Generalized Projective Synchronization of Double-Scroll Chaotic Systems Using Active Feedback Control

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Abstract. This paper deploys active feedback control method for achieving generalized projective synchronization (GPS) of double-scroll chaotic systems, *viz.* identical Li systems (2009), and non-identical Lü-Chen system (2002) and Li system. The synchronization results (GPS) derived in this paper using active feedback control method have been established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active feedback control method is very effective and suitable for achieving the general projective synchronization (GPS) of double-scroll chaotic systems. Numerical simulations are presented to demonstrate the effectiveness of the synchronization results derived in this paper.

Keywords: Chaos, active control, generalized projective synchronization, Li system, Lü-Chen system.

1 Introduction

Chaotic systems are nonlinear dynamical systems, which are highly sensitive to initial conditions. Chaos is an interesting nonlinear phenomenon and has been rigorously studied in the last two decades. In operation, a chaotic system exhibits an irregular behavior and produces broadband, noise-like signals, thus it is found to be very useful in secure communications [1].

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive system* and another chaotic system is called the *slave* or *response system*, then the idea of synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the seminal work by Pecora and Carroll ([2], 1990), a variety of impressive approaches for chaos synchronization have been used for chaos synchronization such as the PC method [2], sampled-data feedback synchronization method [3], OGY method [4], time-delay feedback method [5], backstepping method [6], active control method [7], adaptive control method [8], sliding control method [9], etc.

In generalized projective synchronization [10], the chaotic systems can synchronize up to a constant scaling matrix. Complete synchronization [11], anti-synchronization

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[12], hybrid synchronization [13], projective synchronization [14] and generalized synchronization [15] are special cases of generalized projective synchronization. The generalized projective synchronization (GPS) has important applications in secure communications.

This paper addresses the generalized projective synchronization (GPS) of double-scroll chaotic systems, viz. Li system ([16], 2009) and Lü-Chen system ([17], 2002).

This paper is organized as follows. In Section 2, we derive results for the GPS between identical Li systems (2009). In Section 3, we derive results for the GPS between non-identical Lü-Chen system (2002) and Li system (2009). Section 4 summarizes the main results derived in this paper.

2 Generalized Projective Synchronization of Identical Double-Scroll Systems

2.1 Main Results

In this section, we derive results for the generalized projective synchronization (GPS) of identical Li systems ([16], 2009).

Thus, the master system is described by the Li dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= x_1 x_3 - x_2 \\ \dot{x}_3 &= b - x_1 x_2 - c x_3 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the *state* variables and a, b, c are constant, positive parameters of the system.

The Li system (1) is chaotic when $a = 5,\ \ b = 16$ and $c = 1$. Figure 1 depicts the state orbits of the *double-scroll* attractor given by Li dynamics (1).

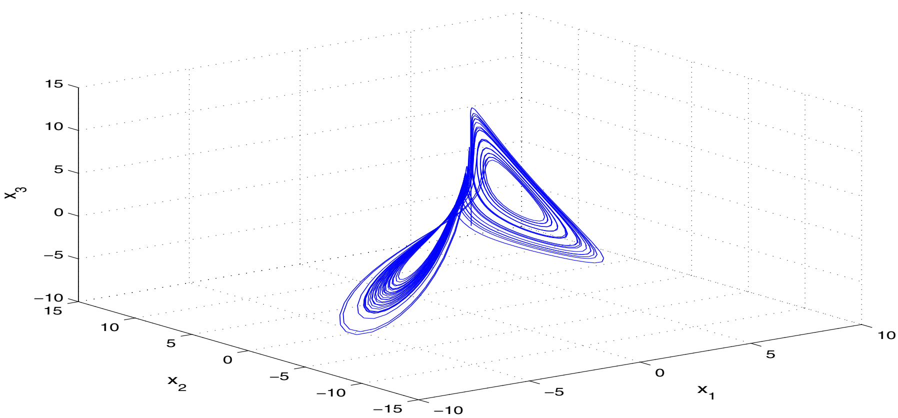

Fig. 1. State Orbits of the Li System

Also, the slave system is described by the controlled Li dynamics

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= y_1 y_3 - y_2 + u_2 \\ \dot{y}_3 &= b - y_1 y_2 - c y_3 + u_3\end{aligned} \tag{2}$$

where y_1, y_2, y_3 are the *state* variables and u_1, u_2, u_3 are the active controls.

