Comparison of Single and Dual Frequency GNSS Receivers in the Presence of Ionospheric and Multipath Errors

Danai Skournetou and Elena-Simona Lohan

Tampere University of Technology, P.O. Box 553 Tampere, Finland
{danai.skournetou,elena-simona.lohan}@tut.fi
http://www.cs.tut.fi/tlt/pos/

Abstract. In this paper, we test the performance of single and dual frequency GPS and Galileo GNSS receivers in terms of satellite-receiver range estimation. In particular, we focus on the effects caused by ionosphere and multipath propagation. Therefore, the available pseudoranges are assumed to be contaminated with first-order ionospheric delay and measurement errors produced in the code tracking stage. We used three dual-frequency methods, two ionospheric models (Klobuchar and NeQuick for GPS and Galileo receivers, respectively) and compared their ionosphere-corrected ranges on a Root Mean Square Error basis. The simulation results showed that a dual-frequency receiver is superior to a single-frequency one, only when the standard deviation of the measurement error is small and when the correlation factor between the two available pseudoranges is higher than −0.4.

Keywords: Global Navigation Satellite System, ionosphere, multipath, dual-frequency receivers.

1 Introduction

Almost three decades have passed since the first Global Navigation Satellite System (GNSS), commonly known as Global Positioning System (GPS), became available to the public. Since then, the amount of devices which are equipped with a GPS receiver has been continuously increasing. The growing interest on the position information has initiated the creation of new GNSSs with improved performance characteristics; among those, Galileo represents Europe's initiative to build it's "own" GNSS, expected to be ready by 2014 [1].

The main principle of satellite-based positioning lies on the trilateration method which requires the computation of at least three satellite-receiver ranges (a minimum of four ranges is needed in order to estimate the receiver clock bias). For an accurate range estimation, it is not enough to measure the difference between the received and transmitted times. Instead, the various error sources affecting the transmitted signal shall be accounted for and mitigated. Among those, the ionosphere is responsible for the signal's biggest delay [2]. More precisely, when the satellite signal travels through the ionospheric layer (located 50-1000 km above the Earth's surface) it is delayed due to

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the presence of charged particles (ions and electrons). The amount of the delay depends on two parameters: the frequency of the signal the Total Electron Content (TEC) [3].

In single-frequency receivers, TEC is estimated with the help of mathematical models whose accuracy is typically counterbalanced by their complexity. Among the various model reported in the literature, Klobuchar model is the one employed in most GPS receivers and which makes use of eight broadcast coefficients [4]. The NeQuick model is adopted by ITU-R and proposed for the future Galileo receivers [5]. Unlike Klobuchar model, NeQuick makes use of only three broadcast coefficients and it is claimed to be more accurate [6].

In dual-frequency receivers no modelling of the ionosphere is required because the availability of two signals which have undergone the same ionospheric effects is exploited. In the absence of measurement errors, first-order ionospheric delay can be estimated and fully mitigated via linear combination of the available pseudorange measurements [2]. The afore-mentioned advantage in combination with the advent of new GNSS signals (e.g., future Galileo and modernised GPS signals) and the decreasing cost of GNSS receivers can meet the growing demand of higher accuracy in mass-market receivers.

However, in practise the presence of measurement errors (i.e., due to multipath propagation effects) degrades the accuracy of ionospheric delay estimation. Moreover, the existing studies on the impact of multipath errors in the estimation accuracy of the ionospheric delay and consequently of the range estimation in the case of dual frequency methods have been rather limited [7]. In this paper, we attempt to shed some light on the above-mentioned problem. More precisely, we examine the effect of multipath errors in the estimation of ionoshere-corrected ranges by theoretically modelling the performance of single- and dual- frequency receivers and comparing the ionosphere-corrected ranges of various methods in terms of Root Mean Square Error (RMSE).

The remainder of this manuscript is organised in the following manner: Section 2 includes the description of the model used to analyse the effects of ionosphere and multipath propagation. Section 3 describes the simulation setup and presents the results. Finally, Section 4 summarises the most important findings of our study.

2 Model Description

In this section we describe the mathematical model used to represent dual-frequency receivers and the methods employed by such receivers to estimate and correct the delay caused by ionosphere. In what follows, we consider only the first-order ionospheric effects since they account for 99% of the total delay [3] and because the effect of the higher order terms can be considered negligible for the accuracy requirements of mass-market GNSS receivers.

The first-order ionospheric delay is defined as [2]

$$I_i = \frac{40.3}{f_i^2} TEC \tag{1}$$

where TEC is the total electron content measured in TEC Units (TECUs) with 1 TECU= 10^{16} electrons/m^2 and f_i is the i−th frequency for $i = 1, 2$. The first order ionospheric delay versus TEC for different carrier frequencies can be seen in Fig. 1.

