

Optimum Beam Bandwidth Allocation Based on Traffic Demands for Multi-spot Beam Satellite System

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Abstract. Multibeam satellite networks can extend the service coverage as deploying its spot beam. It is important to allocate the appropriate resources to downlink multibeams to prevent the unnecessary waste of resources in the satellite system. This paper presents an optimum beam allocation scheme for multi-spot beam satellite system, as beam bandwidth to be allocated is controlled dynamically. We apply the Lagrange theory to obtain the optimization formula for bandwidth allocation of each spot beam in order to meet the total bandwidth constraint. Eventually we can find out the optimum beam profile respect to bandwidth.

Keywords: multi-spot beam, satellite, beam allocation, optimization.

1 Introduction

It is crucial to manage the satellite downlink communication resources effectively in order to maximize the utilization of limited on-board resources over satellite networks.

A future satellite will generate its wide service coverage area by using multiple spot beams. The goal of the satellite system with narrow spot-beams is to support high data rates to user terminals and thus achieve maximum throughput for satellite systems. For this, it needs to study on dynamic resource allocation method among the each beam according to different traffic demands and other channel conditions.

Some attempts have been proposed to adjust the power for optimal resource allocation in [1][2][3]. These techniques have the inherent drawback of high-cost system since it has controllable multi-port travelling wave tube amplifiers (TWTAs) with nonlinearity. To alleviate this problem, we consider the adjustment of the beam bandwidth to maximize the spectral efficiency according to the operation condition instead of the each spot beam power.

In this paper, we propose the adaptive beam bandwidth allocation scheme based on traffic demands and channel condition. The rest of the paper is organized as follows. In section 2, we describe a system configuration of the multi-spot beam satellite network. Section 3 presents how to calculate the optimum bandwidth allocation using a Lagrangian function theory. In the section 4, simulation result shows the validity of the proposed scheme. Finally, the conclusion is drawn in section 5.

2 System Configuration of Multi-spot Beam Satellite

Multi-spot beam antenna techniques can achieve the beam pattern having a high directional gain as well as narrow beam width. Also it is possible to switch among the beams into several areas according to high speed phase conversion. Therefore, it can make a flexible construction of service via effective operation of limited communication resources. In addition, the total system capacity increases by providing appropriate resources according to traffic distribution and channel conditions, and thus we need a means to allocated reasonable beam resources such as power, bandwidth and spot beams.

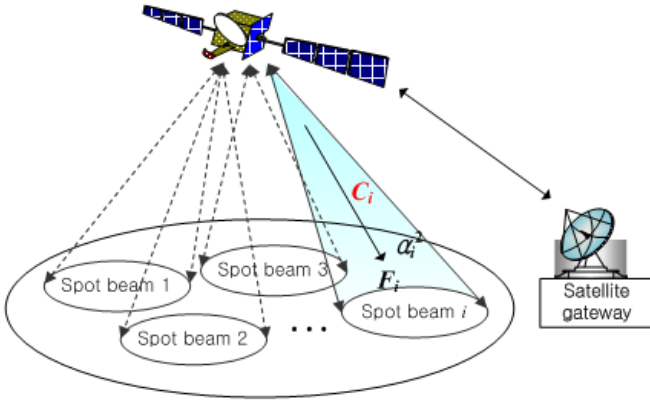


Fig. 1. A multi-spot beam satellite that provides capacity C_i for the i th cell of traffic demand F_i

Figure 1 shows a system configuration of satellite with multiple spot beams. In the network, a multi-spot beam satellite in geostationary orbit and an ensemble of satellite cell sites are deployed. i th beam requires some traffic demand F_i to be served, and multi-beam satellite allocate the capacities to cells. On-board transmission resource (bandwidth in this scheme) is divided and allocated to meet required traffic demand for each spot beam. Using the time sharing scheme for Gaussian broadcast channels, we can obtain the Shannon bounded capacity C_i for i th spot beam as follow [4].

$$C_i = W_i \log_2 \left(1 + \frac{\alpha_i^2 P}{W_i N_0} \right), \quad (1)$$

where α_i^2 represent the signal attenuation and N_0 is noise power density for i th beam. We assume uniform signal attenuation across each narrow spot beam. And also P is the allocated power and uniform for all beams. As the W_i is the i th beam bandwidth to be allocated, it is a considered factor to achieve the reasonable resource allocation and total capacity improvement in this paper. In the next section, we derive the beam

profile for bandwidth based on traffic demands and channel conditions to minimize the waste of the resource and to maximize the spectral efficiency, thereby achieving optimum beam allocation method.

