# The Design of Observers for Nonlinear Control Systems around Equilibria

Sundarapandian Vaidyanathan

R & D Centre, Vel Tech Dr. RR & Dr. SR Technical University
Avadi-Alamathi Road, Avadi, Chennai-600 062, India
sundarvtu@gmail.com
http://www.vel-tech.org/

**Abstract.** This paper investigates the local observer design for nonlinear control systems with real parametric uncertainty around equilibria. In this paper, new results are derived for a general class of nonlinear systems with real parametric uncertainty. In this paper, it is first shown that equilibrium-state detectability is a necessary condition for the existence of local asymptotic observers for any nonlinear system and using this result, it is shown that for the classical case, when the state equilibrium does not change with the real parametric uncertainty, and when the plant output is purely a function of the state, there is no local asymptotic observer for the plant. Next, it is shown that in sharp contrast to this case, for the general case of problems where we allow the state equilibrium to change with the real parametric uncertainty, there generically exist local exponential observers even when the plant output is purely a function of the state. In this paper, a characterization and construction procedure for local exponential observers for a general class of nonlinear systems with real parametric uncertainty has also been derived under some stability assumptions. It is also shown that for the general class of nonlinear systems considered, the existence of local exponential observers in the presence of inputs implies, and is implied by the existence of local exponential observers in the absence of inputs.

**Keywords:** Nonlinear observers, exponential observers, real parametric uncertainty, nonlinear control systems.

## 1 Introduction

The design of observers is an important problem in the control literature because state estimators are needed for system monitoring and for the implementation of state feedback control laws designed for control systems.

For linear control systems, the observer design problem was introduced and fully solved by Luenberger [1]. For nonlinear control systems, the observer design problem was introduced by Thau [2]. During the past three decades, a large research effort has been devoted to the construction of observers for nonlinear control systems ([2]-[16]).

This paper investigates the nonlinear observer design problem for a general class of nonlinear systems with real parametric uncertainty. In this paper, we consider a general class of nonlinear systems described by

N. Meghanathan et al. (Eds.): CCSIT 2012, Part I, LNICST 84, pp. 21–30, 2012.

$$\begin{aligned} \dot{x} &= f(x,\lambda) + g(x,\lambda)u \\ y &= h(x,\lambda) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $\lambda \in \mathbb{R}^l$ is the real parametric uncertainty, $u \in \mathbb{R}^m$ the input and $y \in \mathbb{R}^p$ the output. We assume that the state $x$ is defined in an open neighbourhood of an isolated state equilibrium $\bar{x}$ in $\mathbb{R}^n$ and the input $u$ belongs to a class $\mathcal{U}$ of admissible input functions.

In Sections 2 and 3, we assume that $\mathcal{U}$ consists of all locally $\mathcal{C}^1$ functions $u$ with $u(0) = 0$.

In Section 4, we assume that $\mathcal{U}$ consists of all inputs of the form

$$u = r(\omega) \tag{2}$$

where $\omega$ is the state of a *neutrally stable* exosystem given by

$$\dot{\omega} = s(\omega) \tag{3}$$

We also assume that the parametric uncertainty $\lambda$ takes values in an open neighbourhood $G$ of the origin of $\mathbb{R}^l$. We set $Y = h(X, G)$. We also assume that

$$f(\bar{x}, 0) = 0, \; g(\bar{x}, 0) = 0 \; \text{ and } \; h(\bar{x}, 0) = 0$$

In this paper, it is first shown that *equilibrium-state detectability* is a necessary condition for the existence of local asymptotic observers for the nonlinear system (1). Using this condition, we establish that for the classical case of problems when the state equilibrium does not change with the real parametric uncertainty, there does not exist any local asymptotic observer for the nonlinear plant. Next, we show that in sharp contrast to this case, for the general case of problems where we allow the state equilibrium to change with the real parametric uncertainty, there typically exist local exponential observers even when the plant output is purely a function of the state.

