A Novel Approach for MIMO OFDM Systems to Estimate I/Q Imbalance, CFO, Channel Response Using Training Sequences

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Abstract. Systems using Orthogonal Frequency Division Multiplexing (OFDM) suffer with carrier frequency offset (CFO), in phase and quadrature phase imbalance (I/Q) due to which there will be large performance degradation. The CFO, I/Q imbalance are caused due to mismatch of carrier frequency at the transmitter and local oscillator frequency at the receiver. This paper presents a novel approach for joint estimation of I/Q imbalance, CFO and Channel Estimation for Multiple-Input Multiple-Output (MIMO) OFDM systems. A new energy parameter called φ is introduced, and from this parameter φ we can jointly estimate CFO, I/Q imbalance irrespective of channel estimation. The proposed method uses an optimal training block with one or two training sequences. For estimation of two repeated sequences, a two-step approach is proposed. From the simulation results we show that the Mean-Square Error (MSE) of this method is close to Cramer-Rao Bound (CRB).

Keywords: CFO, Channel response, I/Q imbalance, MIMO, OFDM.

1 Introduction

OFDM is the promising multiplexing scheme for future wireless communication for its good performance in terms of maintaining orthogonality between the cells, protection against inter-symbol interference, and effective utilization of band width. With the addition of MIMO causes high data rate and low complexity. These characteristics of OFDM are achieved when the receiver has exact channel information and the system parameters of transmitter and receiver are perfectly matched. But in real scenario these ideal condition does not prevail. Some of these non-idealities are I/Q imbalance and CFO. The I/Q imbalance is due to amplitude and phase mismatch of I phase and Q phase, where as the CFO is caused due to mismatch of carrier frequency at the transmitter and receiver. It is known from [1] that I/Q imbalance and CFO will cause severe Inter Carrier Interference (ICI) in OFDM which degrades the system performance. Several approaches are presented to compensate for

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the I/Q imbalance and CFO[1]-[5]. The impact of I/Q imbalance and CFO on OFDM are estimated in [2]. A time domain method was proposed in [3], for the joint estimation of I/Q imbalance and channel response using one OFDM. In [4] maximum likelihood method was proposed for CFO estimation. Joint estimation and compensation of I/Q imbalance and CFO in frequency domain is proposed in [5].

To improve the data rates the MIMO system was combined with OFDM [6]. Several approaches are proposed to deal with the channel estimation of MIMO OFDM systems. One of these approaches is to send a optimal training sequence from different antennas which must be orthogonal [7]. The I/Q imbalance for MIMO OFDM systems have been investigated in [8], which cause error flooring. A compensation of I/Q imbalance was proposed in [9] with more number of OFDM blocks. The authors derived Cramer – Rao Bound (CRB) for estimation of CFO and channel response for MIMO systems. Recently, researchers proposed a joint estimation of I/Q imbalance and channel response with only one OFDM training block [11].

In this paper we propose a novel method for joint estimation of I/Q imbalance, CFO and channel response in MIMO OFDM systems by introducing a new energy parameter φ . By minimising the φ we are able to estimate I/Q imbalance and CFO without knowing the channel response. From the estimated I/Q imbalance and CFO we can calculate the channel response easily. This method needs only one OFDM block for training. If the training data consists of two repeated sequences the above approach is to be performed on two training sequences and the average is calculated for estimation.

In Section 2 the MIMO OFDM system model is described. The channel estimation of MIMO is discussed in Section 3. The new method for estimation is studied in Section 4. In Section 5 the estimation for repetitive sequences are discussed. Simulations and results are discussed in Section 6 and we conclude the paper in Section 7.

2 System Model

The MIMO-OFDM transmission model used in this paper is shown in Fig. 1. A Nttransmit / Nr-receive antenna configuration is considered. From the Fig.1 the input vector s_i is of Mx1 containing input symbols. To maintain the orthogonality between these symbols the input vector s_i are fed to IDFT of M-point. Then we obtain the Mx1 vector x_i . After insertion of a cyclic prefix (CP) of length L-1, the signal is transmitted through the ith transmit antenna. Let the channel impulse response of ith transmit antenna to the kth receive antenna be $h_{ksj}(n)$. The length of all the channels are