3 Optimum Bandwidth Allocation (OBA)

3.1 Derivation of Optimum Beam Bandwidth Profile

As one of the metrics to evaluate the system performance of resource allocation over satellite downlinks, the authors in [2] addressed some tradeoff between different objects for system optimization. They derived the downlink multibeam capacity optimization problem and proposed a schematic method. Motivated by this paper, we formulate an optimum beam bandwidth profile of the parallel multibeams with respect to traffic distributions to achieve the reasonable fairness among users.

In this paper, we only focus on the problem of bandwidth allocation when the total traffic demands exceed the total system capacities. To be the best optimum case, it ought to match the traffic demand F_i and capacity C_i and means the gap between the two should be minimized as possible across the all spot beams, for $i=1,2,\dots,n$. In view of this, we adopt a square deviation cost function between capacities and traffic demands and formulate our beam bandwidth allocation problem as given here.

$$\text{Minimize } \sum_{i=1}^n (F_i - C_i)^2. \quad (2)$$

We have some constraints to solve this optimization problem. First, we do not exceed the more bandwidth required by traffic demand from each beam (or $C_i \leq F_i$) to prevent the unnecessary waste of resources. Second condition is subject to total bandwidth supply such as $\sum_{i=1}^n W_i \leq W_{total}$.

Applying the Lagrangian function as $L(W_i, \Lambda) = \sum (F_i - C_i)^2 + \Lambda(\sum W_i - W_{total})$, we have the optimum beam bandwidth profile W_i , which should satisfy as follow equation (3).

$$F_i - W_i \log \left(1 + \frac{\alpha_i^2 P}{W_i N_0} \right) = \frac{\frac{\Lambda W_i \ln 2}{2} \left(1 + \frac{\alpha_i^2 P}{W_i N_0} \right)}{W_i \ln 2 \left(1 + \frac{\alpha_i^2 P}{W_i N_0} \right) \log \left(1 + \frac{\alpha_i^2 P}{W_i N_0} \right) - \frac{\alpha_i^2 P}{N_0}}, \quad (3)$$

where Λ is a Lagrange multiplier which is determined by total bandwidth constraint. Nonnegative Λ means that it satisfies the constraint for $C_i \leq F_i$.

We need a verification process to confirm whether the beam bandwidth W_i which can be obtained from (3) is the optimum case or not, since it does not closed-form

solutions can be found out W_i in terms of F_i . For this, we provide the solution for W_i through the approximation process in the case of low and high signal-to-noise ratios (SNRs) region.

3.2 Approximation Formula to Verify Optimization Bounds

At the low SNR region of $\alpha_i^2 P/W_i N_0 \ll 1$ using the property $\log_2(1+x) \approx x/\ln 2$ for x is very small, we can derive the first order approximation formula from (3) as follow.

$$W_i = \begin{cases} \left(\frac{2\alpha_i^2 P}{\lambda N_0 \ln 2} \right) \left(F_i - \frac{\alpha_i^2 P}{N_0 \ln 2} \right), & \text{if } F_i > \frac{\alpha_i^2 P}{N_0 \ln 2} \\ 0, & \text{if } F_i \leq \frac{\alpha_i^2 P}{N_0 \ln 2} \end{cases} \quad (4)$$

Next, for the high SNR region of $\alpha_i^2 P/W_i N_0 \gg 1$, we use a truncated part of the Taylor series of the $\log_2(1+x) \approx (x/\ln 2) - (x^2/2\ln 2)$, and then we also get the third order approximation, given as

$$\frac{\lambda(\ln 2)^2}{(\alpha_i^2 P)^3} W_i^3 + \left[\frac{2}{(\alpha_i^2 P) N_0^2} - \frac{F_i(\ln 2)}{(\alpha_i^2 P)^2 N_0} \right] W_i^2 + \left[\frac{F_i(\ln 2)}{(\alpha_i^2 P) N_0^2} - \frac{2}{N_0^3} \right] W_i + \frac{(\alpha_i^2 P)}{2N_0^4} = 0. \quad (5)$$

In general, there are many methods to solve cubic polynomials and the form of the roots of cubic equation is determined by discriminant. After investigating the discriminant of (5), we know that it has a real root and two imaginary roots. Here we adopt the only real root to decide the optimization boundary. In the section for simulation part, we will compare these two approximations of (4), (5) and numerical solution (3).

