Design of MIMO Satellite System: Inter-antenna Spacing Determination and Possible Enhancement of Capacity

Boujnah Noureddine[1] and Issaoui Leyla[2]

[1] Politecnico di Torino, Electronic Department, Turin, Italy
boujnah_noureddine@yahoo.fr
[2] Ecole Superieure des Communications de Tunis, MEDIATRAN, Tunisie
leyla.issaoui@gmail.com

Abstract. MIMO(Multiple Input Multiple Output) technology was found to exploit the channel richness and spatial diversity, its possible to increase data throughput and reduce the error probability[1]. MIMO channel capacity was derived by Teletar in [2], MIMO find its application in many technology such as HSPA and WLAN. Recent research were devoted to applicability of MIMO to satellite communication and for high altitude platforms (HAPs)[3]. Previous works were concentrated on describing the MIMO model [4] by mean of ray tracing. This paper is organized as follow: first, we investigate MIMO-SAT model for clear sky case and presents new method of computation of the covariance matrix in order to determine the inter-antenna spacing for the MIMO-SAT system at ground and in the satellite side by mean of capacity maximization. The second part will be devoted to MIMO-SAT model with atmospheric effect and impairment mitigation using precoding and dual polarization scheme.

Keywords: MIMO satellite, antenna inter-spacing, Covariance matrix, Capacity maximization.

1 Introduction

MIMO satellite system consist of one or more satellite in the same orbit communicating with one or more fixed ground station. MIMO enables system capacity to be increased in proportion to the number of transmitting and receiving antennas, a result of which is improved spectrum efficiency. application of MIMO to satellite system, with appropriate model definition, raise capacity of links and reduce error probability. In satellite communication, frequency is always above 10 GHz where wave attenuation in free space is much higher than for terrestrial wireless system, distance between transmitter and receiver is at least 500Km for low orbit satellite(LEO) and reach 36000 for Geostationary orbits(GEO), the radio link depend also on the earth to satellite observation angle which is variable from LEO satellite. In this paper we assume that satellites and ground stations are in the same orbital plane containing the center of the earth and the

G. Giambene and C. Sacchi (Eds.): PSATS 2011, LNICST 71, pp. 351–364, 2011.

satellite orbits is circular. Waves for frequency above 10 GHz are subject of deep mitigation such as attenuation and scattering from rain and snow, exploiting spatial diversity can reduce this effect.

2 MIMO Channel with No Atmospheric Effect

2.1 MIMO Model

In MIMO system the discrete time model is commonly used:

$$\mathrm{Y} = \mathrm{HX} + \mathrm{B} \tag{1}$$

Where, Y is $r \times 1$ received signal, X is $t \times 1$ transmitted signal. H is the $r \times t$ channel matrix. B is $r \times 1$ is the additive noise vector.

Using results from Information theory, MIMO system achieve the maximum capacity when the elements of the transmitted vector X are zero mean independent identically distributed Complex Gaussian variables. The covariance matrix of the transmitted signal is given by:

$$\Gamma_{\mathrm{XX}} = \mathrm{E}(\mathrm{XX}^*) \tag{2}$$

The total transmitted power is given by:

$$P = tr(\Gamma_{\mathrm{XX}}) \tag{3}$$

When the channel is unknown, components of X are independent and have equal power distribution, thus the covariance is:

$$\Gamma_{\mathrm{XX}} = \frac{\mathrm{P}}{\mathrm{t}} \mathrm{I_t} \tag{4}$$

In general we assume that components of B are zero mean Gaussian variable with independent and equal variance real and imaginary part. The covariance matrix of the noise is:

$$\Gamma_{\mathrm{BB}} = \mathrm{E}(\mathrm{BB}^*) = \mathrm{N_0 I_r} \tag{5}$$

Where, N_0 is the noise level at receiver side.

Let define the signal to noise ratio by: $\gamma = \frac{E(||X||^2)}{tE(||B||^2)}$

2.2 Covariance Matrix: First Approach

The block diagram of MIMO system is presented in Fig.1. To describe the channel by its covariance matrix let first transform H into a colon vector $\tilde{H}$ with length $rt \times 1$ as follow:

$$\tilde{H}_{t(j-1)+i} = H_{i,j} \tag{6}$$

The covariance matrix of the channel is given by:

$$\Gamma_{\tilde{\mathrm{H}}\tilde{\mathrm{H}}} = \mathrm{E}(\tilde{\mathrm{H}}\tilde{\mathrm{H}}^*) \tag{7}$$

