

# Unified Multibeam Satellite System Model for Payload Performance Analysis

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**Abstract.** This paper presents a novel unified multibeam satellite system model for the performance analysis of different satellite payloads. The model allows the analysis in terms of Signal to Interference plus Noise Ratio (SINR) and Co-Channel Interference (CCI). Specifically we formulate the SINR as a function of the multibeam geometry for a given user location granularity. Furthermore, we apply our model to analyze the performance of two novel satellite payloads with respect to current conventional (CONV) ones using fixed frequency reuse and per-beam frequency/time assignment: the so-called “flexible” (FLEX) payload and the “beam-hopping” (BH) which allow a flexible per-beam frequency assignment and a flexible per-beam time assignment respectively. Our results show that CONV payloads achieve higher SINR values than BH and FLEX payloads at the expense of lower bandwidth assignment to the beams. Leading, therefore, to a trade-off, between received signal quality and resource management flexibility.

**Keywords:** Co-channel, interference, payload, multibeam and model.

## 1 Introduction

Current trends of multibeam satellite systems focus on the design of more efficient systems in order to achieve not only larger throughputs but also flexible resource management. There already exists an amount of work corresponding to this topic, such as the implementation of new techniques, e.g. power control [1], Forward Error Correction (FEC) codes at physical or link layer [2] and Adaptive Coding and Modulation (ACM) techniques [2]. In addition, another way to achieve larger throughputs is by increasing the number of beams. However, this leads to an increment of the CCI since the same frequency is reused by a subset of beams.

This effect was noticed in reference [3] where pre-coding schemes were used in order to overcome the CCI. Also references [1] and [4] focus on algorithms for satisfying user requirements and performing the multiple access respectively taking into account the minimization of the CCI. Therefore, studying the CCI is of relevant importance in satellite systems in order to validate new multibeam satellite payload models.

The aim of this paper is to formulate the unified expressions for multibeam satellite systems in order to compare the performances of any payload models. Based on the general expressions we compare the performance of three different satellite payloads, a conventional payload model (CONV) and two novel payload models named flexible (FLEX) and beam-hopping (BH). All three models are designed for the multimedia broadband satellite services. This comparison is carried out in terms of SINR and CCI.

The remainder of the paper is organized as follows: Section 2 introduces the derivation of the general system model. In Section 3 we introduce three payload models which are designed for the broadband multimedia and IP services. Finally in Section 4 we evaluate the performance of the payload models. Section 5 draws the conclusions.

## 2 Derivation of a Unified System Model

In this section, we first depict preliminary issues for the general system model derivation, i.e. the multibeam geometry and chosen antenna models. Subsequently, we express the steps to model the system in a general and unified way.

### 2.1 Multibeam Satellite Geometry

The multibeam satellite geometry interested in this paper is shown in Fig. 1, without loss of generality we focus on an example with two beams. Assuming that the position of a specific user (e.g.  $P$ ) in a beam  $i$ , a beam center  $BC_j$  and the satellite  $SL$  is known, we can compute the distances  $(P, BC_j)$ ,  $(P, SL)$  and  $(SL, BC_j)$ . Therefore, we can derive the angle  $\theta_{ij}$  between the link  $(P, SL)$  and  $(SL, BC_j)$  by applying the cosine law.

$$\theta_{ij} = \arccos\left(\frac{(P, BC_j)^2 - (SL, BC_j)^2 - (SL, P)^2}{- (SL, BC_j)(SL, P)}\right). \tag{1}$$

where  $\theta_{ij}$  is the angle between the user location inside the beam  $i$  and the corresponding beam center  $BC_j$  (e.g.  $\theta_{12}$  shown in Fig. 1). Note that we can also obtain  $\theta_{ii}$  (e.g.  $\theta_{11}$  in Fig. 1) in the same way.

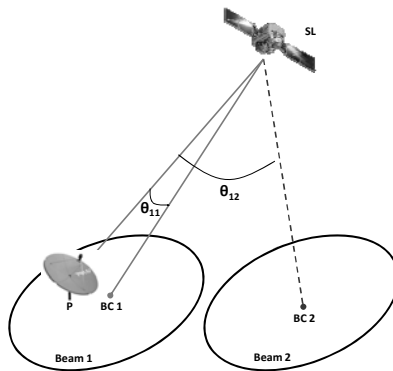


Fig. 1. Considered geometry

## 2.2 Analytical Antenna Models

We now present the two analytical antenna models that generate the multibeam coverage.

