identity matrix.

To reduce the complexity caused by matrix inversion of ideal MMSE equalizer, we propose an adaptive MMSE equalizer algorithm. In code-multiplexed pilot CDMA systems, conventional adaptive equalizer is difficult to implement for lack of reference signal. In this paper, the steepest descent method [4] is used to derive adaptive equalizer algorithm in code-multiplexed pilot CDMA systems.

According to Eqn.3 and Eqn.5, the mean square error (MSE) J can be expressed as

$$J(w) = E[e(i)e(i)^*] = \sigma_s^2 - w^H p - p^H w + w^H R w \tag{8}$$

where autocorrelation matrix $R = E[y(i)y^H(i)]$; cross-correlation vector $p = E[y(i)d^*(i)]$, σ_s^2 denotes the signal power; $(.)^*$ represents conjugate operation. Because the wireless channel is time-varying, the equalizer coefficients vector w must be updated real time. Conventional adaptive algorithm requires reference signal $d(i)$, while in the downlink of code-multiplexed pilot CDMA systems, $d(i)$ is

difficult to distill. To resolve this problem, the steepest decent method is used. From Eqn.8, the gradient vector is

$$\frac{\partial J(w)}{w} = -2p + 2Rw \quad (9)$$

then the equalizer coefficients updating equation is

$$w(i+1) = w(i) + 2\mu[p - Rw(i)] \quad (10)$$

where parameter μ is a positive real-valued constant which controls the size of the incremental correction applied to the equalizer coefficients vector.

For the autocorrelation matrix:

$$\begin{aligned} R &= E[y(i)y^H(i)] \\ R &= E[s(i)s^H(i)]\{H^H(i)H(i)\}^T + E[n(i)n^H(i)] \\ R &= \sigma_s^2 \{H^H(i)H(i)\}^T + \sigma_n^2 I \end{aligned} \quad (11)$$

the cross-correlation vector

$$\begin{aligned} p &= E[y(i)d^*(i)] = E[(H^T(i)s(i) + n(s))s^*(i-D)] \\ p &= \sigma_s^2 H^T(i)\delta_D \end{aligned} \quad (12)$$

From Eqn.7,8,9, we can obtain the time recursive equation of MMSE equalizer by:

$$w(i+1) = w(i) + 2\mu\sigma_s^2 [H^T(i)\delta_D - \{H^H(i)H(i)\}^T + \frac{1}{SNR} I w(i)] \quad (13)$$

As can be seen from Eqn.13, the updating process avoids the matrix inversion operation. On the other hand, the updating process abstains the requirement to store the autocorrelation matrix $\mathbf{R}(i)$ and only the equalizer coefficients vector of last time is needed. From Eqn.13 we know, the channel convolution matrix $\mathbf{H}(i)$ is required to update the equalizer coefficients vector.

For CMA, channel response can be estimated through code-multiplexed pilot. In this paper, the low complexity sliding-window method is used to estimate the channel coefficients, which can be expressed

$$\hat{\beta}_l(i) = \frac{1}{2\sqrt{\alpha p w(i+1)T_s}} \int_{\tau_l + (i - \frac{w}{2})T_s}^{\tau_l + (i + \frac{w}{2})T_s} y(t)c_p^*(t - \tau_l) dt \quad (14)$$

where $\hat{\beta}_l(i)$ is estimation of the complex gain of l -th path; w is the length of sliding-window in symbols and should be selected properly according to the varying speed of the channel.

6 Experimental Result

The Simulation Results shows that CMA Algorithm offers substantial performance gains over the traditional Adaptive MMSE and LMS algorithm, figures from 1-8.

It is observer from these plots that perform of the channel estimation is analyzed by transmitter and a receiver bit which shows the equalizers convergences.