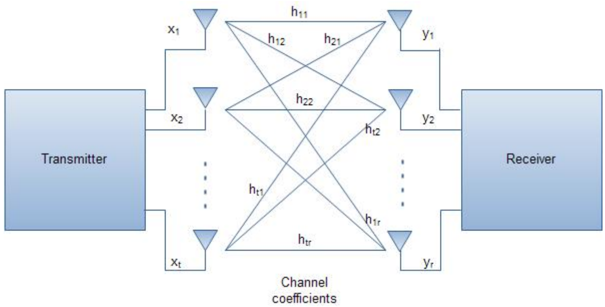

Fig. 1. Block diagram of MIMO system

and,

$$\Gamma_{\tilde{\mathrm{H}}\tilde{\mathrm{H}}}(\mathrm{a}, \mathrm{b}) = \mathrm{E}(\tilde{\mathrm{H}}_{\mathrm{a}}\tilde{\mathrm{H}}_{\mathrm{b}}^{*}) = \mathrm{E}(\mathrm{H}_{\mathrm{kl}}\mathrm{H}_{\mathrm{mn}}{}^{*}) \tag{8}$$

with $a = t(l-1)+k$ and $b = r(n-1)+m$. Size of $\Gamma_{\tilde{\mathrm{H}}\tilde{\mathrm{H}}}$ is $rt \times rt$. The computation of $\Gamma_{\tilde{\mathrm{H}}\tilde{\mathrm{H}}}$ depend on path difference of MIMO channel.

$$\Gamma_{\tilde{\mathrm{H}}\tilde{\mathrm{H}}}(\mathrm{a}, \mathrm{b}) = \mathrm{E}(\{\frac{\mathrm{c}_0}{2\pi \mathrm{f_c} \mathrm{R}}\}^2 \exp\{-\mathrm{j}\frac{2\pi \mathrm{f_c}}{\mathrm{c}_0}(\mathrm{R_{lk}} - \mathrm{R_{lk}})\}) \tag{9}$$

Consider a MIMO channel with t transmit and r receive antennas, with antenna separation of d_t and d_r, let assume that transmitter and receiver are separated by a distance h.

Let ϵ_{kl}^{t} the algebraic angle of the direction of departure (AOD) and ϵ_{lk}^{r} be the algebraic angle of the direction of arrival (AOA), and denote by R_{kl} the distance between antenna k from one TX/RX side and antenna l from the other side.

Using the model with only free space attenuation $H_{kl} = \frac{c_0}{2\pi f_c R_{kl}} \exp\{-j\frac{2\pi f_c}{c_0} R_{kl}\}$, using the fact that $h >> d_t$ and $h >> d_r$, each entry of the channel matrix, taking into account previous conditions, can be approximated by:

$$H_{kl} = K \exp\{-j\frac{2\pi f_c}{c_0} R_{kl}\} \tag{10}$$

Were, $K = \frac{c_0}{2\pi f_c h}$. The entries of the covariance matrix $\mathrm{R}_{\tilde{\mathrm{H}}\tilde{\mathrm{H}}}$ are given by:

$$\Gamma_{\tilde{\mathrm{H}}\tilde{\mathrm{H}}}(\mathrm{a}, \mathrm{b}) = \mathrm{K}^2\mathrm{E}(\exp\{-\mathrm{j}\frac{2\pi \mathrm{f_c}}{\mathrm{c}_0}(\mathrm{R_{kl}} - \mathrm{R_{mn}})\}) \tag{11}$$

the computation of $R_{kl} - R_{mn}$ could be performed geometrically and using some approximation.

$$R_{kl} - R_{mn} = (R_{kl} - R_{ml}) + (R_{ml} - R_{mn}) \tag{12}$$

Let compute first $R_{kl}^2 - R_{ml}^2$ and $R_{ml}^2 - R_{mn}^2$:

$$R_{kl}^2 - R_{ml}^2 = (k-m)^2 d_t^2 + 2(m-k) d_t R_{ml} \sin(\epsilon_{ml}^{t}) \tag{13}$$

And,

$$R_{ml}^2 - R_{mn}^2 = (l-n)^2 d_r^2 + 2(l-n) d_r R_{mn} \sin(\epsilon_{mn}^r) \tag{14}$$

Eq.13 can be simplified as:

$$R_{kl} - R_{ml} = \frac{(k-m)^2 d_t^2}{R_{kl} + R_{ml}} + 2(m-k) d_t \frac{R_{ml}}{R_{kl} + R_{ml}} \sin(\epsilon_{ml}^t) \tag{15}$$

using previous approximation, Eq.15 become:

$$R_{kl} - R_{ml} = (m-k) d_t \sin(\epsilon_{ml}^t) \tag{16}$$

Using the same approach to compute $(R_{ml} - R_{mn})$, we obtain:

$$R_{ml} - R_{mn} = (n-l) d_r \sin(\epsilon_{mn}^r) \tag{17}$$

and finally,

$$R_{kl} - R_{mn} = (m-k) d_t \sin(\epsilon_{ml}^t) + (n-l) d_r \sin(\epsilon_{mn}^r) \tag{18}$$