The first one is the Single Feed per Beam Network (SFBN) antenna model with combined transmission and reception antennas, i.e. each of the beams has a dedicated feed element to generate the beam and the same antennas are used for signal transmission and reception. In the analyzed payload models, both CONV and BH structures implement SFBN antenna model which is analytically expressed as in [3]:

$$G(\theta) = G_{\max} \left( \frac{J_1(u)}{2u} + 36 \frac{J_3(u)}{u^3} \right)^2 \quad (2)$$

where  $u = 2.07123 \frac{\sin \theta}{\sin \theta_{-3dB}}$ , being  $\theta_{-3dB}$  the half angle power bandwidth,  $G_{\max}$  the antenna boresight gain and  $J_1$  and  $J_3$  are the Bessel functions of first and third kind respectively.

The other one is the Array Fed Reflector (AFR) antenna model using separated transmission and reception antennas, i.e. we have fewer antenna elements than beams, and the beams are generated through a Digital Beam Forming Network (DBFN). Different antennas are used for transmission and reception of the signal. FLEX payload structure implement the AFR model which can be modeled as in [5]:

$$G(\theta, \phi) = \sum_{i=1}^N c_i g_i(\theta, \phi) \quad \text{for} \quad \sum_{i=1}^N |c_i|^2 \quad (3)$$

where  $N$  are the number of elements in the AFR antenna,  $c_i$  is a complex excitation coefficient and  $g_i(\theta, \phi)$  is the secondary component beam directivity.

## 2.3 SINR Derivation

In this subsection we introduce the formulation of the general multibeam satellite system model. We first define the overall channel matrix  $\mathbf{H} \in \mathbb{C}^{k \times k}$  which is composed of two terms: (1) the satellite antenna gains matrix  $\mathbf{G} \in \mathbb{C}^{k \times k}$  which depends on the angle  $\theta$ , (2) the link budget matrix  $\mathbf{A} \in \mathbb{C}^{k \times k}$ . Subsequently, the received signal model and SINR can be formulated to study the CCI.

We adopt following notations:

- Vectors are set in bold lowercase letters.
- Matrixes are set in bold uppercase letters.
- Superscript  $(\cdot)^T$  denotes the transpose of a vector or matrix in  $(\cdot)$ .
- $\text{diag}(\mathbf{x})$  stands for a diagonal matrix with the elements of  $\mathbf{x}$  on its main diagonal.

The scenario is shown in Fig. 2, where a user in the interested beam  $i$  (e.g. beam 1 in the figure) is being interfered by any number of beams, e.g.  $k$ . The desired signal power level depends on the angle,  $\theta_{ij}$ , where  $i = j$  ( $\theta_{11}$  in the figure), of the user with its beam center. The interference signal power level depends on the  $\theta_{ij}$ 's where  $i \neq j$  ( $\theta_{12}$  to  $\theta_{1K}$  in the figure).

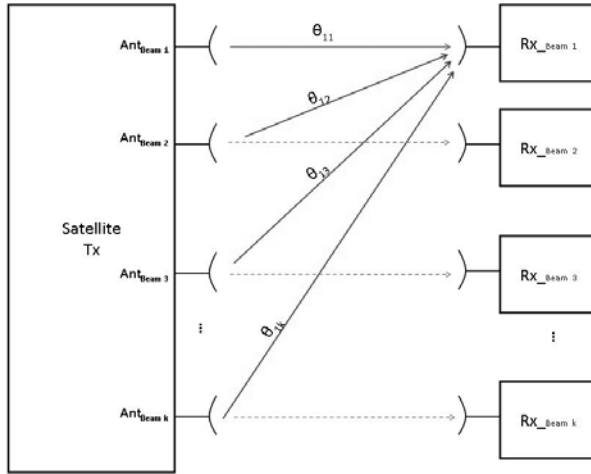


Fig. 2. Considered scenario

Let the symbols transmitted to user  $i$  inside the coverage of beam  $i$  be defined as  $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{iM}]$ .