Fig. 1. A MIMO OFDM system

assumed to L, so there is no inter block interference .Then the received signal at the k^{th} receiving antenna after removal of CP is given as

$$r_{k} = \begin{bmatrix} H_{K,0} & H_{K,1} & H_{K,2} & \dots & H_{K,N_{r}-1} \end{bmatrix} \begin{bmatrix} X_{0} \\ X_{1} \\ X_{2} \\ \vdots \\ \vdots \\ \vdots \\ X_{N_{r}-1} \end{bmatrix} + q_{k}$$
(1)

Where $H_{k,i}$ is an MxM circulant matrix with first column

$$h_{k,i} \equiv \left[h_{K,i(0)}h_{K,i(1)}h_{K,i(2)}\dots h_{K,i(L-1)}0\dots 0\right]^{T}$$
(2)

And q_k is the channel noise vector of length Mx1.The output of the DFT block received vector is passed through a Frequency domain Equaliser (FEQ) to recover the transmit signals s_{i} .

If the system suffers with carrier frequency offset (CFO) Δf_k then the normalised frequency offset is given as

$$\theta_{k} = \frac{\Delta f_{k}}{\frac{1}{MT}} = \Delta f_{k} MT$$
(3)

Where M,T are given as the size of the DFT matrix and T sample spacing. The vector due to CFO is

$$y_k = E_k r_k \tag{4}$$

where r_k is desired vector and E_k is the MxM diagonal matrix given by

$$E_{k} \stackrel{\Delta}{=} diag \left[1 \quad e^{j\frac{2\pi}{M}\theta_{k}} \dots e^{j\frac{2\pi}{M}(M-1)\theta_{k}} \right]$$
(5)

Suppose if there is I/Q mismatch at the receiver, then received vector due to it is

$$Z_k = \mu_k y_k + v_k y_k^* \tag{6}$$

• 4

Where μ_k and v_k are I/Q parameters due to amplitude mismatch $\epsilon_{k,i}$, phase mismatch Φ_k and they are given as

$$\mu_k = \frac{1 + \varepsilon_k e^{-j\phi_k}}{2} \text{ and } v_k = \frac{1 - \varepsilon_k e^{-j\phi_k}}{2}. \tag{7}$$

Substituting the value of y_k in (6)

$$Z_k = \mu_k E_k r_k + \nu_k E_k^* r_k^* \tag{8}$$

From (8) It is clear that the received vector consists of not only the desired base band vector r_k but also its complex conjugate r_k^* , and the E_k is due to CFO which also destroy the sub carrier orthogonality.

 \boldsymbol{z}_k is a received vector in the presence of I/Q imbalance and CFO. To recover \boldsymbol{r}_k from \boldsymbol{z}_k we define a parameter

$$\alpha_{k} = \frac{v_{k}}{\mu_{k}}.$$
(9)

If α_k is estimated correctly at the receiver then

$$\mu_k y_k = \frac{z_k - \alpha_k z_k}{1 - |\alpha_k|^2}.$$
(10)

If the normalised CFO θ_k is known at the receiver then the recovered vector from the received vector is given by

$$\mu_{k} r_{k} = E_{k}^{*} \mu_{k} y_{k} = E_{k}^{*} \frac{z_{k} - \alpha_{k} z_{k}^{*}}{1 - |\alpha_{k}|^{2}}$$
(11)

3 MIMO Channel Estimation

An MIMO channel is one where the antenna arrays are available at transmitter and receiver as shown in Fig.1. Channel estimation of these channels using training sequences are described in [7] If a MIMO channel with N_t transmit and N_r receiving antennas are selected then the received vector r is given as

$$r = \begin{bmatrix} X_0 & X_1 & \dots & X_{N_{t-1}} \end{bmatrix} \begin{bmatrix} h_o \\ h_1 \\ \vdots \\ h_{N_{t-1}} \end{bmatrix} + q$$
(12)

Where h_j is the Mx1 vector defined in (12). x_j is the first column of MxM circulant matrix . Let us define a MxL sub matrix A_j , which consist of the first L columns of x_j and c_j is defined as

$$c_i = [h_i(0)h_i(1)\dots\dots h_i(L-1)]^T$$
(13)

And

$$A = \begin{bmatrix} A_0 & A_1 \dots \dots & A_{N_{t-1}} \end{bmatrix}$$
(14)

The r may be re written as

$$r=Ac+q$$
 (15)

In the above equation the channel vector c is identifiable if and only if M x LN_t matrix A has full column rank. Thus a necessary condition for channel identifiability is $M \ge LN_t$. Then the least square estimator \hat{c} is given by

$$\hat{c} = (A^+ A)^{-1} A^+ r$$
 (16)