In this paper, we also derive necessary and sufficient conditions for local exponential observers and using this, we deduce a simple construction procedure for the design of exponential observers for the nonlinear plants with exogenous inputs. In this context, we also derive a new result which states that under some stability assumptions on the plant, the existence of local exponential observers for the nonlinear plant (1) in the presence of inputs implies and is implied by the existence of local exponential observers for the plant (1) in the absence of inputs. Thus, this new result simplifies the nonlinear observer design problem significantly.

## 2 Basic Definitions

In this paper, we study the nonlinear observer design problem for the nonlinear plant (1). Since $\lambda$ is a real parametric uncertainty, it may not be available for measurement. Thus, we may consider $\lambda$ as an additional state variable and estimate $\lambda$ as well.

Thus, we consider the plant (1) in an extended form as

$$\begin{aligned} \dot{x} &= f(x,\lambda) + g(x,\lambda)u \\ \dot{\lambda} &= 0 \\ y &= h(x,\lambda) \end{aligned} \tag{4}$$

In this paper, we derive new results for local asymptotic observers and exponential observers for the nonlinear plant (4) with real parametric uncertainty around the equilibria $(x, \lambda) = (\bar{x}, 0) \in \mathbb{R}^n \times \mathbb{R}^l$.

**Definition 1.** *[16] Consider the nonlinear system (*candidate observer*) defined by*

$$\begin{aligned} \dot{z} &= \phi(z, \mu, y, u) \\ \dot{\mu} &= \psi(z, \mu, y, u) \end{aligned} \tag{5}$$

*where the state $z$ of the candidate observer (5) is defined locally (say, in the neighbourhood $X$ of $\bar{x}$ of $\mathbb{R}^n$) and the state $\mu$ of the candidate observer (5) is defined locally (say, in the neighbourhood $G$ of the origin of $\mathbb{R}^l$). We assume that $\phi$ and $\psi$ are locally $\mathcal{C}^1$ mappings such that*

$$\phi(\bar{x}, 0, 0, 0) = 0 \;\; \textit{and} \;\; \psi(\bar{x}, 0, 0, 0) = 0$$

*We say that the candidate observer (5) is a* **local asymptotic** *(resp.* **local exponential***) observer for the plant (4) if the following conditions are satisfied:*

*(O1) If $(x(0), \lambda(0)) = (z(0), \mu(0))$, then $(x(t), \lambda(t)) = (z(t), \mu(t))$ for all $t \geq 0$ and for all $u \in \mathcal{U}$.*
*(O2) There exists a neighbourhood $V$ of the origin of $\mathbb{R}^n \times \mathbb{R}^l$ such that for all values of $(z(0), \mu(0)) - (x(0), \lambda(0))$ in $V$, the measurement error $(z(t) - x(t), \mu(t) - \lambda(t))$ decays to zero asymptotically (resp. exponentially) as $t \to \infty$.* □

We define the estimation error by

$$e \triangleq \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} z \\ \mu \end{bmatrix} - \begin{bmatrix} x \\ \lambda \end{bmatrix} \tag{6}$$

Then the error satisfies the differential equation

$$\begin{aligned} \dot{e}_1 &= \phi(x + e_1, \lambda + e_2, y, u) - f(x, \lambda) - g(x, \lambda)u \\ \dot{e}_2 &= \psi(x + e_1, \lambda + e_2, y, u) \end{aligned}$$

We consider the composite system

$$\begin{aligned} \dot{x} &= f(x, \lambda) + g(x, \lambda)u \\ \dot{\lambda} &= 0 \\ \dot{e}_1 &= \phi(x + e_1, \lambda + e_2, y, u) - f(x, \lambda) - g(x, \lambda)u \\ \dot{e}_2 &= \psi(x + e_1, \lambda + e_2, y, u) \end{aligned} \tag{7}$$

Next, we state a simple lemma which provides a geometric characterization of the condition (O1) in Definition 1.