Let also define the link budget matrix  $\mathbf{A} \in C^{k \times k}$  and the channel matrix gain  $\mathbf{G} \in C^{k \times k}$  which includes the satellite antennas gains as:

$$\mathbf{A} = \text{diag}(\sqrt{\beta_1}, \sqrt{\beta_2}, \dots, \sqrt{\beta_k}). \tag{4}$$

$$\mathbf{G} = \begin{pmatrix} g_{11} & g_{12} & \dots & g_{1k} \\ g_{21} & g_{22} & & g_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ g_{k1} & g_{k2} & \dots & g_{kk} \end{pmatrix}. \tag{5}$$

where:

- $\beta_i = OBO_{hpa} L_{sat} L_{down} G_{gt}$  where the parameters show the gain and losses which is the gain and losses terms that do not depend on the angle  $\theta$ ,  $OBO_{hpa}$  is the Output Back-Off of the High Power Amplifier (HPA),  $L_{sat}$  is the satellite repeater output losses,  $L_{down}$  is the free space losses and the additional rain, polarization, atmospheric and scintillation losses of the FWD downlink, and  $G_{gt}$  is the ground terminal antenna gain .
- $g_{ij} := \sqrt{g(\theta_{ij})}$  is the square root of the antenna gain between the satellite transmitter antenna for beam  $j$  and beam  $i$ , being  $\theta_{ij}$  the angle that forms the receiver in beam  $i$  towards the spot beam center  $j$  as seen from the satellite.

Hence, we can formulate the overall channel matrix  $\mathbf{H}$  as:

$$\mathbf{H}=\mathbf{A}\mathbf{G}=\begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1k} \\ h_{21} & h_{22} & & h_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ h_{k1} & h_{k2} & \cdots & h_{kk} \end{pmatrix}. \quad (6)$$

where the element of  $\mathbf{H}$ , e.g.  $h_{ij}$  is defined as  $h_{ij} = \sqrt{\beta_i g(\theta_{ij})}$ . Note that the definition of matrix  $\mathbf{A}$  and matrix  $\mathbf{G}$  can help us to separate the overall channel matrix into two terms, one does not depend on the  $\theta$  (i.e.  $\beta_i$ ) whilst the other one depends on  $\theta$  (i.e.  $g(\theta_{ij})$ ). The reason being, we can evaluate the CCI at beam level as a function of the angle  $\theta$ .

Subsequently we can express the received symbols  $\mathbf{y}_i(\theta) \in C^{M \times 1}$  for a user in a beam  $i$  by separating the received signal from non-desired signal as in equation (7).

$$\mathbf{y}_i(\theta) = \sqrt{P_{sat}} h_{ii} \mathbf{x}_i + \sum_{j=1, j \neq i}^{j=k} \sqrt{P_{sat}} h_{ij} \mathbf{x}_j + \mathbf{n}_i. \quad (7)$$

where the term  $\sqrt{P_{sat}} h_{ii} \mathbf{x}_i$  is our desired signal, the term  $\sum_{j=1, j \neq i}^{j=k} \sqrt{P_{sat}} h_{ij} \mathbf{x}_j$  is the co-channel interference and the term  $\mathbf{n}_i$  is a column vector of zero mean and complex circular noise with variance  $N$ .

By replacing  $h_{ij}$  with  $\sqrt{P_{sat} \beta_i g(\theta_{ij})}$  we can obtain the following expression:

$$\mathbf{y}_i(\theta) = \sqrt{P_{sat} \beta_i g(\theta_{ii})} \mathbf{x}_i + \sum_{j=1, j \neq i}^{j=k} \sqrt{P_{sat} \beta_i g(\theta_{ij})} \mathbf{x}_j + \mathbf{n}_i. \quad (8)$$

The SINR can be derived from equation (7) for a specific user in beam  $i$  by assuming that the power of the transmitted symbols is normalized,  $E[|\mathbf{x}_i|^2]=1$ .

$$SINR_i(\theta) = \frac{P_{sat} |h_{ii}(\theta)|^2}{\sum_{j=1, j \neq i}^{j=k} (P_{sat} |h_{ij}(\theta)|^2) + N_i} \quad (9)$$