3.3 Updating the Lagrangian Multiplier

From (3) and the constraint for Lagrangian multiplier which is determined by total bandwidth constraint, we have a formula for the Lagrangian multiplier Λ as follow.

$$\Lambda = \frac{2}{\ln 2} \left[F_i - W_i \log \left(1 + \frac{\alpha_i^2 P}{W_i N_0} \right) \right] * \frac{\ln 2 \left(1 + \frac{\alpha_i^2 P}{W_i N_0} \right) \log \left(1 + \frac{\alpha_i^2 P}{W_i N_0} \right) - \frac{\alpha_i^2 P}{W_i N_0}}{1 + \frac{\alpha_i^2 P}{W_i N_0}}, \quad (6)$$

As mentioned earlier, since the beam bandwidth profile is not yield close-form solutions, it needs an intuitive approximation method to find a closed form solution for W_i by using the relationship between total traffic demand and beam bandwidth profile. In

[3], the heuristic method to search the Lagrangian multiplier for the optimal power allocation is presented, and we use this method to find the optimal bandwidth W_i and Λ . First, if a beam requires sum of the traffic demands, F_{sum} , and then, total bandwidth W_{total} will be allocated the beam, we can calculate an initial value Λ_0 . Using binary search as rule of thumb, we set $\Lambda_{min}=\Lambda_0/10$ and $\Lambda_{max}=\Lambda_0*10$. We undergo a process to find the optimal Λ in the range of $\Lambda_{min}, \Lambda_{max}$ according to several different simulation scenarios.

$$\Lambda_0 = \frac{2}{\ln 2} \left[F_{sum} - W_{total} \log_2 \left(1 + \frac{\alpha_i^2 P}{W_{total} N_0} \right) \right] * \frac{\ln 2 \left(1 + \frac{\alpha_i^2 P}{W_{total} N_0} \right) \log_2 \left(1 + \frac{\alpha_i^2 P}{W_{total} N_0} \right) - \frac{\alpha_i^2 P}{W_{total} N_0}}{1 + \frac{\alpha_i^2 P}{W_{total} N_0}} \quad (7)$$

Inserting the Λ_0 to (3), the initial optimum beam bandwidth W_i^{opt} for each spot beam can be calculated. However, it is not definitive optimal values, and thus, we need the process to update these allocated bandwidths. A set of updating process is as follow.

- i) Save the sum of the allocated bandwidth $\sum W_i^{opt}$.
- ii) If $\sum W_i^{opt} > W_{total}$, then set $\Lambda_{min}=\Lambda$ and let $\Lambda=(\Lambda_{min}+\Lambda_{max})/2$. Find W_i^{opt} again using (3).
- iii) If $\sum W_i^{opt} < W_{total}$, then set $\Lambda_{max}=\Lambda$ and let $\Lambda=(\Lambda_{min}+\Lambda_{max})/2$. Find W_i^{opt} again using (3).
- iv) The updating process is repeated iteratively until $\sum W_i^{opt}=W_{total}$.

The optimum solution W_i^{opt} achieves the reasonable proportional fairness according to traffic demand for all spot beams. Eventually, we expect to improve the total capacity.

4 Simulation Result

This section presents the simulation results. For the simulation, we assume signal attenuation is uniform across each narrow spot beam and $\alpha_i^2=1$. In addition, the number of beams to be allocated is 28, and total bandwidth constraint is 100Hz. The signal power to noise power spectral density(P/N_0) for each beam is 200w. The minimum traffic demand is 5bps at the first beam, and it goes up by 5bps at each beam up to 140bps(at the 28th beam) like $\{F_i | F_1=5, F_2=10, \dots, F_{17}=135, F_{18}=140\}$.

Table 1. Total allocated bandwidth after optimum bandwidth allocation algorithm

j	1	2	3	4	5	6
$\sum W_i^{opt}$	327.4	77.8	130.5	98.6	112.4	105
j	7	8	9	10	11	12
$\sum W_i^{opt}$	101.7	100.1	99.2	99.5	99.7	99.9
j	13	14	15	16		
$\sum W_i^{opt}$	99.9	100.1	100.1	100		

Table 1 shows the results of total allocated bandwidth after optimum algorithm. It means we obtain the final optimum result at the 16th step ($\sum W_i^{opt} = W_{total}$). Next table 2 and figure 2 shows the updating process to search the optimal Λ finally. It looks unstable in the early stage, and then it has stable value gradually.

Table 2. Mortification process to find the optimal Lagrangian multiplier Λ

j	1	2	3	4	5	6
Λ	25.604	140.82	83.213	112.02	97.616	104.82
j	7	8	9	10	11	12
Λ	108.42	110.22	111.12	110.67	110.44	110.33
j	13	14	15	16		
Λ	110.27	110.25	110.26	110.27		