Then Eq.11 become:

$$\Gamma_{\mathrm{HH}}(\mathrm{a}, \mathrm{b}) = K^2 E(\exp\{-j\frac{2\pi f_c}{c_0}((m-k) d_t \sin(\epsilon_{ml}^t))\} \tag{19}$$

$$\exp\{-j\frac{2\pi f_c}{c_0}((n-l) d_r \sin(\epsilon_{mn}^r))\}) \tag{20}$$

Equation Eq.19 point out that the covariance matrix of the channel depend on distances between antennas in transmitter and receiver side and arrival and departure angles.

Expectation operator concern only angles, distance between antenna are fixed. In practical MIMO communication system d_r is approaching to Zeros, the covariance matrix will depend only on the distances between antennas in the transmitter side and the AOD statistics in presence of scatters.

When the channel is random and AOD and AOA are assumed to be statistically independent, the covariance matrix expressed in Eq.19 is often assumed to have a simpler separable Kronecker structure[8]. In rich scattering environment the Non-Line of Sight(NLOS) dominate, such as in indoor and in some outdoor scenarios [9] [10].

2.3 Covariance Matrix: Second Approach

Let first study the case where only the free space attenuation is considered.

The MIMO system consist of t ground stations and r satellites in the same orbits, the system model is given by:

$$\mathrm{Y} = \mathrm{HX} + \mathrm{B} \tag{21}$$

Y, H, X and B have the same dimension as considered previously. Fig.2 show the geometrical presentation of 2×2 MIMO satellite system.

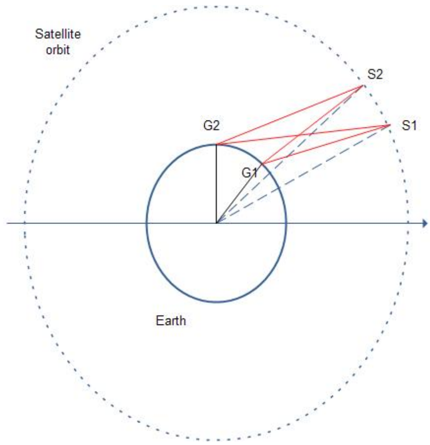

Fig. 2. Block diagram of 2 × 2 MIMO satellite system

Each entry of H, H_{kl} with $k \in 1 \dots r$ and $l \in 1 \dots t$, is given by:

$$H_{lk} = \frac{c_0}{2\pi f_c R_{lk}} \exp\{-j\frac{2\pi f_c}{c_0} R_{lk}\} \tag{22}$$

The distance R_{lk} between the l^{th} ground station to the k^{th} satellite antenna is given by:

$$R_{lk} = \sqrt{(h + r_e)^2 + r_e^2 - 2(h + r_e) r_e \cos(\theta_{G_l} - \theta_{S_k})} \tag{23}$$

Where G stand for ground station and S antenna satellite, one satellite can contain more than one antenna, h is the satellite height and r_e the effective earth radius(8500Km).

Distance between earth grounds and satellites is higher than distance between antenna in each side of MIMO system, H_{lk} is then approximated by:

$$H_{lk} = \frac{c_0}{2\pi f_c R} \exp\{-j\frac{2\pi f_c}{c_0} R_{lk}\} \tag{24}$$

Where $R_{lk} \approx R, \forall l, k$ For sake of simplicity we choose $R = h$. For the MIMO satellite communication distance R_{lk} depend on two independents angle θ_{G_l} and θ_{S_k}, for circular orbit, θ_{S_k} is a linear function of time t and the angle of the ground station is fixed.

Ground stations are uniformly separated by the same distance d_G, and satellite antenna are separated by the same distance d_S. Using a geometrical approach, we compute the covariance matrix of the channel Γ_{HH} given by Eq.8. Using the same approach as in paragraph 2.2, we need first to calculate $R_{lk} - R_{nm}$. Denote by $\psi_{nm} = \cos(\theta_{G_n} - \theta_{S_m})$

$$R_{lk}^2 - R_{nm}^2 = 2(h + r_e) r_e [\psi_{nm} - \psi_{lk}] = 2(h + r_e) r_e [\psi_{nm} - \psi_{lm} + \psi_{lm} - \psi_{lk}] \tag{25}$$