**Lemma 1.** *([16]) The following statements are equivalent.*

*(a) The condition (O1) in Definition 1 holds for the composite system (4)-(5).*

*(b)* *For all* $x \in X, \lambda \in G$ *and for all* $u \in \mathcal{U}$*, we have*

$$\phi(x, h(x, \lambda), u) = f(x, \lambda) + g(x, \lambda)u \;\; \textit{and} \;\; \phi(x, h(x, \lambda), u) = 0$$

*(c)* *The submanifold defined via* $e = 0$ *is invariant under the flow of the composite system (7).* □

**Lemma 2.** *([16]) Consider the plant (4) and the candidate observer (5). Then the condition (O1) of Definition 1 holds if and only if* $\phi$ *and* $\psi$ *have the following form:*

$$\begin{aligned} \phi(z, \mu, y, u) &= f(z, \mu) + g(z, \mu)u + \alpha(z, \mu, y, u) \\ \psi(z, \mu, y, u) &= \beta(z, \mu, y, u) \end{aligned}$$

*where* $\alpha$ *and* $\beta$ *are locally* $\mathcal{C}^1$ *mappings with*

$$\alpha(\bar{x}, 0, 0, 0) = 0 \;\; \textit{and} \;\; \beta(\bar{x}, 0, 0, 0) = 0$$

*and also such that*

$$\alpha(x, \lambda, h(x, \lambda), u) = 0 \;\; \textit{and} \;\; \beta(x, \lambda, h(x, \lambda), u) = 0$$ □

## 3 A Necessary Condition for Local Asymptotic Observers for Nonlinear Systems

In this section, we shall show that if the plant (4) has a local exponential observer of the form (5), then the plant (4) must be *equilibrium detectable*, *i.e.* if $(x(t), \lambda(t))$ is the solution of the system (4) with small initial condition $(x(0), \lambda(0)) = (x_0, \lambda_0)$ near the equilibrium $(\bar{x}, 0)$ satisfying $y(t) = h(x(t), \lambda(t) \equiv 0$, then $(x(t), \lambda(t))$ must converge to $(\bar{x}, 0)$ asymptotically as $t \to \infty$.

Since $\lambda(t) \equiv \lambda_0$, the equilibrium-state detectability requirement is equivalent to requiring that the solution $(x(t), \lambda(t))$ yielding zero-output for the plant (4) must be such that $x(t) \to \bar{x}$ asymptotically as $t \to \infty$ and $\lambda_0 = 0$.

**Theorem 1.** *A necessary condition for the existence of a local exponential observer for the plant (4) is that the plant (4) is equilibrium-state detectable, i.e. any solution trajectory* $(x(t), \lambda(t))$ *of (4) with small initial condition* $(x_0, \lambda_0)$ *near the equilibrium* $(\bar{x}, 0)$ *satisfying*

$$y(t) = h(x(t), \lambda(t)) \equiv 0$$

*must be such that* $x(t) \to \bar{x}$ *asymptotically as* $t \to \infty$ *and* $\lambda_0 = 0$.

*Proof.* This is a simple consequence of Lemma 2 for local asymptotic observers.

In classical bifurcation theory, a standard assumption is that there is a trivial solution from which the bifurcation is to occur ([17], p149). Thus, in the classical bifurcation case, the control plant (4) is often assumed to satisfy

$$f(\bar{x}, \lambda) = 0 \quad \text{and} \quad g(\bar{x}, \lambda) = 0 \tag{8}$$

Next, as a consequence of Theorem 1, we establish the following result.