Then the error vector is defined as $e = \hat{c} - c = (A^+ A)^{-1}A^+ q$. The design of a optimal sequence that minimize the mean square error (MSE) $E ||e||^2$ is given in [7]. The optimal training sequence from different antennas must satisfy

$$A_k^+ A_i = \delta(k-i)I \tag{17}$$

That means the training sequence from different transmitter antenna must be orthogonal .When the training sequence are orthogonal the least square estimate of channel response becomes

$$\hat{\mathbf{c}} = \mathbf{A}^+ \mathbf{r} \tag{18}$$

In this paper the above estimation is formulated in a different way, for joint estimation of I/Q imbalance, CFO and channel response that is

$$\rho = \frac{M}{Nt} \tag{19}$$

We assume that M is a multiple of N_t and also this can be further extended to the case where M is not a multiple of N_t by simple Modification. Furthermore we assume that $\rho \ge L$. To obtain the channel identifiability condition M \ge N_t we add the (ρ -L)zeros to the length L vectors of c_i to obtain ρ x1 vector given by

$$d_i = \binom{c_i}{0}$$
 for i=0,1,....N_{t-1} (20)

From all the vectors of d_i for $0 \le i \le N_t$ we form the Mx1 vector

$$d = \left[d_0^{\ T} d_1^{\ T} \dots \dots \dots d_{Nt-1}^{\ T}\right]^T$$
(21)

Let A_k^1 be any M x (ρ -L) matrix such that the following MxM matrix B is invertible

$$A = \begin{bmatrix} A_0 & A_0^1 & A_1 & A_1^1 & \dots & \dots & A_{N_{t-1}} & A_{N_{t-1}}^1 \end{bmatrix}$$
(22)

From (21)&(22) the vector can be rewritten as

$$r=Bd+q$$
 (23)

The estimate MIMO channel response is given by

$$\hat{d} = \left[\hat{d}_0^T \hat{d}_1^T \dots \dots \dots \hat{d}_{Nt-1}^T\right]^T$$
(24)

$$=B^{-1}r\tag{25}$$

The estimated channel response \hat{c}_j is given by the first L entries of \hat{d}_j . When the training sequences are orthogonal then the columns of A_k are orthogonal .Thus the channel estimate is given by $\hat{d} = B^{-1}r$. It means the orthogonal sequences are the optimal sequences. B can be chosen to be unitary and circulant.

4 Proposed Joint Estimation Method

In this section we propose a new method to estimate the channel response in the presence of I/Q imbalance and CFO. The estimation done in two sections in section 4.1 we estimate channel response in the presence of I/Q imbalance and assume that no CFO is present. In section 4.2 we estimate channel response in the presence of CFO and I/Q imbalance. In both section we estimate the optimal solution α_k and θ_k from the received vector z_k at the k^{th} receiving antenna. The estimation is performed for both simple sequence and repeated sequences.

4.1 Joint Estimation of Channel Response and I/Q Imbalance

In this section we assume that the carrier frequency at the transmitter and receiver are orthogonal means no CFO $\theta = 0$ and E = I in (11). Then the μ r is related to the received vector and it is given as

$$\mu r = \frac{z - \alpha z^*}{1 - |\alpha|^2} \tag{26}$$

If α is known in (26) then the channel estimation can obtained as

$$\mu \hat{d} = \mu [\hat{d}_0 \, \hat{d}_1 \, \dots \, \dots \, \hat{d}_{Nt-1}]^T$$

= $B^{-1} r$
= $B^{-1} \frac{z - \alpha z^*}{1 - |\alpha|^2}$ (27)

In the above expression when α is estimated perfectly at the receiver then the L entries of the \hat{h}_j gives channel estimation, and rest of the entries ρ -L of \hat{d}_j are due to channel noise. To obtain adequate SNR the energy of the other entries must be small. Then defining a new energy parameter φ as

$$\varphi = \sum_{l=0}^{N_{t-1}} \sum_{i=L}^{\rho-1} \left| \left[\mu \hat{d}_l \right]_i \right|^2$$
(28)

Where $[\mu \hat{d}_l]_i$ denotes the ith entry of $\mu \hat{d}_l$. If α is not estimated perfectly φ will increase. By minimizing the φ we can estimate α without knowing channel response .Let us define a new matrix S of $(M - N_t L)XN$ where

$$s = \begin{bmatrix} 0 & I_{\rho-L} & 0 & \cdots & 0 \\ 0 & \dots & I_{\rho-L} & \ddots & \vdots \\ 0 & & \cdots & I_{\rho-L} \end{bmatrix}$$
(29)