Applying the identity:

$$\cos{(p)} - \cos{(q)} = 2\sin{(\frac{p+q}{2})}\sin{(\frac{q-p}{2})}$$

We get:

$$\psi_{nm} - \psi_{lm} = 2\sin{(\frac{\theta_{G_l} + \theta_{G_n}}{2} - \theta_{S_m})}\sin{(\frac{\theta_{G_l} - \theta_{G_n}}{2})} \tag{26}$$

and

$$\psi_{lm} - \psi_{lk} = 2\sin{(\theta_{G_l} - \frac{\theta_{S_m} + \theta_{S_k}}{2})}\sin{(\frac{\theta_{S_m} - \theta_{S_k}}{2})} \tag{27}$$

Let d_{ln} the distance between G_l and G_n and D_{mk} distance between antenna satellite S_m and S_k. We have:

$$d_{ln} = (n-l)d_G = 2r_e\sin{(\frac{\theta_{G_l} - \theta_{G_n}}{2})} \tag{28}$$

And

$$D_{mk} = (m-k)d_S = 2(r_e + h)\sin{(\frac{\theta_{S_m} - \theta_{S_k}}{2})} \tag{29}$$

Combining Eq.27 and Eq.28 we get:

$$\psi_{nm} - \psi_{lm} = \frac{d_{ln}}{r_e}\sin{(\frac{\theta_{G_l} + \theta_{G_n}}{2} - \theta_{S_m})} \tag{30}$$

And combining Eq.26 and Eq.29 we get:

$$\psi_{lm} - \psi_{lk} = \frac{D_{mk}}{r_e + h}\sin{(\theta_{G_l} - \frac{\theta_{S_m} + \theta_{S_k}}{2})} \tag{31}$$

Let θ_1 and θ_2 the angular separation between two consecutives ground stations and two satellites antenna respectively, we have:

$$\begin{cases} \theta_{G_l} = (l-1)\theta_1 + \theta_{G_1} \\ \theta_{G_n} = (n-1)\theta_1 + \theta_{G_1} \\ \theta_{S_m} = (m-1)\theta_2 + \theta_{S_1} \\ \theta_{G_k} = (k-1)\theta_2 + \theta_{S_1} \end{cases} \tag{32}$$

Eq.30 become:

$$\psi_{nm} - \psi_{lm} = \frac{(n-l)d_G}{r_e}\sin\left[\theta_{G_1} - \theta_{S_1} + \frac{l+n-2}{2}\theta_1 + (m-1)\theta_2\right] \tag{33}$$

and, Eq.31 become:

$$\psi_{lm} - \psi_{lk} = \frac{(m-k)d_S}{r_e + h}\sin\left[\theta_{G_1} - \theta_{S_1} + (l-1)\theta_1 + \frac{m+k-2}{2}\theta_2\right] \tag{34}$$

Assume that $d_G << r_e$ and $d_S << (r_e + h)$, Eq.33 and Eq.34 can be writen as:

$$\begin{aligned} &\psi_{nm} - \psi_{lm} = \\ &\tfrac{(n-l)d_G}{r_e} \sin\left[\theta_{G_1} - \theta_{S_1} + \tfrac{(l+n-2)d_G}{2r_e} + \tfrac{(m-1)d_S}{h+r_e}\right] \end{aligned} \tag{35}$$

$$\begin{aligned} &\psi_{lm} - \psi_{lk} = \\ &\tfrac{(m-k)d_S}{h+r_e} \sin\left[\theta_{G_1} - \theta_{S_1} + \tfrac{(l-1)d_G}{r_e} + \tfrac{(m+k-2)d_S}{2(h+r_e)}\right] \end{aligned} \tag{36}$$

$$\begin{aligned} &R_{lk}^2 - R_{nm}^2 = \\ &2(n-l)d_G(h+r_e) \sin\left[\theta_{G_1} - \theta_{S_1} + \tfrac{(l+n-2)d_G}{2r_e} + \tfrac{(m-1)d_S}{h+r_e}\right] \\ &+2(m-k)d_S r_e \sin\left[\theta_{G_1} - \theta_{S_1} + \tfrac{(l-1)d_G}{r_e} + \tfrac{(m+k-2)d_S}{2(h+r_e)}\right] \end{aligned} \tag{37}$$

Let assume that $R_{kl} + R_{mn} \approx 2h$ [23] and for sake of simplicity let assume that $\theta_{G_1} = 0$, Eq.37 become:

$$\begin{aligned} &R_{lk} - R_{nm} = \\ &(n-l)d_G(1 + \tfrac{r_e}{h}) \sin\left[\tfrac{(l+n-2)d_G}{2r_e} + \tfrac{(m-1)d_S}{h+r_e} - \theta_{S_1}\right] \\ &+(m-k)d_S \tfrac{r_e}{h} \sin\left[\tfrac{(l-1)d_G}{r_e} + \tfrac{(m+k-2)d_S}{2(h+r_e)} - \theta_{S_1}\right] \end{aligned} \tag{38}$$

The first approach is convenient in presence of scatter in atmosphere such as rain, in this case, angle of arrival and angle of departure are random variable, the second approach is adequate for clear sky scenarios in the MIMO-SaT links.