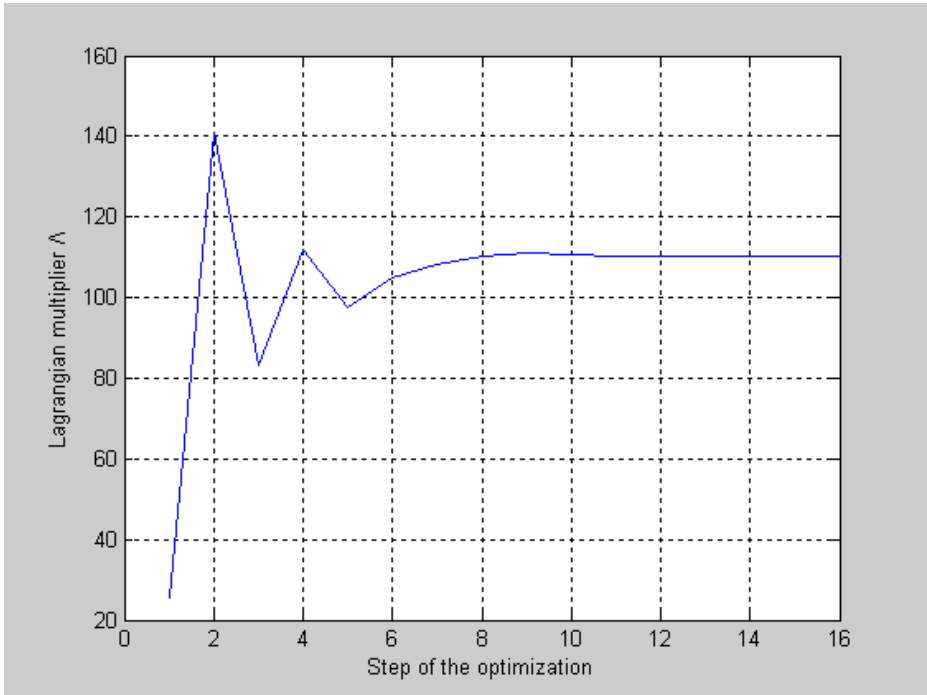


Fig. 2. Mortification process to find the optimal Lagrangian multiplier Λ

We will compare the approximated close-form answers for low and high SNR in (4) and (5) with the numerical solution of (3) to confirm the optimum distribution of beam bandwidth W_i as follow figure 3. We can confirm the bandwidth allocation of (3) is under the optimization boundary regions.

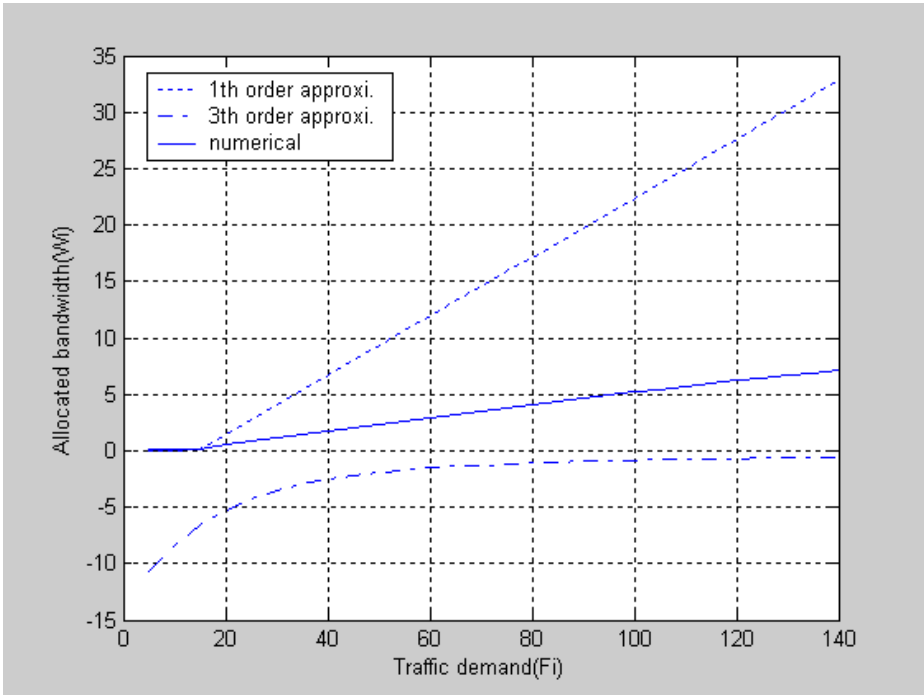


Fig. 3. Optimum beam bandwidth allocation W_i for demand F_i in (3) and its approximated close-form solutions in (4), (5)

5 Conclusion

In this paper, we formulate an optimum beam bandwidth profile of the parallel multibeams with respect to traffic distributions to achieve the reasonable fairness among users. In addition, we show the process to search the optimal beam bandwidth updating the Lagrangian multiplier by heuristics method. The simulation result show the proposed resource allocation scheme is under optimized boundary region to minimize the gap of the traffic demands and total capacity.

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