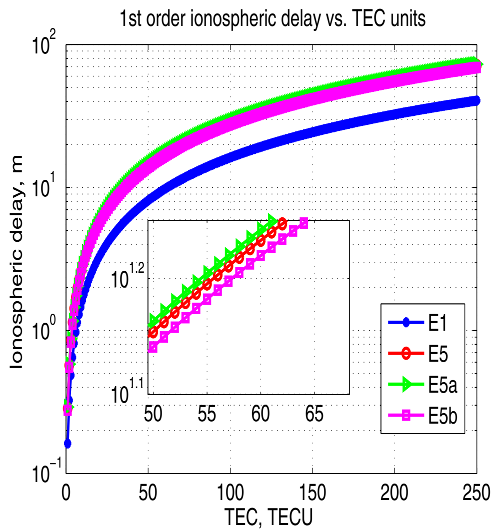

Fig. 1. First order ionospheric delay vs. TEC for E1 (1575.42 MHz), E5 (1189 MHz), E5a (1176.45 MHz) and E5b (1207.14 MHz) carrier frequencies

Considering a dual-frequency GNSS receiver, it is possible to model the available pseudoranges into matricial format as [7]

$$\begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{40.3}{f_1^2} \\ 1 & \frac{40.3}{f_2^2} \end{bmatrix} \begin{bmatrix} \rho \\ TEC \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{2}$$

For simplicity, we neglect other error sources in Eq. (2) because we want to focus on the impact of multipath errors in the estimation of the satellite-receiver range which is characterised by ionospheric delay. Moreover, we notice that most of the errors sources affecting the transmitted signals from a single satellite are the same since they travel through the same medium. So, common errors (e.g., ephemeris, tropospheric and clock errors) can be easily removed by subtracting one of the two pseudoranges from the other [2].

Equivalently, Eq. (2) can be represented in a compact manner as

$$\mathbf{r} = \mathbf{A}\mathbf{x} + \mathbf{e} \tag{3}$$

where $\mathbf{r}$ is the observation vector that contains the pseudorange measurements, $\mathbf{A}$ is a 2×2 matrix, $\mathbf{x}$ is the unknown parameter vector to be estimated and $\mathbf{e}$ is the measurement error vector. The measurement error represents the residue of the processing done in the code tracking stage. We notice that the code tracking error is different for different signals because it depends on signal-specific characteristics such as type (i.e. data or pilot), modulation, frequency, etc. and it represents mostly the effects of multipath propagation [7].

Unlike in the case of single-frequency receivers where the total electron content has to be modelled, dual-frequency receivers exploit the availability of two pseudoranges and discard the need for TEC modelling. In the absence of errors, TEC can be accurately estimated by a proper linear combination of the pseudoranges [8]. Then, the ionospheric-free ranges can be computed in a straightforward manner.

In the presence of errors (i.e., due to multipath delay tracking errors), the linear Least Square (LS) solution can be used. In this case, the unknown TEC and true range parameters can be estimated as [9]

$$\hat{\mathbf{x}}_{LS} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{r} \tag{4}$$

where T denotes the operation of transposition.

One limitation of LS method is that it does not account for physically invalid solutions. For example, it was found that the estimated TEC parameter can be a negative value which is against its physical meaning (i.e., the electron content can be only equal or greater to zero) [7]. In order to avoid the above-mentioned scenario, we can impose certain constraints for the estimated vector. This leads to the method commonly known as Constrained Least Square (CLS). More precisely, the idea is to minimise the squared difference between the observed data and $\mathbf{Ax}$, subject to the linear inequality constraint $\mathbf{A}\hat{\mathbf{x}}_{CLS} \geq \mathbf{b}$ (see Section 3 for the constraint chosen in our study).

In order to avoid the computationally heavy approach of CLS, a new method, called Brute Force Constraint (BFC), was proposed in [7]. The main idea of BFC is that within only two iterations we are able to estimate TEC and true range subject to the constraint of non-negative TEC. So, the complexity of BFC is reduced compared to the one of CLS method the BFC-based TEC estimates do not violate any physical rule.

3 Simulation Profile and Results

In this section, we compare the range estimation performance of single and dual-frequency receiver methods in terms of Root Mean Square Error (RMSE). The satellite systems of interest are the existing Navstar GPS and future Galileo. In the case of single frequency receivers, we assume that a GPS receiver employs the Klobuchar model [10, 4] for the estimation of TEC and a Galileo receiver utilises the Ne-Quick model [6, 11]. Unlike Klobuchar model, NeQuick does not make use of the thin-shell assumption; instead, it is a three-dimensional model that exhibits higher degree of realism.