By replacing  $h_{ij}$  with  $h_{ij} = \sqrt{\beta_i g(\theta_{ij})}$ , equation (9) can be reformulated as:

$$SINR_i(\theta) = \frac{P_{sat} \beta_i g(\theta_{ii})}{\sum_{j=1, j \neq i}^{j=k} (P_{sat} \beta_i g(\theta_{ij})) + N_i} \quad (10)$$

Regarding the obtained expressions (9) and (10) we have to note that:

- The expression of the received signal, i.e.  $\mathbf{y}_i(\theta)$ , and the signal to interference plus noise ratio, i.e.  $SINR_i(\theta)$ , depend not only on the  $\theta_{ii}$  where  $i = j$  (i.e. the angle between user  $i$  and its beam center  $i$ ) but also on the  $\theta_{ij}$  where  $i \neq j$  (i.e. the angle between user  $i$  and each of the interferers  $j$ ). Thus, we have expressed the received signal and the SINR as a function of  $\theta$ , which is the objective of this subsection.
- $\beta_i = OBO_{hpa} L_{sat} L_{down} G_{gt}$  depends on the system payload design. We can extract specific SINR expressions for each of the payloads,  $SINR_i^{CONV}(\theta)$ ,  $SINR_i^{FLEX}(\theta)$  and  $SINR_i^{BH}(\theta)$ , by replacing  $\beta_i$  with  $\beta_i^{CONV}$ ,  $\beta_i^{FLEX}$  and  $\beta_i^{BH}$  respectively where the superscripts *CONV*, *FLEX* and *BH* stand for the acronym of each of the payloads we will present in Section 3.

### 3 Payload Models

The aim of this section is to describe three different payloads models which are designed for the multimedia and IP broadcasting services in a multi-star access network, using Digital Video Broadcasting over Satellite second generation (DVB-S2) in the FWD link and Digital Video Broadcasting Return Channel over Satellite (DVB-RCS) in the return (RTN) link.

We first study the current operating payloads in multibeam satellite systems (Conventional payload or CONV in the equations) in order to have reference for the comparison with the other two payloads. Subsequently, we study the flexible payload model where the carrier allocation is fully flexible for each beam (Flexible Payload or FLEX in the equations). Finally we introduce the beam-hopping payload model, in which a subset of beams can be illuminated simultaneously during each timeslot (Beam-hopping payload or BH in the equations).

Regarding to the satellite payloads configuration and performance evaluation, more results can be found in [6].

#### 3.1 Conventional Payload

Conventional payload, abbreviated CONV, is used for the classical MF-TDM transmission schemes where the total bandwidth is divided into a fixed number of portions. Each beam can be assigned one of the portions. Portions of the bandwidth (carrier slots) can be reused or not. The elements forming part of the conventional FWD link payload can be seen in Fig. 3.

After the uplink signal filtering of each polarization output, the antenna elements are connected to a 2 for 1 redundant Low Noise Amplifier (LNA). Depending on the frequency plan, more than one type of Down Converter (DOCON) could be needed, so the splitter performs the action of sending the signal to the correct DOCON. Then, the DOCONs down-convert each of the frequency segments. Depending on the number of gateways and the number of polarizations, the number of inputs and outputs of the DOCONs could change.