3 Capacity Optimization

3.1 MIMO-SAT Capacity for Single User Communication

MIMO Capacity is the maximum of mutual information between the transmitted vector X and received vector Y, when component of the transmitted vector X have equal power distribution the explicit form of capacity, derived by Teletar in [2], and given by: [17]

$$C(H) = \log(\det(I_{r\times r} + \gamma H H^*)) \tag{39}$$

Using the property: $\det(I + AB) = \det(I + BA)$ we can write:

$$C(H) = \log(\det(I_{t\times t} + \gamma H^* H)) \tag{40}$$

Let denote by:

$$\begin{cases} W_1 = HH^* \\ \\ W_2 = H^*H \end{cases} \tag{41}$$

entries of W_1 and W_2 are given by:

$$W_1(k,l) = \sum_{m=1}^{t} H_{k,m} H_{l,m}^* \tag{42}$$

And

$$W_2(k,l) = \sum_{m=1}^{r} H_{m,k} H_{m,l}^* \tag{43}$$

Matrix W_1 and W_2 are both definite positive.

Link between ground and satellite is fixed for GEO system and variable for NGEO system with respect to elevation angle in clear sky condition.

3.2 Inter-spacing Computation for Clear Sky Conditions

Inter antenna spacing is a convex optimization problem, where the objective function is the system capacity. Using results from Hadamrad inequality for definite positive matrix, when $\min(r,t) \geq 2$ capacity reach its maximum when W_1 is diagonal. For the particular case of 2×2 MIMO system, the optimization problem can expressed by the following equations:

$$\begin{array}{l} \{\frac{2\pi f_c}{c_0}(R_{11} - R_{12})\} - \{\frac{2\pi f_c}{c_0}(R_{21} - R_{22})\} = (2\mu + 1)\pi \\ \{\frac{2\pi f_c}{c_0}(R_{11} - R_{21})\} - \{\frac{2\pi f_c}{c_0}(R_{12} - R_{22})\} = (2\nu + 1)\pi \end{array} \tag{44}$$

where, $\nu, \mu \in \mathbf{Z}$. For GEO system $\theta_{S_1} = \frac{\pi}{36}$, solution to Eq.51 and Eq.49 d_G and d_S can be performed numerically.

The equation system of Eq.44 can be reduced to the following equation:

$$R_{11} - R_{12} - R_{21} + R_{22} = (\mu + \frac{1}{2})\frac{c_0}{f_0} \tag{45}$$

GEO System. AOD and AOA for GEO system are fixed, hence, capacity is maximized when $W_1 = tI_{t \times t}$, for 2×2 MIMO-SAT system, inter-spacing distance d_G and d_S are solution of the following equations:

$$W_1(1,2) = 0 \tag{46}$$

and

$$W_2(1,2) = 0 \tag{47}$$

this is equivalent to the equations:

$$\frac{2\pi f_c}{c_0} d_S \frac{r_e}{h} \left[\sin(\theta_{S_1}) + \sin(\frac{d_G}{r_e} - \theta_{S_1}) \right] = (2\nu + 1)\pi \tag{48}$$

And

$$\frac{2\pi f_c}{c_0} d_G (1 + \frac{r_e}{h}) \left[\sin(\theta_{S_1}) + \sin(\frac{d_S}{h + r_e} - \theta_{S_1}) \right] = (2\mu + 1)\pi \tag{49}$$

Using first order Taylor's expansion of $\sin(\frac{d_S}{h+r_e} - \theta_{S_1})$ and $\sin(\frac{d_G}{r_e} - \theta_{S_1})$ we get:

$$\frac{f_c}{c_0} \frac{d_S d_G \cos(\theta_{S_1})}{h} = (\nu + \frac{1}{2}) \tag{50}$$

Finally,

$$d_S d_G = \frac{h c_0}{\cos(\theta_{S_1}) f_c} (\nu + \frac{1}{2}) \tag{51}$$

NGEO System. For NGEO system satellite describe an elliptical orbit(only circular case will be considered), and the orbit period is less than 24 hours, the satellite is in orbit lower than for GEO system. Using the concavity property of $C(H)$, capacity is upper bounded by its Jensen bound:

$$\mathbf{E}(C(H)) < C(\hat{W}_1) \tag{52}$$

Where, $\hat{W}_1$ is the mean value over the angle θ_{S_1} which can variate from two angular positions: $\theta_{min} = \theta_0 - \Delta\theta$ and $\theta_{max} = \theta_0 + \Delta\theta$, $\Delta\theta$ is positive value that depend on the height of the satellite.