For the testing of single-frequency methods, real data are required as the inputs to a certain ionospheric model. However, because we want to compare the theoretical performance of range estimation methods, we model the ionospheric delay estimation error of the single frequency methods as $e_{I,i} = \alpha I_i$, where α represents the percentage of the ionospheric delay which is not corrected (i.e., the estimation error of ionospheric delay). While the reported percentage values vary in the literature, we have here the most representative ones. More precisely, in the case of Klobuchar model, it is claimed that 50% of the ionospheric delay is corrected [12] (so, $\alpha = 50\%$). When NeQuick model is employed, up to 70% of the ionospheric delay can be corrected [13], leading to an ionospheric delay estimation error of $\alpha = 30\%$.

For the case of dual-frequency receivers, we focus on mass-market Galileo receivers. More precisely, we have chosen the E1-E5a frequency combination which appears to be

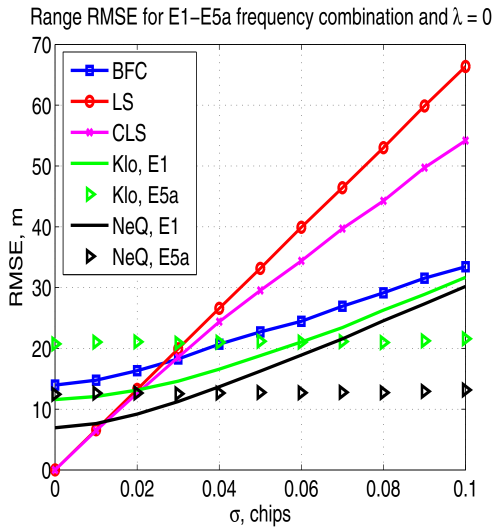

Fig. 2. RMSE vs. standard deviation of error for $i = 1, 2$) and $\lambda = 0$

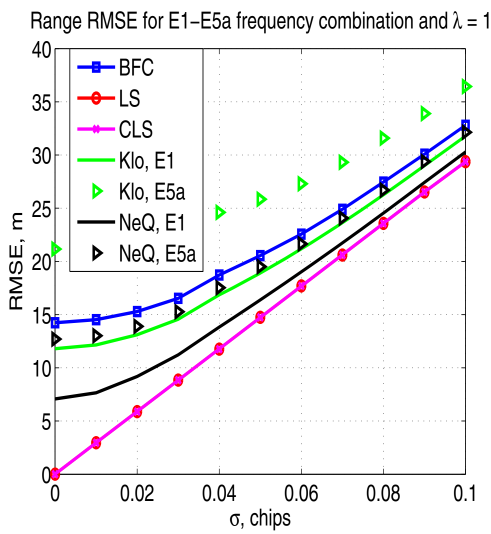

Fig. 3. RMSE vs. standard deviation of error for $\lambda = 1$

the most suitable [7]. In addition,this choice of frequencies is advantageous because it is overlapping with the existing L1 and L5 bands, thus facilitating the design of a joint GPS/Galileo receiver [14]. For the mitigation of ionospheric delay we used the three methods described in Section 2: LS, CLS with constraint vector $\mathbf{b} = [0\ 0]^T$ and BFC (more detailed description of these algorithms and the reasoning for choosing the above constraint vector are included in [7]).