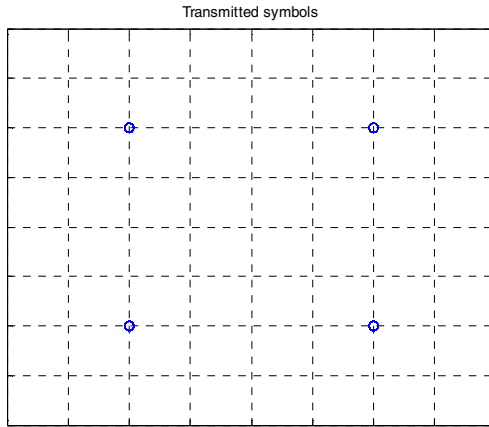


Fig. 1. Transmitter side of Adaptive CMA

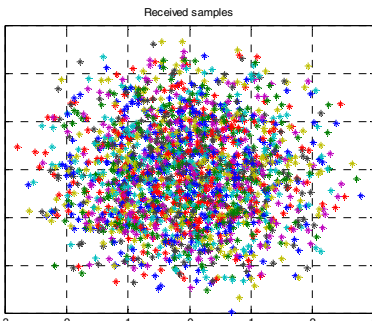


Fig. 2. Receiver side of Adaptive CMA

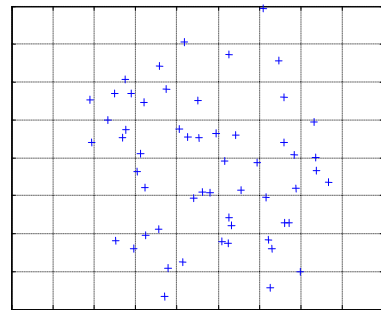


Fig. 3. Adaptive CMA Equalizer

The figures from 1-4 are obtained for Adaptive CMA Equalizer, with more efficient for equalization and convergences. Secondly from figure 5-8 are obtained for an Adaptive MMSE equalizer through LMS algorithm. The efficient of equalization and convergences is too good. The time complexity is very less and more efficient for advance communication systems.

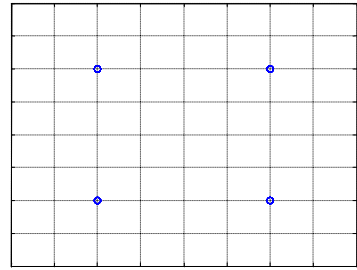
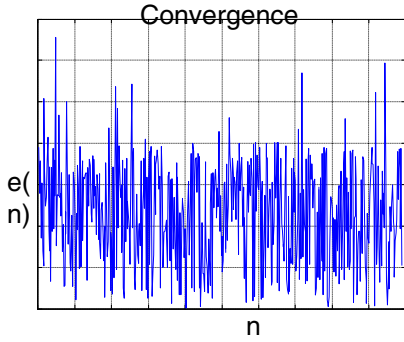


Fig. 4. Convergence of Adaptive CMA equalizer **Fig. 5.** Transmitter side of Adaptive MMSE Equalizer

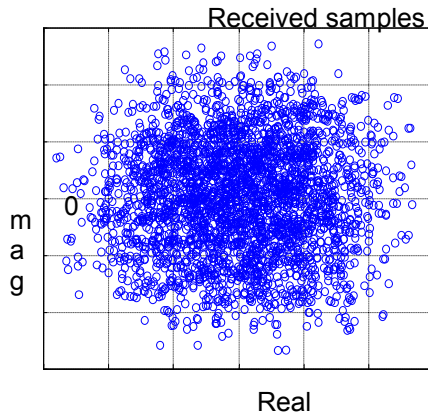


Fig. 6. Receiver side of Adaptive MMSE Equalizer

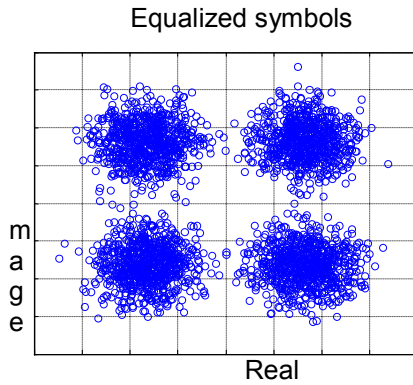


Fig. 7. Adaptive MMSE Equalizer through LMS

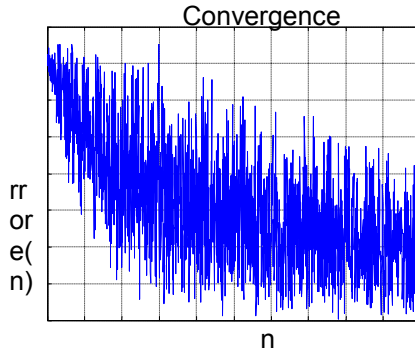


Fig. 8. Convergence of Adaptive MMSE Via LMS algorithm

7 Conclusion

In this paper, Aim at conventional Rake receiver can't satisfy the performance requirement in high data rate transmission, while ideal MMSE equalizer is difficult to real-time implement because its large computational complexity, a low complexity adaptive MMSE equalizer algorithm is proposed. In future conclusion, the proposed low complexity adaptive MMSE equalizer in code-multiplexed CDMA system can be proposed and this system has better practical application value.

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