For the particular case of 2×2 MIMO, let compute first $\hat{W}_1(2,1)$. Using results from paragraph 2.3, one can write:

$$\begin{aligned} &\hat{W}_1(2,1) = \\ &\tfrac{1}{2\Delta\theta} \left(\tfrac{c_0}{2\pi f_c R} \right)^2 \int_{\theta_{min}}^{\theta_{max}} \left[\exp\left(j \tfrac{2\pi f_c}{c_0} \tfrac{d_S r_e}{h} \sin\theta \right) \right. \\ &\left. + \exp\left(-j \tfrac{2\pi f_c}{c_0} \tfrac{d_S r_e}{h} \sin(\tfrac{d_G}{r_e} - \theta) \right) d\theta \right. \end{aligned} \tag{53}$$

Let $\Psi(d_S, d_G, \theta) = \exp\left(j \frac{2\pi f_c}{c_0} \frac{d_S r_e}{h} \sin\theta \right) + \exp\left(-j \frac{2\pi f_c}{c_0} \frac{d_S r_e}{h} \sin(\frac{d_G}{r_e} - \theta) \right)$ Applying the triangular inequality to Eq.53, one get:

$$|\hat{W}_1(2,1)| \leq \tfrac{1}{2\Delta\theta} \left(\tfrac{c_0}{2\pi f_c R} \right)^2 \int_{\theta_{min}}^{\theta_{max}} |\Psi(d_S, d_G, \theta)| \, d\theta \tag{54}$$

The expression inside the integral of Eq.54 can be simplified as:

$$|\Psi(d_S, d_G, \theta)| = 2 \left| \cos\left(\frac{c}{c_0} \frac{d_S d_G \cos(\theta)}{h} \right) \right|$$

Eq.54 became:

$$|\hat{W}_1(2,1)| \leq \tfrac{1}{\Delta\theta} \left(\tfrac{c_0}{2\pi f_c R} \right)^2 \int_{\theta_0 - \Delta\theta}^{\theta_0 + \Delta\theta} \left| \cos\left(\tfrac{\pi f_c}{c_0} \tfrac{d_S d_G \cos(\theta)}{h} \right) \right| d\theta \tag{55}$$

4 MIMO SAT Channel with Atmospheric Effect

By contrast to MIMO for terrestrial communication, MIMO satellite depend on the geographic position of the ground station on the earth, climatic condition variate from one region to another, recent research are performed in some countries to draw their rain or snow map, in order to determine the average rain fall rate and rainy period [22].

Using inter-distances results from the previous section, its possible to choose, by mean of similation, the best design that mitigate the best the atmospheric effect.

4.1 Impact of Rain

Impact of rain links was studied for the scenario where one satellite with 2 transmitting antenna (inter antenna-spacing is very small) and it is was shown [23] that capacity depend only on modulus of the rain attenuation. The rain attenuation for a given path p_R is given by [15]:

$$L^r(dB) = aR^b sp_R \tag{56}$$

And the attenuation coefficient T^r:

$$T^r = 10^{\frac{-L^r}{20}} \tag{57}$$

Where, a and b two constant depending on frequency, polarization and elevation angle, p_R the path length and s the path correcting factor.

4.2 Depolarization

By passing through raindrops, the electric field tend to have a new component along the orthogonal axis. Depolarization is strongly correlated with rain attenuation, and standard models of depolarization use this fact to predict L^d directly from the attenuation L^r . One such model takes the form:

$$L^d(dB) = \alpha - \beta \log(L^r) \tag{58}$$

α and β two constants depending on frequency. For frequencies above 10 GHz, $\alpha = 35,8$ and $\beta = 13,4$ [7].

5 Mitigation to Atmospheric Effect

5.1 Impact on System Capacity

In presence of atmospheric impairment, antenna inter-spacing to get optimal capacity is not sufficient, because matrices W_1 and W_2 are not diagonals. Two technique can be proposed to reduce this effect and enhance the capacity.

5.2 Dual Polarization

Transmission of dual independent orthogonal polarized channels in the same frequency band increase channel capacity. The transmitted electromagnetic wave can be impaired by atmospheric medium by transferring an amount of energy from one polarization to another, resulting in interference between the two channels.