If ρ >L then S is non zero matrix. Multiplying μd with S the φ can be given as

$$\varphi = \left| \left| S \mu \hat{d} \right| \right|^2 \tag{30}$$

$$\varphi = \left| \left| SB^{-1} \frac{z - \alpha z^*}{1 - |\alpha|^2} \right| \right|^2 \tag{31}$$

" α " is estimated in such a way to minimize the energy parameter $\phi.$ When the α is small, then ϕ is given as

$$\varphi \cong \left| \left| sB^{-1}(z - \alpha z^*) \right| \right|^2 \tag{32}$$

By this the optimal value of α which may minimize the ϕ is given by

$$\alpha_{\rm opt} = \frac{(sB^{-1}z^*)(sB^{-1}z)}{||sB^{-1}z^*||^2}$$
(33)

4.2 Joint Estimation of Channel Response, CFO and I/Q Imbalance

If the CFO and I/Q imbalance both are present in the system then the received vector r in (11) is given as

$$\mu r = E^* \mu y = E^* \frac{z - \alpha z^*}{1 - |\alpha|^2}$$
(34)

Where E is due to CFO. Then the estimation of channel response can be obtained by

$$\mu \hat{d} = B^{-1} \mu r = B^{-1} E^* \frac{z - \alpha z^*}{1 - |\alpha|^2}$$
(35)

Similarly in the previous section if α and θ are estimated perfectly then ρ -L entries \hat{d}_j are due to minimizing the φ is

$$\varphi(\alpha,\theta) = \left| \left| S\mu \hat{d} \right| \right|^2 \tag{36}$$

Substitute (35) in (36)

$$\varphi(\alpha,\theta) = \left| \left| SB^{-1}E^* \frac{z - \alpha z^*}{1 - |\alpha|^2} \right| \right|^2$$
(37)

If $F = SB^{-1}E^*$ then it may be

$$\varphi(\alpha,\theta) = \left| \left| F \frac{z - \alpha z^*}{1 - |\alpha|^2} \right| \right|^2$$
(38)

Here we can estimate α and θ by minimizing φ . We need to calculate first optimal α for a given θ and based on that optimized θ assuming α is small then

$$\varphi(\alpha,\theta) \simeq \left| \left| Fz - \alpha Fz^* \right| \right|^2 \tag{39}$$

For given θ the optimal value of α is estimated as

$$\alpha_{\text{opt}}(\theta) = \frac{|Fz|^*(Fz)}{\left||Fz^*\right||^2} \tag{40}$$

 $\alpha_{opt}(\theta)$ is a function of CFO(θ), because F depends on E substituting $\alpha_{opt}(\theta)$ in (39) ϕ can be written as

$$\varphi(\theta) = \left| |Fz^*| \right|^2 - \frac{|(Fz)^*(Fz^*)|^2}{\|Fz^*\|^2}$$
(41)

Then

$$\theta_{\rm opt} = \min \ \varphi(\theta) \tag{42}$$

After obtaining α_{opt} and θ_{opt} , the channel response estimation can be obtained by substituting these values in (39). In many practical applications the training data consists of repeated sequences so another approach is a two step approach for joint optimization.

5 Proposed Method for Repeated Sequence

If two repeated sequences are available in the in the training data [12] that means the block is of length (M+L-1)x1 vector. Out of which the training sequence x_i is of M/2 x1 vector and cyclic prefix of length L-1. If the above specified sequence suffers with I/Q imbalance, CFO then the channel estimation procedure is as follows.

There are two received vectors z_a , z_b and they are given as

$$Z_a = \mu y + \nu y^* + q_a \quad and \quad Z_b = \mu e^{j\pi\theta} y + \nu (e^{j\pi\theta} y)^* + q_b \tag{43}$$

Where a&b are OFDM blocks and y is an M/2 x 1 vector. For joint estimation of I/Q imbalance, CFO and channel Estimation we propose two cases *Case1: Estimate CFO for given* α .