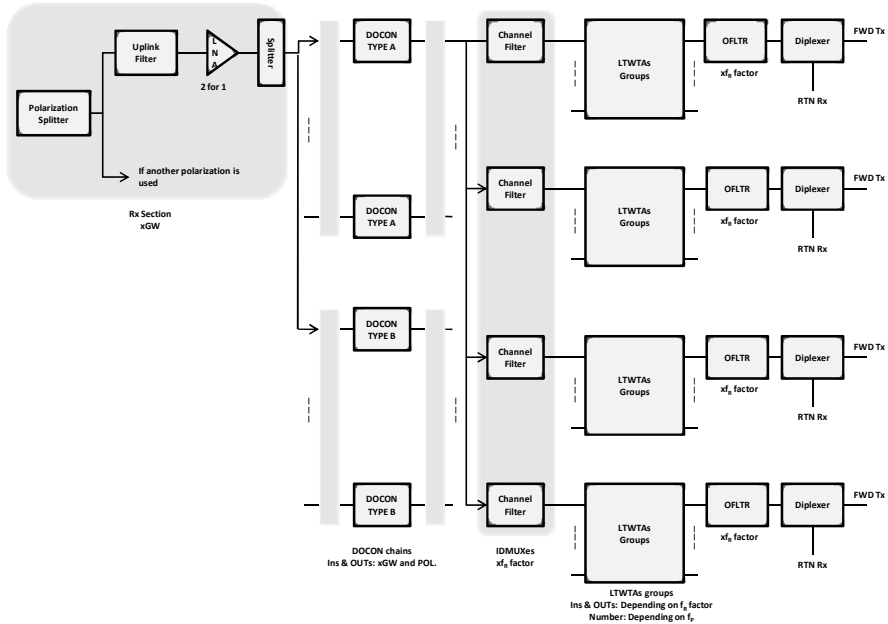


Fig. 3. Conventional payload

Then Input Demultiplexers (IDMUXes) separate the channels assigned to each user link beam, the needed number of IDMUXes is at least the same as the frequency reuse factor. A group of Linear Traveling Wave Tube Amplifiers (LTWTAs) are used to provide the final amplification of the channels and Output Filters (OFLTRs) are used to limit the inter-modulation and harmonics high amplification effects.

### 3.2 Flexible Payload

Flexible payload, or just FLEX, is used in Non Orthogonal Frequency Reuse (NOFR) air interfaces where a ground cell can allocate a variable number of carriers depending on the traffic requirement. The elements constituting the flexible FWD link payload can be seen in Fig. 4.

In the FWD link, firstly each polarization output signal is amplified by a LNA, then the DOCONS down-convert the received signals to the C-band frequency, consequently the On Board Processor (OBP) can process the converted signals. The Intermediate Filters (IFLTRs) are applied to limit the out of band spurious emissions.

The OBP performs the following actions:

- Spectral isolation of the individual modulated user channels that compose each Frequency Division Multiplexing (FDM) multiplexed, multicarrier, gateway signal.
- Routing and steering of the complex samples that compose the uplink carriers signals received on FWD uplink to the destined FWD downlink Digital Beam Forming Network (DBFN) in order to generate the subsequent FWD downlink signals.

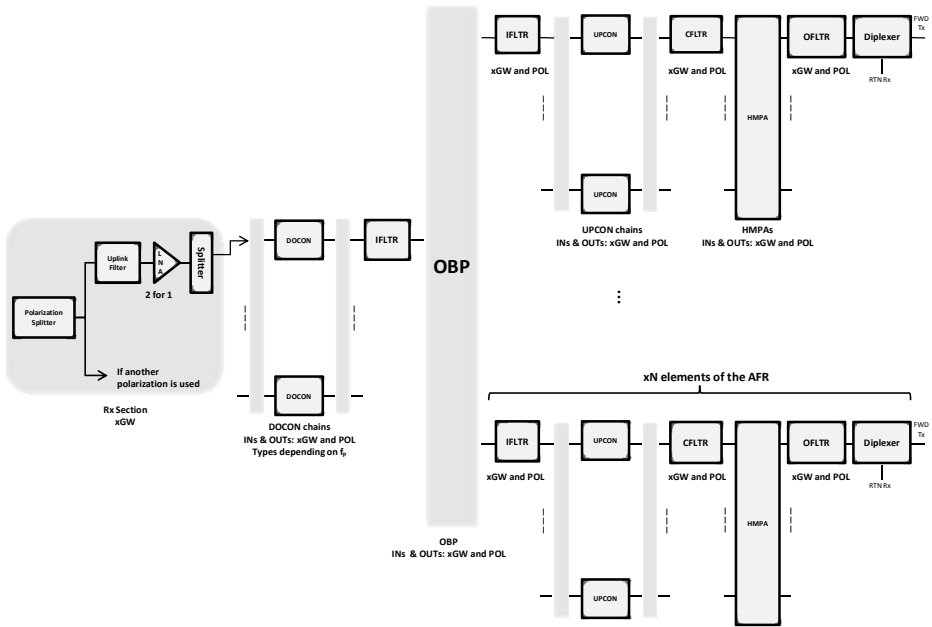


Fig. 4. Flexible payload

- Spatial filtering of the complex samples that compose the uplink carriers signals to generate the subsequent constituent beam signals to be applied to the antenna elements.
- Frequency synthesis of the spatially filtered element beam signals to generate the FDM multiplexed, multicarrier element signal to be applied to each of the antenna elements that compose the transmission antenna array.