For the GPS of (1) and (2), the synchronization errors are defined as

$$\begin{aligned}e_1 &= y_1 - \alpha_1 x_1 \\ e_2 &= y_2 - \alpha_2 x_2 \\ e_3 &= y_3 - \alpha_3 x_3\end{aligned} \tag{3}$$

where the scales $\alpha_1, \alpha_2, \alpha_3$ are real numbers.

A simple calculation yields the error dynamics

$$\begin{aligned}\dot{e}_1 &= a(y_2 - y_1) - \alpha_1 a(x_2 - x_1) + u_1 \\ \dot{e}_2 &= y_1 y_3 - y_2 - \alpha_2(x_1 x_3 - x_2) + u_2 \\ \dot{e}_3 &= b - y_1 y_2 - c y_3 - \alpha_3(b - x_1 x_2 - c x_3) + u_3\end{aligned} \tag{4}$$

We consider the active nonlinear controller defined by

$$\begin{aligned}u_1 &= -a(y_2 - y_1) + \alpha_1 a(x_2 - x_1) - k_1 e_1 \\ u_2 &= -y_1 y_3 + y_2 + \alpha_2(x_1 x_3 - x_2) - k_2 e_2 \\ u_3 &= -b + y_1 y_2 + c y_3 + \alpha_3(b - x_1 x_2 - c x_3) - k_3 e_3\end{aligned} \tag{5}$$

where the gains k_1, k_2, k_3 are positive constants.

Substitution of (5) into (4) yields the closed-loop error dynamics

$$\begin{aligned}\dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3\end{aligned} \tag{6}$$

We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2}\, e^T e = \frac{1}{2}\, \left(e_1^2 + e_2^2 + e_3^2\right) \tag{7}$$

which is positive definite on $\mathbb{R}^3$.

Differentiating (7) along the trajectories of the system (6), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{8}$$

which is a negative definite function on $\mathbb{R}^3$, since k_1, k_2, k_3 are positive constants.

Thus, by Lyapunov stability theory [18], the error dynamics (6) is globally exponentially stable. Hence, we obtain the following result.

Theorem 1. *The active feedback controller (5) achieves global chaos generalized projective synchronization (GPS) between the identical Li systems (1) and (2).* ■

2.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (1) and (2) with the active controller (5).

The parameters of the identical Li systems are selected as $a = 5, \ b = 16, \ c = 1$ so that the systems (1) and (2) exhibit chaotic behaviour.

The initial values for the master system (1) are taken as

$$x_1(0) = 4, \ x_2(0) = 12, \ x_3(0) = 6$$

The initial values for the slave system (2) are taken as

$$y_1(0) = 20, \ y_2(0) = 5, \ y_3(0) = 14$$

The GPS scales α_i are taken as $\alpha_1 = -2.3, \ \alpha_2 = 0.5, \quad \alpha_3 = 1.8.$

We take the state feedback gains as $k_1 = 4, \ k_2 = 4$ and $k_3 = 4$.

Figure 2 shows the time response of the error states e_1, e_2, e_3 of the error dynamical system (4) when the active nonlinear controller (5) is deployed. From this figure, it is clear that all the error states decay to zero exponentially in 2 sec and thus, generalized projective synchronization is achieved between the identical Li systems (1) and (2).

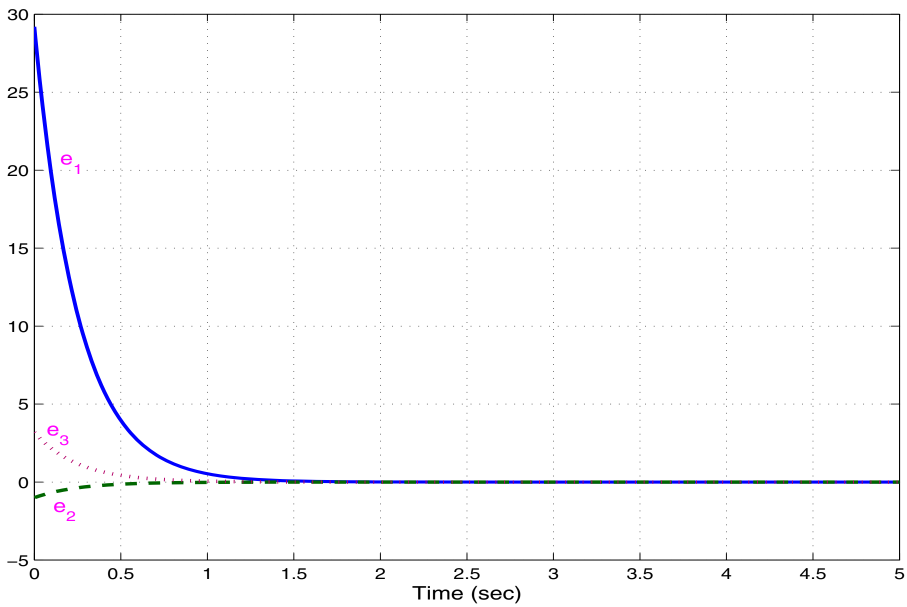

Fig. 2. Time Responses of the Error States of (4)