The 2×2 channel matrix $H^a(k,l)$ when using dual polarization for each link (k,l) in MIMO-SAT system in presence of atmospheric impairment is expressed as function of the polarization matrix $T(k,l)$ by:

$$H^a = H(k,l)T \tag{59}$$

Where,

$$T = \begin{pmatrix} \tilde{T}_r^{(1)} & \frac{\tilde{T}_r^{(2)}}{\tilde{T}_d^{(2)}} \\ \frac{\tilde{T}_r^{(1)}}{\tilde{T}_d^{(1)}} & \tilde{T}_r^{(2)} \end{pmatrix} \tag{60}$$

Where,

- (1): horizontal polarization
- (2): vertical polarization
- $\tilde{T}_r$:complexe rain attenuation coefficient in the link (k,l)
- $\tilde{T}_d$: complexe depolarization loss in the link (k,l)

Using climatic parameters such as, rain rate, rain region dimension it is possible to determine the modulus of rain attenuation and the modulus of polarization loss, we assume that phase shift caused by atmospheric impairment is uniformly distributed in $[0, 2\pi]$ [21] and hence can be added to the AOA or AOD for each polarization. The resulting channel has dimension of $2t \times 2r$.

5.3 Power Allocation: Linear Precoding

Precoding design for MIMO wireless has been an active research area in recent year and is now finding applications in emerging wireless standard, the linear precoder functions as an input shaper and a beam-former with one or multiple beams with per-beam power allocation at the transmitting side. Precoder must match signal from Space-Time encoder and transmit antenna, a precoder which is matrix L is a subject of constraint optimization of a certain function, for example channel capacity, pairwise error probability. Generally we assume that the precoder satisfy the following power constraint:

$$tr(LL^*) = 1 \tag{61}$$

Assuming a power normalized code word C with covariance matrix Γ_{CC}, the transmitted signal at the transmitter is decomposed as : $\mathbf{X} = \sqrt{\frac{P}{t}}\mathbf{LC}$, capacity is expressed as:

$$C(H) = \log(\det(I_{t\times t} + \gamma H L \Gamma_{CC} L^* H^*)) \tag{62}$$

Where P is the total transmitted signal power.

6 Simulation Results

6.1 GEO System

Simulation parameters for 2×2 MIMO for GEO system are fixed as follow:

- $f_c = 12GhZ$
- GEO satellite $\theta_{S_1} = \frac{\pi}{36}$(in earth position near equatorial region)
- $d_S = 6m$ antenna are embedded in the same satellite

The distance between antenna at ground is then given by: $d_G(Km) = 90.3438 \times (\mu + \frac{1}{2})$ and $\mu = 0, 1, 2, ...$ The antenna separation depend mainly on the region and taking into account installation cost of the ground station, its possible to determine the best choice of μ.

6.2 NGEO System

Simulation parameters for 2×2 MIMO for NGEO system are fixed as follow:

- $f_c = 12GhZ$
- NGEO satellite: The angular position of the satellite toward earth center is variating between $\theta_0 - \Delta\theta$ and $\theta_0 + \Delta\theta$
- $d_S = 6m$ antenna are embedded the same satellite

The determination of distance between the earth station d_G can be performed numerically by minimization of the upper bound of $|\hat{W}_1(2,1)|$, in figure 3 are presented the variation of the upper bound of $|\hat{W}_1(2,1)|$ as function of $K = \frac{\pi f_c d_S d_G}{c_0 h}$.

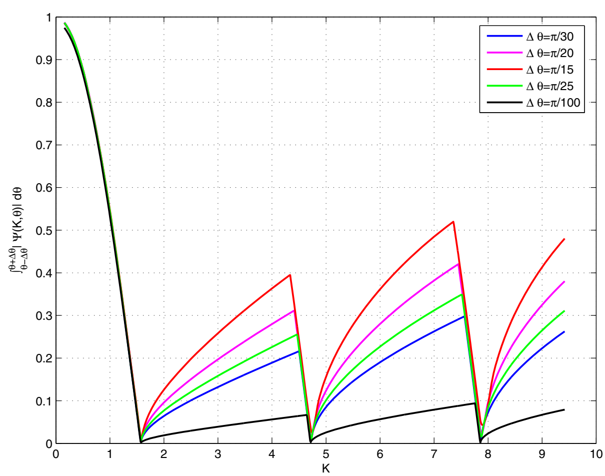

Fig. 3. Variation of $|\hat{W}_1(2,1)|$ as function of K for deferents values of $\Delta\theta$

7 Conclusion

We present in this paper a theoretical study of the design of MIMO-SAT system by computing the inter-spacing distance between antenna maximize the system capacity for both GEO an NGEO system in the case of clear sky conditions, the solution of the optimization problem is not unique and depend on geographical position. It is possible to perform a second stage optimization of capacity in order to reduce effects of the atmosphere on communication link, two techniques are proposed in this case: linear precoding and dual polarization techniques.