**Theorem 2.** *Suppose that the plant (4) satisfies the assumption (8) so that $x = \bar{x}$ is an equilibrium for all values of the parameter $\lambda$ and also that the output function $y$ is purely a function of $x$,* i.e. *it has the form $y = \gamma(x)$. Then there is no local asymptotic observer for the plant (4).*

*Proof.* We show that the plant (4) is not equilibrium-state detectable. Suppose that we take $x(0) = \bar{x}$ and $\lambda(0) = \lambda_0$, where $\lambda_0 \neq 0$ is any small initial condition. Then we have $x(t) \equiv \bar{x}$ for all $t$ and it follows that

$$y(t) = h(x(t), \lambda(t)) = \gamma(x(t)) = \gamma(\bar{x}) = 0$$

However, $\lambda(t) = \lambda_0 \neq 0$. This shows that the plant (4) is not equilibrium-state detectable. From the necessary condition given in Theorem 1, it is then immediate that there is no local asymptotic observer for the plant (4). □

## 4 Observer Design for Nonlinear Systems around Equilibria

In this section, we suppose that the class $\mathcal{U}$ consists of inputs $u$ of the form

$$u = r(\omega), \tag{9}$$

where $\omega$ satisfies the autonomous system (*exosystem*)

$$\dot{\omega} = s(\omega) \quad \text{with} \quad s(0) = 0 \tag{10}$$

The state $\omega$ of the exosystem (10) lies in an open neighbourhood $W$ of the origin of $\mathbb{R}^q$. One can view the equations (9) and (10) as an *input generator*. We assume that the exosystem dynamics (10) is *neutrally stable* at $\omega = 0$. Basically, this requirement means that the exosystem (10) is Lyapunov stable in both forward and backward time at $\omega = 0$.

In this section, we first derive a basic theorem that completely characterizes the existence of local exponential observers of the form (5) for nonlinear plants of the form (4). We note that this result holds for both classical and general cases of systems with real parametric uncertainty.

Using (9) and (10), the plant (4) can be expressed as

$$\begin{aligned} \dot{x} &= f(x, \lambda) + g(x, \lambda) r(\omega) \\ \dot{\lambda} &= 0 \\ \dot{\omega} &= s(\omega) \\ y &= h(x, \lambda) \end{aligned} \tag{11}$$

Also, the composite system (7) can be written as

$$\begin{aligned} \dot{x} &= f(x, \lambda) + g(x, \lambda) r(\omega) \\ \dot{\lambda} &= 0 \\ \dot{\omega} &= s(\omega) \\ \dot{e}_1 &= \phi(x + e_1, \lambda + e_2, h(x, \lambda), r(\omega)) - f(x, \lambda) - g(x, \lambda) r(\omega) \\ \dot{e}_2 &= \psi(x + e_1, \lambda + e_2, h(x, \lambda), r(\omega)) \end{aligned} \tag{12}$$

**Theorem 3.** *Suppose that the plant dynamics in (11) is Lyapunov stable at the equilibrium* $(x, \lambda, \omega) = (\bar{x}, 0, 0)$. *Then the candidate observer (5) is a local exponential observer for the plant (11) if and only if*

*(a) The submanifold defined via* $e = 0$ *is invariant under the flow of the composite system (12).*
*(b) The dynamics*

$$\begin{aligned} \dot{e}_1 &= \phi(e_1, e_2, 0, 0) \\ \dot{e}_2 &= \psi(e_1, e_2, 0, 0) \end{aligned} \tag{13}$$

*is locally exponentially stable at* $e = 0$.

*Proof.* The necessity follows immediately from the Definition 1 for local exponential observers and Lemma 1. The sufficiency can be established using Lyapunov stability theory as in [16]. □

As an application of Theorem 3, we establish the following result which states that when the plant dynamics in (11) is Lyapunov stable at $(x, \lambda, \omega) = (\bar{x}, 0, 0)$, the existence of a local exponential observer for the plant (11) in the presence of inputs implies and is implied by the existence of a local exponential observer for the plant (11) in the absence of inputs.