3 Generalized Projective Synchronization of Non-identical Double-Scroll Systems

3.1 Main Results

In this section, we derive results for the generalized projective synchronization (GPS) of non-identical double-scroll systems, *viz.* Lü-Chen system ([17], 2002) and Li system ([16], 2009).

Thus, the master system is described by the Lü-Chen dynamics

$$\begin{aligned} \dot{x}_1 &= p(x_2 - x_1) \\ \dot{x}_2 &= -x_1 x_3 + r x_2 \\ \dot{x}_3 &= x_1 x_2 - q x_3 \end{aligned} \tag{9}$$

where x_1, x_2, x_3 are the *state* variables and p, q, r are constant, positive parameters of the system.

The Lü-Chen system (9) is chaotic when $p = 36, q = 3$ and $r = 15$.

Figure 3 depicts the state orbits of the *double-scroll* attractor given by Lü-Chen dynamics (9).

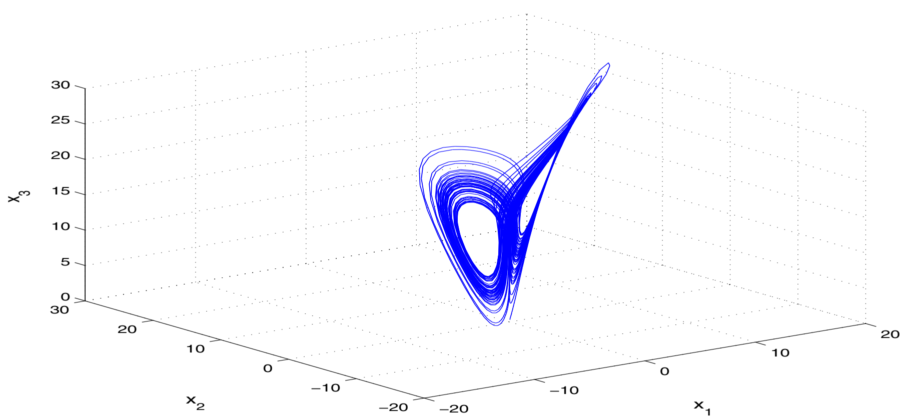

Fig. 3. State Orbits of the Lü-Chen System

Also, the slave system is described by the controlled Li dynamics

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + u_1 \\ \dot{y}_2 &= y_1 y_3 - y_2 + u_2 \\ \dot{y}_3 &= b - y_1 y_2 - c y_3 + u_3 \end{aligned} \tag{10}$$

where y_1, y_2, y_3 are the *state* variables and u_1, u_2, u_3 are the active controls.

For the GPS of (9) and (10), the synchronization errors are defined as

$$\begin{aligned} e_1 &= y_1 - \alpha_1 x_1 \\ e_2 &= y_2 - \alpha_2 x_2 \\ e_3 &= y_3 - \alpha_3 x_3 \end{aligned} \tag{11}$$

where the scales $\alpha_1, \alpha_2, \alpha_3$ are real numbers.

A simple calculation yields the error dynamics

$$\begin{aligned} \dot{e}_1 &= a(y_2 - y_1) - \alpha_1 p(x_2 - x_1) + u_1 \\ \dot{e}_2 &= y_1 y_3 - y_2 - \alpha_2(-x_1 x_3 + r x_2) + u_2 \\ \dot{e}_3 &= b - y_1 y_2 - c y_3 - \alpha_3(x_1 x_2 - q x_3) + u_3 \end{aligned} \tag{12}$$

We consider the active nonlinear controller defined by

$$\begin{aligned} u_1 &= -a(y_2 - y_1) + \alpha_1 p(x_2 - x_1) - k_1 e_1 \\ u_2 &= -y_1 y_3 + y_2 + \alpha_2(-x_1 x_3 + r x_2) - k_2 e_2 \\ u_3 &= -b + y_1 y_2 + c y_3 + \alpha_3(x_1 x_2 - q x_3) - k_3 e_3 \end{aligned} \tag{13}$$

where the gains k_1, k_2, k_3 are positive constants.