For a given value of α we can estimate CFO

$$\mu y = \frac{z_a - \alpha z_a^*}{1 - |\alpha|^2} \tag{44}$$

$$e^{j\pi\theta}\mu y = \frac{z_{\rm b} - \alpha z_{\rm b}^{*}}{1 - |\alpha|^2} \tag{45}$$

If \propto is given the CFO can be estimated

$$\hat{\theta} = \frac{1}{\pi} \text{angle}(z_a - \propto z_a^*)(z_b - \propto z_b^*)$$
(46)

Case2: *Estimate* $h(n), \alpha$ *for given* θ From(35)

$$\mu \hat{d}_a = B^{-1} E^* \frac{z_a - \alpha z_a^*}{1 - |\alpha|^2} \text{ and } \mu \hat{d}_b = e^{j\pi\theta} B^{-1} E^* \frac{z_b - \alpha z_b^*}{1 - |\alpha|^2}$$
(47)

Now for two repeated sequences the B and E are assumed to be of dimensions M/2 x M/2, and rest of the properties are same. Then the energy parameter ϕ is given as

$$\varphi_a = \left| \left| s \mu \hat{d}_a \right| \right|^2$$
 and $\varphi_b = \left| \left| s \mu \hat{d}_b \right| \right|^2$ (48)

Similarly optimum value of α to minimize ϕ is

$$\widehat{\alpha}_{a} = \frac{(Fz^{*}_{a})^{+}(Fz_{a})}{\left||Fz^{*}_{a}|\right|^{2}} \qquad \text{and } \widehat{\alpha}_{b} = \frac{(Fz^{*}_{b})^{+}(Fz_{b})}{\left||Fz^{*}_{b}|\right|^{2}}$$
(49)

By taking average

$$\widehat{\alpha} = \frac{1}{2} \left(\widehat{\alpha}_{a} + \widehat{\alpha}_{b} \right) \tag{50}$$

Substituting the value of $\hat{\alpha}$ in $\mu \hat{d}$ then the estimated Channel response is given as

$$\mu \hat{d} = \frac{1}{2} (\mu \hat{d}_a + \mu \hat{d}_b) \tag{51}$$

6 Simulations

In this paper the joint estimation of CFO, I/Q imbalance and channel response is proposed and these are performed on computer simulations. The simulations are conducted on MIMO OFDM systems with various parameters.

These simulations are performed on 64 random channels and the channel taps are assumed to be complex Gaussian random variables with variance equal to unity. The channel length is of L=65,CP length of L-1=64 and training data are QPSK symbols and channel taps are correlated, with 2 cases of parameters.

Case i: $N_t=2$, $N_r=2$ with an amplitude mismatch $\epsilon=1$, phase mismatch $\Phi=10^0$ and CFO $\theta=1$.

Case ii: $N_t=4$, $N_r=2$ with an amplitude mismatch of $\varepsilon=1$, phase mismatch $\Phi=10^0$ and CFO $\theta=3$



Fig. 2. MSE Vs SNR for I/Q in Case i

These two cases of simulations are conducted for both proposed models and compared with the method of IQ_CFO_FD [5]. Fig.2&3 shows the MSE of I/Q parameter estimation for case I &II. From these simulations it is clear that the proposed method provide good performance. In the result for IQ_CFO_FD the MSE [5] become flat, but for the proposed method the error flooring is reduced for high SNR values. Similarly Fig.4 &5. shows the MSE of CFO estimation for Case I &II and these simulations are very close to Cramer-Rao bound (CRB) [10]. The proposed method provides good estimation of CFO, whereas the method [5]cannot estimate the values of CFO greater than 1.In case II the CFO is considered to be 3. Fig.6&7 shows the MSE of channel estimation and the performance of this also close to CRB. If these are compared with IQ-CFO-FD for Case I parameters the MSE is good for less values of SNR but as the SNR increases the error flooring remains same, where as in case of proposed methods the error flooring reduces for high SNR values.







Fig. 4. MSE Vs SNR for CFO in Case i



Fig. 5. MSE Vs SNR for CFO in Case ii



Fig. 6. MSE Vs SNR for Channel response in Case i



Fig. 7. MSE Vs SNR for Channel response in Case ii

7 Conclusion

In this paper, we propose a new parameter φ for joint estimation of CFO, I/Q imbalance and channel response for MIMO based OFDM systems. The proposed methods in section IV and V measure mean square error accurately compared with earlier approaches. Moreover, in this method the MSE's can be estimated for higher values of CFO. When the single OFDM block is available then the method proposed in section IV is suggested, if repetitive sequences are available then the method proposed in section V is suggested. In both the methods the simulation results shows that the MSE's are close to CRB.

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