For the purpose of stating this result, we note that the unforced plant corresponding to $\omega = 0$ is given by

$$\begin{aligned} \dot{x} &= f(x, \lambda) \\ \dot{\lambda} &= 0 \\ y &= h(x, \lambda) \end{aligned} \tag{14}$$

**Theorem 4.** *Suppose that the plant dynamics in (11) is Lyapunov stable at* $(x, \lambda, \omega) = (\bar{x}, 0, 0)$. *If the system*

$$\begin{aligned} \dot{z} &= \phi(z, \mu, y, u) \\ \dot{\mu} &= \psi(z, \mu, y, u) \end{aligned}$$

*is a local exponential observer for the full plant (11), then the system defined by*

$$\begin{aligned} \dot{z} &= \phi(z, \mu, y, 0) \\ \dot{\mu} &= \psi(z, \mu, y, 0) \end{aligned}$$

*is a local exponential observer for the unforced plant (14). Conversely, if the system*

$$\begin{aligned} \dot{z} &= \eta(z, \mu, y) \\ \dot{\mu} &= \sigma(z, \mu, y) \end{aligned}$$

*is a local exponential observer for the unforced plant (14) near* $(x, \lambda) = (\bar{x}, 0)$, *then the system defined by*

$$\begin{bmatrix} \dot{z} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} \phi(z, \mu, y, u) \\ \psi(z, \mu, y, u) \end{bmatrix} \stackrel{\Delta}{=} \begin{bmatrix} \eta(z, \mu, y) \\ \sigma(z, \mu, y) \end{bmatrix} + \begin{bmatrix} g(z, \mu)u \\ 0 \end{bmatrix}$$

*is a local exponential observer for the full plant (11) near* $(x, \lambda, \omega) = (\bar{x}, 0, 0)$.

*Proof.* The first part of this theorem is straightforward. The second part of the theorem follows by verifying the conditions (a) and (b) given in Theorem 3. □

Let $(C^\star, A^\star)$ denote the linearization pair for the unforced plant (14), *i.e.*

$$C^\star = [\, C \quad Z \,] \text{ and } A^\star = \begin{bmatrix} A & P \\ 0 & 0 \end{bmatrix}$$

where

$$C = \frac{\partial h}{\partial x}(\bar{x}, 0), \;\; Z = \frac{\partial h}{\partial \lambda}(\bar{x}, 0), \;\; A = \frac{\partial f}{\partial x}(\bar{x}, 0), \;\; P = \frac{\partial f}{\partial \lambda}(\bar{x}, 0),$$

In view of the reduction procedure outlined in Theorem 4, we first derive some important results on the exponential observer design for the unforced plant (14). First, we state the following necessary condition for the local exponential observers that can be established in [16].

**Theorem 5.** *If the unforced plant (14) has a local exponential observer near the equilibrium* $(x, \lambda) = (\bar{x}, 0)$*, then the pair* $(C^\star, A^\star)$ *is detectable.* □

**Corollary 1.** *If the full plant (11) has a local exponential observer near the equilibrium* $(x, \lambda, \omega) = (\bar{x}, 0, 0)$*, then the pair* $(C^\star, A^\star)$ *is detectable.*

*Proof.* The assertion follows immediately from Theorems 4 and 5. □

Using the necessary condition given in Theorem 5, we establish the following result, which gives a simple necessary condition for the existence of local exponential observers for the unforced plant (14).

**Theorem 6.** *If the unforced plant (14) has a local exponential observer near the equilibrium* $(x, \lambda) = (\bar{x}, 0)$*, then the pair* $(C, A)$ *is detectable and*

$$rank \begin{bmatrix} Z \\ P \end{bmatrix} = l = dim(\lambda)$$

*Proof.* Suppose that the unforced plant (14) has a local exponential observer near the equilibrium $(x, \lambda) = (\bar{x}, 0)$. Then by Theorem 5, the pair $(C^\star, A^\star)$ is detectable. Note that by PBH rank test [19], a necessary and sufficient condition for $(C^\star, A^\star)$ to be detectable is that

$$\text{rank} \begin{bmatrix} C^\star \\ \xi I - A^\star \end{bmatrix} = n + l \tag{15}$$

for all complex numbers $\xi$ in the closed right-half plane (RHP), *i.e.* in the region, where $\text{Re}(\xi) \geq 0$.