Substitution of (13) into (12) yields the closed-loop error dynamics

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 \\ \dot{e}_2 &= -k_2 e_2 \\ \dot{e}_3 &= -k_3 e_3 \end{aligned} \tag{14}$$

We consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2}\, e^T e = \frac{1}{2}\, \left(e_1^2 + e_2^2 + e_3^2\right) \tag{15}$$

which is positive definite on $\mathbb{R}^3$.

Differentiating (15) along the trajectories of the system (14), we get

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{16}$$

which is a negative definite function on $\mathbb{R}^3$, since k_1, k_2, k_3 are positive constants.

Thus, by Lyapunov stability theory [18], the error dynamics (14) is globally exponentially stable. Hence, we obtain the following result.

Theorem 2. *The active feedback controller (13) achieves global chaos generalized projective synchronization (GPS) between the non-identical Lü-Chen system (9) and Li system (10).* ■

3.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (9) and (10) with the active controller (13).

The parameters of the Lü-Chen system (9) and Li system (10) are taken as in the chaotic case.

The initial values for the master system (9) are taken as

$$x_1(0) = 14, \ x_2(0) = 7, \ x_3(0) = 4$$

The initial values for the slave system (10) are taken as

$$y_1(0) = 3, \ y_2(0) = 15, \ y_3(0) = 22$$

The GPS scales α_i are taken as $\alpha_1 = 3.8, \ \ \alpha_2 = -0.3, \ \ \ \alpha_3 = -2.7.$

We take the state feedback gains as $k_1 = 4, \ \ k_2 = 4$ and $k_3 = 4.$

Figure 4 shows the time response of the error states e_1, e_2, e_3 of the error dynamical system (12) when the active nonlinear controller (13) is deployed. From this figure, it is clear that all the error states decay to zero exponentially in 2 sec and thus, generalized projective synchronization is achieved between the non-identical Lü-Chen system (9) and Li system (10).

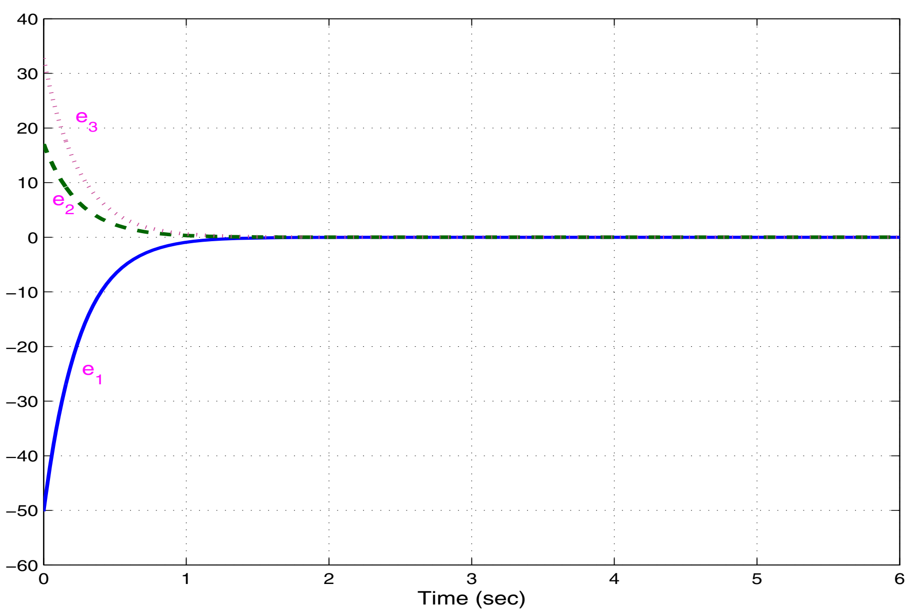

Fig. 4. Time Responses of the Error States of (12)

4 Conclusions

In this paper, active feedback control method has been deployed to achieve generalized projective synchronization (GPS) of double-scroll chaotic attractors, *viz.* identical Li systems (2009), and non-identical double-scroll attractors, *viz.* Lü-Chen system (2002) and Li system (2009). The synchronization results derived in this paper have been proved using Lyapunov stability theory. Since Lyapunov exponents are not required for these calculations, the proposed active control method is very effective and suitable for achieving GPS of the double-scroll chaotic attractors addressed in this paper. Numerical simulations are presented to demonstrate the effectiveness of the synchronization results derived in this paper.

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