References

1. Marzetta, T.L., Hochwald, B.M.: Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading. IEEE Trans. Inform. Theory 45, 139–157 (1999)
2. Telatar, E.: Capacity of multi-antenna Gaussian channels, Bell Lab. Technical Memorandum (October 1995), `http://mars.bell-labs.com/papers/proof/`, (Europe Trans. Telecomm. ETT 10(6), 585–596 (November 1999)
3. Perez-Neira, A.I., Zorba, N., Realp, M.: TR2-MIMO Applicability to Satellite Systems, internal report of MIMO Applicability to Satellite Networks with reference: AO/1-5146/06/NL/JD (July 2007)
4. Michailidis, E.T., Kanatas, A.G.: Capacity Optimized Line-of-Sight HAP-MIMO Channels for Fixed Wireless Access. In: International Workshop on Satellite and Space Communications IWSSC 2009, Siena-Tuscany, Italy (September 9-11, 2009)
5. Cover, T., Thomas, J.: Elements of Information Theory. John Wiley & Sons, Chichester (1991)
6. Sampath, H., Paulraj, A.: Linear precoding for space-time coded systems with known fading correlations. IEEE Communications Letters 6(6), 239–241 (2002)
7. Flock, W.L.: Propagation effects on satellite systems at frequencies below 10 GHz, NASA Reference Publication 1108 (1983)
8. Shiu, D., Foschini, G., Gans, M., Kahn, J.: Fading correlation and its effect on the capacity of multielement antenna systems. IEEE Trans. Comm. 48(3), 502–513 (2000)
9. Yu, K., Bengtsson, M., Ottersten, B., McNamara, D., Karlsson, P., Beach, M.: Second order statistics of NLOS indoor MIMO channels based on 5.2 GHz measurements. In: Proc. IEEE Global Telecomm. Conf., vol. 1, pp. 25–29 (November 2001)
10. Kermoal, J., Schumacher, L., Pedersen, K., Mogensen, P., Frederiksen, F.: A stochastic MIMO radio channel model with experimental validation. IEEE J. Selected Areas in Comm. 20(6), 1211–1226 (2002)
11. Tarokh, V., Seshadri, N., Calderbank, A.R.: Space-time codes for high data rate wireless communication: Performance criterion and code construction. IEEE Trans. Inform. Theory 44, 744–765 (1998)
12. Alamouti, S.M.: A simple transmit diversity technique for wireless communications. IEEE J. Select. Areas Commun. 16, 1451–1458 (1998)
13. Alamouti, S.M.: A simple transmit diversity technique for wireless communications. IEEE J. Sel. Areas Comm. 16(8), 1451–1458 (1998)
14. Tarokh, V., Jafarkhani, H., Calderbank, A.: Spacetime block codes from orthogonal designs. IEEE Trans. Inform. Theory 45, 1456–1467 (1999)

15. Saunders, S.R., Zavala, A.A.: Antenna and propagation for wireless communication systems, 2nd edn. JohnWiley & Sons Ltd., The Atrium (2007)
16. Tarokh, V., Jafarkhani, H., Calderbank, A.: Spacetime block codes from orthogonal designs. IEEE Trans. Inform. Theory 45, 1456–1467 (1999)
17. Cover, T., Thomas, J.: Elements of Information Theory. John Wiley and Sons, Chichester (1991)
18. Jngren, G., Skoglund, M., Ottersten, B.: Combining beamforming and orthogonal space-time block coding. IEEE Trans. on Info. Theory 48(3), 611–627 (2002)
19. Liu, L., Jafarkhani, H.: Application of quasi-orthogonal space-time block codes in beamforming. IEEE Trans. on Signal Processing 53(1), 54–63 (2005)
20. Gallager, R.: Information Theory and Reliable Communication. Wiley and Sons, Chichester (1968)
21. Conrat, J.M., Pajusco, P.: A Versatile Propagation Channel Simulator for MIMO Link Level Simulation. EURASIP Journal on Wireless Communications and Networking 2007, Article ID 80194, 13 (2007)
22. Moupfouma, F.: Improvement of rain attenuation prediction method for terrestrial microwave links. IEEE Trans. Antennas Propag. 32(12), 1368–1372 (1984)
23. Schwarz, R.T., Knopp, A., Lankl, B.: The Channel Capacity of MIMO Satellite Links in a Fading Environment: A Probabilistic Analysis. In: International Workshop on Satellite and Space Communications, IWSSC 2009, Siena-Tuscany, Italy (September 9-11, 2009)