The signals from the output of the OBP are then up-converted to the downlink frequencies by the Up Converters (UPCONs) and filtered by the Chanel Filters (CFLTRs) to limit the out of band spurious emissions. Hybrid Matrix Power Amplifiers (HMPAs) composed of LTWTAs are used to amplify the signals that feed the antenna elements. Signals are filtered with OFLTRs before transmitted to limit the noise in the receive frequency band and to limit the spurious emissions.

### 3.3 Beam-Hopping Payload

Beam-hopping payload, abbreviated BH, is used in air interfaces where the total bandwidth is used in some specific beams during a timeslot. The elements in the beam-hopping FWD link payload can be seen in Fig. 5.

In the FWD link the signals go through the 2 for 1 LNAs, then are down-converted to the OBP C-band and processed by the IFLTRs to limit the out of band spurious emissions.



The OBP performs the following actions:

- Spectral isolation of the individual, phase modulated carriers signals that constitute each FDM multiplexed, multicarrier gateway signal.
- Grouping the carriers received on the FWD uplink into FWD downlink sets.
- Frequency synthesis of the FWD downlink carrier sets to generate the subsequent FDM multiplexed, multicarrier signals. These synthesized multicarrier signals are identified as beam-hopping signals.
- Application of the beam-hopping signals to the antenna elements.

The signal at the output of the OBP is up-converted from the OBP C-band to the FWD downlink frequency by the UPCON, filtered and amplified by HMPAs. The signal is filtered with OFLTRs before sending to the antenna feed elements to limit noise and harmonic distortion.

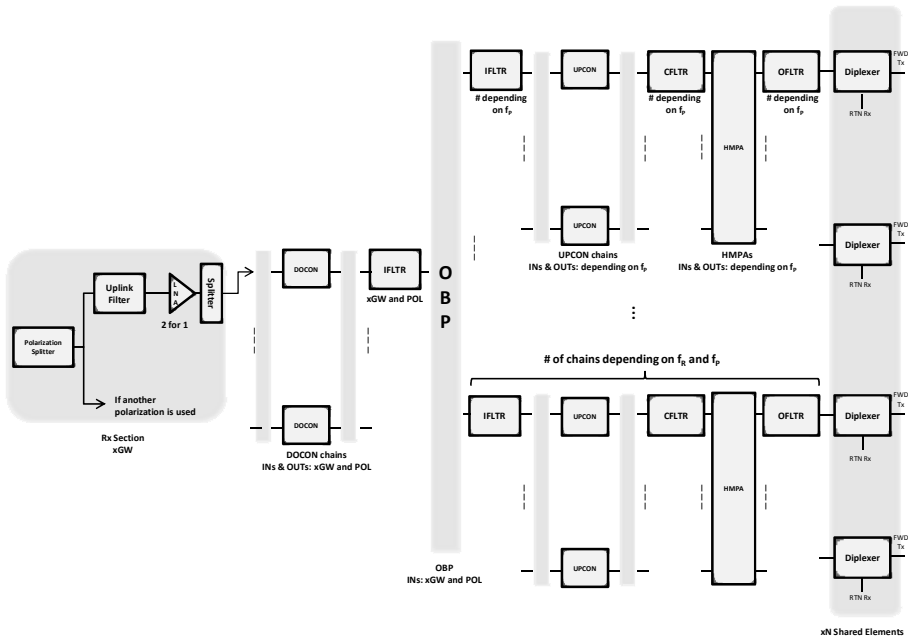


Fig. 5. Beam-hopping payload

## 4 Numerical Results

In order to evaluate and compare performance of the payload models presented above, in a realistic multibeam scenario, we will study the CCI and the  $SINR(\theta)$ . Note that, given space constraints, herein we only discuss the comparative performance evaluation of CONV and BH payloads. However, we note that the authors in [7] have shown that the flexible payload and beam-hopping payload are dual of each other.