We note that

$$\begin{bmatrix} C^\star \\ \xi I - A^\star \end{bmatrix} = \begin{bmatrix} C & Z \\ \xi I - A & -P \\ 0 & \xi I \end{bmatrix}$$

Thus, it is immediate that (15) holds for all complex numbers $\xi$ in the closed RHP only if

$$\begin{bmatrix} C \\ \xi I - A \end{bmatrix} = n$$

for all complex numbers $\xi$ in the closed RHP and

$$\text{rank} \begin{bmatrix} Z \\ P \end{bmatrix} = l.$$

In view of the PBH rank test for detectable [19], the above necessary condition is the same as requiring that $(C, A)$ is detectable and

$$\text{rank} \begin{bmatrix} Z \\ P \end{bmatrix} = l.$$

This completes the proof. □

**Corollary 2.** *If the full plant (11) has a local exponential observer near the equilibrium* $(x, \lambda, \omega) = (\bar{x}, 0, 0)$*, then the pair* $(C, A)$ *is detectable and*

$$\textit{rank} \begin{bmatrix} Z \\ P \end{bmatrix} = l = \textit{dim}(\lambda)$$

*Proof.* This is a simple consequence of Theorems 4 and 6. □

Next, we show that the necessary condition given in Theorem 5 is also sufficient for the existence of a local exponential observer for the unforced plant (14) when the unforced plant dynamics in (14) is Lyapunov stable.

**Theorem 7.** *Suppose that the plant dynamics in (14) is Lyapunov stable at* $(\bar{x}, 0)$ *and suppose also that the matrix* $A^\star - K^\star C^\star$ *is Hurwitz for some matrix* $K^\star$*. Then the system defined by*

$$\begin{bmatrix} \dot{z} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} f(z, \mu) \\ 0 \end{bmatrix} + K^\star \left[ y - h(z, \mu) \right] \tag{16}$$

*is a local exponential observer for the unforced plant (14) near* $(x, \lambda) = (\bar{x}, 0)$.

*Proof.* It is easy to check that the candidate observer (16) satisfies the conditions (a) and (b) of Theorem 3.

When $(C^\star, A^\star)$ is detectable, by the reduction procedure outlined in Theorem 4, we can use the local exponential observer (16) constructed for the unforced plant (14) to construct a local exponential observer for the full plant (11).

**Theorem 8.** *Suppose that the plant dynamics in (11) is Lyapunov stable at* $(\bar{x}, 0, 0)$ *and suppose also that the matrix* $A^\star - K^\star C^\star$ *is Hurwitz for some matrix* $K^\star$*. Then the system defined by*

$$\begin{bmatrix} \dot{z} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} f(z, \mu) + g(z, \mu) u \\ 0 \end{bmatrix} + K^\star \left[ y - h(z, \mu) \right] \tag{17}$$

*is a local exponential observer for the full plant (11) near the equilibrium* $(\bar{x}, 0, 0)$.

*Proof.* The assertion is a simple consequence of the reduction procedure outlined in Theorems 4 and 7 . □

**Corollary 3.** *Suppose that the plant dynamics in (11) is Lyapunov stable at* $(\bar{x}, 0, 0)$ *and that the output function* $y$ *is purely a function of* $x$, i.e. *it has the form* $y = \gamma(x)$. *Assume that equilibrium* $x = \bar{x}$ *of the plant dynamics of* $x$ *in (11) changes with the real parametric uncertainty* $\lambda$. *In this case, the system linearization pair* $(C^\star, A^\star)$ *has the form*

$$C^\star = [\,C \quad 0\,] \;\; \text{and} \;\; A^\star = \begin{bmatrix} A & P \\ 0 & 0 \end{bmatrix}.$$

*If the pair* $(C^\star, A^\star)$ *is detectable, then the full plant (11) has a local exponential observer given by Eq. (17), where* $K^\star$ *is any matrix such that* $A^\star - K^\star C^\star$ *is Hurwitz.* □