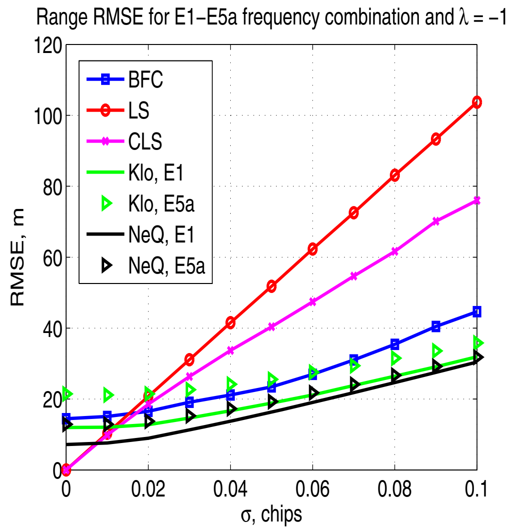

Fig. 4. RMSE vs. standard deviation of error for $\lambda = -1$

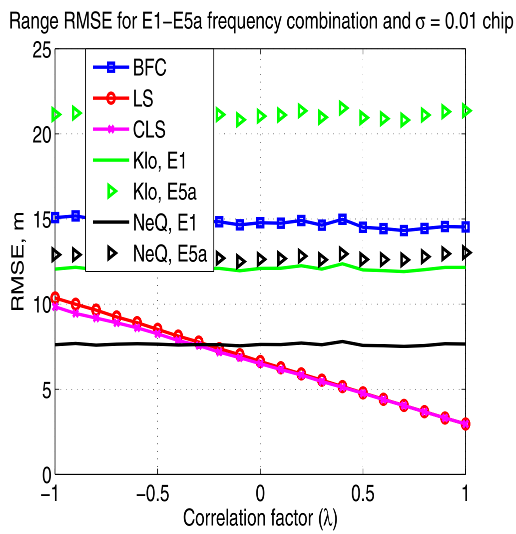

Fig. 5. RMSE vs. correlation factor for $\sigma = 0.01$ chip

In order to compute the RMSE performance values, we generate 2000 random realisations of the signal and of the measurement errors. More precisely, the true range ρ is uniformly distributed between 18000 and 25000 km and the TEC is uniformly distributed between 1 and 250 TECU. The limits of the TEC parameter have been chosen in such a way that typical values encountered in various latitudes are included [15, 16, 17]. In our simulations, we don't employ a specific multipath channel profile. Instead, we model the measurement error as the tracking error attributed to the multipath propagation effects. More precisely, the error e is modelled as a random variable that follows the normal distribution (the assumption of normal distribution has been commonly encountered in the literature [18, 19, 20, 21] and is used here for simplicity; study of different error distributions belongs to our future plans). More precisely, the errors are distributed according to $e_i \sim \mathcal{N}(\mu_{e_i}, \sigma_{e_i})$, where $\mu_{e_i} = 0$ and σ_{e_i} takes values from 0 to 0.1 chips with a step of 0.01 chip. We remark that for the sake of simplicity, the measurement errors in E1 and E5a are assumed to follow the same distributions (however, it was recently found in [22] that the tracking error distributions of L1 and E5a signals may differ in different channel profiles and the effect of such difference is another interesting research topic). In addition, we notice that because E1 signal has smaller chip rate than the other three, a standard deviation error of 0.01 chips translates into 2.932 m. of error for E1 signal and into 0.293 m. for E5a (for the sake of fair comparison, we modelled the measurement errors at chip level because E1 and E5a signals have different chip rates).

In Fig. 2 we see the RMSE values for the case of uncorrelated pseudorange measurements and zero mean error. We observe that when the standard deviation of error is smaller than 0.01 chip the dual-frequency methods (LS and CLS) perform the best. However, when the standard deviation increases further, a single-frequency Galileo receiver operating in E5a carrier frequency would perform the best. When the pseudoranges are characterised by a full positive correlation (see Fig. 3), LS and CLS dual-frequency methods perform always better than the single-frequency ones (we notice that while the assumption of fully positively correlated pseudoranges might be extreme, it has been reported in [23]).

When the pseudoranges have a full negative correlation, the relative performance of the dual-frequency methods remains the same than in the case of no correlation (see Fig. 4). On the other hand, we notice that in case of dual-frequency receivers the choice of the best frequency changes. Finally, the performance of the methods for different correlation factors can be see in Fig. 5. In particular, we observe that the RMSE of LS and CLS methods decreases with increasing degree of correlation while in the case of BFC and single-frequency methods, the RMSE is not affected by the varying correlation factor.

4 Conclusions and Future Plans

In this paper, we investigated the performance of single- and dual- frequency receivers in terms of satellite-receiver range estimation and under the assumption that the received signals have been contaminated due to ionospheric and multipath propagation effects. More precisely, we examined the performance of three dual-frequency receiver methods: The first one is the Least Squares (LS) method which estimates the unknown total electron content and range by trying to minimise the sum of squared distances between the observed responses in the observation set, and the responses predicted by the linear approximation. The second method is a variant of LS, called Constraint LS (CLS) which imposes certain constraints on the solution and therefore, it is more complex. The third method is called Brute Force Constraint (BFC) and it was proposed by the authors due to its low computational burden.

The performance of the above-mentioned methods was compared with the performance of single-frequency GPS and Galileo receivers in term of Root Mean Square Error (RMSE). Furthermore, we assumed that the Klobuchar and Ne-Quick model were used to estimate the electron content, in the single- and dual- frequency receivers, respectively. The results showed that when the pseudoranges are uncorrelated, the LS and CLS methods are superior to single frequency methods only in the case when the standard deviation of the error is smaller than 0.01 chip. If the standard deviation is higher than 0.01 chip, a single-frequency Galileo receiver operating at E5a carrier frequency would perform the best. Moreover, dual-frequency methods perform the best when the pseudoranges are characterised by a full positive correlation. Finally, the simulation results showed that the RMSE of LS and CLS methods decreases with increasing correlation factor while the in the case of the other methods, RMSE is only weakly affected.

We notice that our simulations were done under limited assumptions, such as theoretical modelling of multipath errors and equal error variances on E1 and E5a signals; more remains to be investigated about the possible advantage of a dual-frequency receiver under more realistic assumptions.

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