We assume a 70-beam multi-star access system scenario. For each of the beams we analyze:

- The effect of the interference in the received SINR with respect to the number of adjacent interfering beams.
- The effect of the interference in the SINR with respect to all non-adjacent beams.

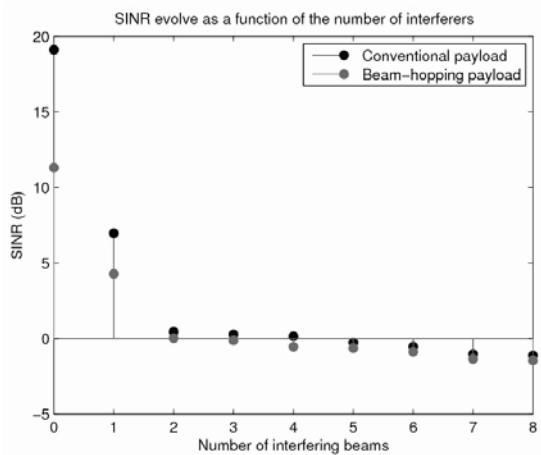
The system parameters are shown in Table 1 and the payload parameters of the CONV and BH payload parameters are extracted from [6].

**Table 1.** System parameters for the simulations

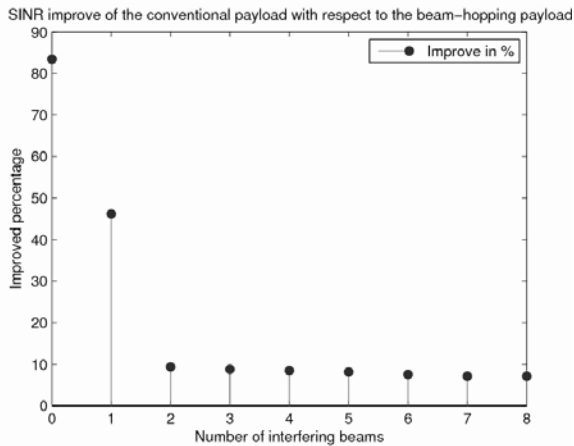
Parameter	Value
Orbit	GEO
Satellite Position	0° Long, 0° Lat
Frequency Band	19.50GHz
Modulation	8PSK
System Bandwidth	500MHz
Frequency reuse factor	17.5
$\theta_{-3dB}$	0.249°

### 4.1 Effect of Adjacent Interfering Beams

Fig. 6 shows the average received SINR for the conventional CONV and BH payload as a function of the number of adjacent interfering beams. Fig. 7 shows the improvement of the average SINR in percentage in the conventional payload with respect to the BH payload for a user located inside the coverage of the beam with coordinates 14.25° Latitude and 50.75° Longitude.



**Fig. 6.** Average SINR as a function of the number of interferers



**Fig. 7.** SINR increase of the conventional payload with respect to the beam-hopping payload

From the figures above it can be observed:

- Case with 0 interferers is equivalent to the received SNR in the beam of interest.
- Adjacent interfering beams cause a fast decrease of the received SINR for both payloads; however, differences are lower as the number of adjacent beams increase.
- For any number of adjacent interfering beams CONV payload is less affected by CCI than BH payload and hence by results obtained in [7] than FLEX payload since the bandwidth assigned to beams is higher in BH and FLEX schemes.