*Remark 1.* It is a well-known result in Control Systems that the system linearization pair $(C^\star, A^\star)$ is generically observable [20]. Thus, from Corollary 3, there generically exist local exponential observers of the form (17) for the full plant (11) under the following conditions:

(a) The plant dynamics in (11) is Lyapunov stable at $(x, \lambda, \omega) = (\bar{x}, 0, 0)$.
(b) The equilibrium $x = \bar{x}$ of the plant dynamics in $x$ changes with the real parametric uncertainty.
(c) The output function $y$ is purely a function of $x$, *i.e.* it has the form $y = \gamma(x)$. □

## 5 Conclusions

In this paper, we showed that *equilibrium-state detectability* is a necessary condition for the existence of local asymptotic observers for any nonlinear system. Using this result, we established that for the classical case, when the equilibrium does not change with the parametric uncertainty and when the plant output is purely a function of the state, there is no local asymptotic observer for the plant. We also showed that in sharp contrast to this case, for the general case of problems where we allow the state equilibrium to change with the parametric uncertainty, there typically exist local exponential observers even when the plant output is purely a function of the state. Next, we derived a procedure for local exponential observers for a general class of nonlinear systems with real parametric uncertainty under some stability assumptions and showed that the existence of local exponential observers in the presence of inputs implies, and is implied by the existence of local exponential observers in the absence of inputs.

## References

1. Luenberger, D.: Observing the state of a linear system. IEEE Trans. Military Electronics 8, 74–80 (1964)
2. Thau, F.E.: Observing the states of nonlinear dynamical systems. Internat. J. Control 18, 471–479 (1973)
3. Kou, S.R., Elliott, D.L., Tarn, T.J.: Exponential observers for nonlinear dynamical systems. Inform. Control 29, 204–216 (1975)

4. Krener, A.J., Isidori, A.: Linearization by output injection and nonlinear observers. Systems & Control Letters 3, 47–52 (1983)
5. Bestle, D., Zeitz, M.: Canonical form observer design for nonlinear time-variable systems. Internat. J. Control 38, 419–431 (1983)
6. Krener, A.J., Respondek, W.: Nonlinear observers with linearizable error dynamics. SIAM J. Control & Optimiz. 23, 197–216 (1985)
7. Xia, X.H., Gao, W.B.: Nonlinear observer design by canonical form. Internat. J. Control 47, 1081–1100 (1988)
8. Xia, X.H., Gao, W.B.: On exponential observers for nonlinear systems. Systems & Control Letters 11, 319–325 (1988)
9. Gauthier, J.P., Hammouri, H., Othman, S.: A simple observer for nonlinear systems - Applications to bioreactors. IEEE Trans. Automatic Control 37, 875–880 (1992)
10. Tsinias, J.: Observer design for nonlinear systems. Systems & Control Lett. 13, 135–142 (1989)
11. Tsinias, J.: Further results on the observer design problem. Systems & Control Lett. 14, 411–418 (1990)
12. Phelps, A.R.: On constructing nonlinear observers. SIAM J. Control Optimiz. 29, 516–534 (1991)
13. Gauthier, J.P., Kupka, I.A.K.: Observability and observers for nonlinear systems. SIAM J. Control Optimiz. 32, 975–994 (1994)
14. Krener, A.J., Kang, W.: Locally convergent nonlinear systems. SIAM J. Control Optimiz. 42, 155–177 (2003)
15. Sundarapandian, V.: Observers for nonlinear systems (D.Sc. Dissertation). Washington University, St. Louis (1996)
16. Sundarapandian, V.: Local observer design for nonlinear systems. Math. Computer Modelling 35, 25–36 (2002)
17. Guckenheimer, J., Holmes, P.: Nonlinear oscillations, Dynamical Systems and Bifurcations of Vector Fields. Springer, New York (1983)
18. Isidori, A.: Nonlinear Control Systems. Sprigner, New York (1989)
19. Rugh, W.J.: Linear System Theory. Prentice, New Jersey (1996)
20. Wonham, W.M.: Linear Multivariable Control. Springer, Berlin (1974)