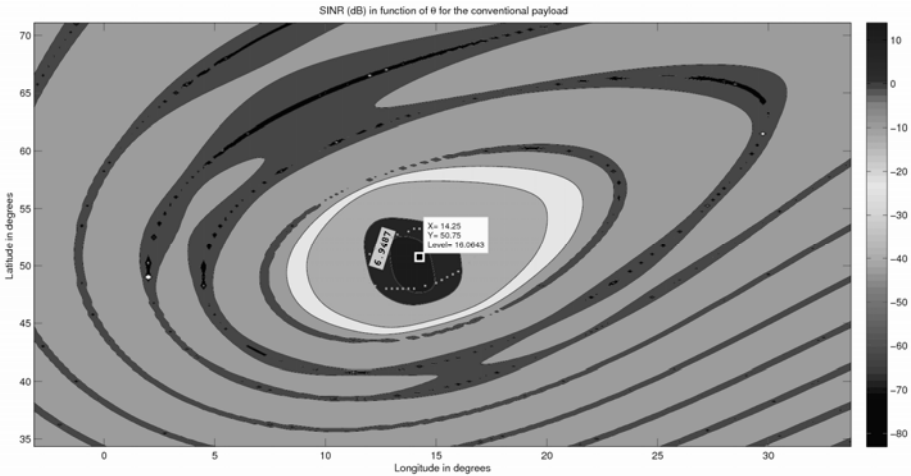
Therefore, it should be avoided to assign the same frequency band to adjacent beams in the CONV payload, and to illuminate at the same time adjacent beams in the BH payload. Besides we can note that CONV payload achieves higher SINR's basically because the amount of bandwidth assigned to each beam is lower than in the BH payload where we assign all the bandwidth to each beam. This bandwidth assignment is done in order to satisfy user requirements in a more efficient and flexible way rather the fixed conventional way used in CONV model. Hence there is a clear trade-off between bandwidth assignment and signal strength and by extension between throughput and signal strength. This means, if we want to achieve larger throughputs, we have to assign more bandwidth to each beam, in order to deal with broadband traffic, but received signal power will be lower because of the noise bandwidth. It is worth mentioning that this trade-off is not a bad feature for the novel payloads, as a uniform quality throughout the coverage might not be necessary.

#### 4.2 Effect of Non-adjacent Interfering Beams

In this subsection we show the effect on the SINR in the beam of interest when we set non-adjacent beams following a typical 4 colored frequency reuse scheme in the conventional payload (for 70 beam frequency reuse factor 17.5). In order to obtain

comparison we will illuminate the same beams in the beam-hopping payload and compare the obtained results.

Obtained SINR for CONV payload can be seen in Fig. 8.



**Fig. 8.** SINR in the beam of interest for a set of non-adjacent interfering beams using a four colored frequency reuse scheme

In the Fig. 8, the grey line indicates the original contour of the beam when there are no interferers (3dB loss). Note that within the original contour of the beam, now the SINR values can differ in 10dB as show the contours. However the SINR levels received in zones close to the center of the beam are big enough to ensure the correct reception of the signal.

Simulations results (not shown here for matter of lack of space) let us draw the same conclusions for the BH payload, a similar interference pattern is obtained. Nevertheless, when illuminating the same set of beams assigning all the bandwidth to each of them, the SINR value obtained in the center of the beam is 6dB under the value of the conventional payload, i.e. as in the adjacent interfering beams case, BH payload is more affected by the CCI than CONV payload. As explained before, this is produced because BH and FLEX schemes assign larger bandwidth to each beam, hence the noise bandwidth is bigger. So, under this scenario we also find the bandwidth signal strength throughput trade-off only that the SINR decrease is not produced in such a drastic way as the interfering beams are now further.

## 5 Conclusions

In this paper we have presented a unified system model for multibeam satellite systems. The model allows the performance analysis of different payloads in terms of received signal strength and co-channel interference. The model is easy to use as it identifies the key parameters of the payloads to be analyzed and how they should be included in the model.

We have applied our model for the performance analysis of two novel satellite payloads with respect to current conventional (CONV) ones: the so-called “flexible” (FLEX) payload and the “beam-hopping” (BH), both described in the paper. The first one allows a flexible per-beam frequency assignment while the second one allows a flexible per-beam time assignment. This flexibility is lacking in current CONV payloads, with fixed frequency reuse and per-beam frequency/time assignment.

The numerical results we have obtained with our developed unified model indicate that the CONV payload achieves better received signal strength and co-channel interference management throughout the coverage than the BH novel payload. This means that a trade-off exists between received signal quality and resource management flexibility. The reason for this is that the new payloads can accommodate larger bandwidths per beam, which is an advantageous feature for handling broadband traffic. This trade-off is actually not a bad feature for the novel payloads as a uniform quality throughout the coverage might not be necessary. For lack of space we have not included the numerical results for the FLEX payload, which show the same trend as the BH and it can be further justified by the duality between both payloads ([7]).

Our results are fully in line with the results in the related papers [1] and [7], which focus on the resource management algorithms of the proposed payloads.

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