Maximum Sum-Rate Interference Alignment Schemes for the 3-User Deterministic MIMO Channel

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Abstract. Closed-form solutions exist for the interference alignment (IA) problem in the multiple-input multiple-output (MIMO) interference channel when there are exactly $K = 3$ users. Specifically, when each user wishes to send d streams and is equipped with $N = 2d$ antennas at both sides of the link, a finite number of IA solutions exist. Exploiting this observation, in this paper we find the maximum sum-rate solution by exhaustive search over the finite set of IA solutions and evaluate its performance. As an alternative, the solution that maximizes the received sum-power in the interference free subspace is also considered. Simulation results show the improvement achieved by both IA strategies in comparison with the conventional scheme proposed by Cadambe and Jafar, which randomly picks one of the IA solutions. Furthermore, the impact of channel correlation in these interference management techniques has also been studied.

Keywords: sum-rate, interference management, interference alignment, interference channel, multiple-input multiple-output (MIMO).

1 Introduction

Interference alignment (IA) is a recently proposed technique to achieve the maximum degrees of freedom (DoF) for K-user interference channels [1]. The degrees of freedom (also referred to as network multiplexing gain) approximates the capacity of a network as

$$C(SNR) = d \log(SNR) + o(\log(SNR))$$

where d is the number of degrees of freedom and $C(SNR)$ represents the capacity of the network as a function of the signal to noise ratio (SNR). At high SNR, the $o(\log(SNR))$ term becomes negligible in comparison to $\log(SNR)$. Therefore, d, represents the asymptotic slope of the $C(SNR)$ vs $\log(SNR)$ curve. By studying wireless networks at high SNR, the degrees of freedom approach de-emphasizes noise and explicitly addresses the effects of interference in a wireless network.

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The DoF also indicate the number of simultaneous data streams for which the interference can be completely cancelled.

For the K-user interference channel, IA has been shown to achieve almost surely $K/2$ DoF per time, frequency or spatial dimension. Basically, IA schemes jointly design the signals transmitted by all users in such a way that the interfering signals at each receiver fall into a reduced-dimensional subspace. The receivers can then extract the projection of the desired signal that lies in the interference-free subspace.

In this paper we consider IA schemes based on linear precoding for the 3-user multiple-input multiple-output (MIMO) interference channel with constant coefficients, where every transmitter and receiver has an even number of antennas ($N = 2d$) and each user wishes to send d streams of data. For such systems, whose closed-form solutions are known, it follows that each user achieves $d = N/2$ DoF [1]. When N is odd, a two time-slot symbol extension is required. These systems[1] are usually denoted as $(2d \times 2d, d)^3$.

The first contribution of this paper is to show that for these MIMO interference channels there is a finite number of IA solutions. Although this result can easily be derived from the IA scheme described in [1] and [2], the existence of a finite number of IA solutions has not been widely acknowledged in the literature. The second contribution is to exploit this fact to find the IA solution that maximizes either the sum-rate or the sum-power.[2] For the 3-user case and a reasonable number of transmitted streams, the max sum-rate and max sum-power solutions can be obtained by exhaustive search over the finite set of IA solutions. The performance of the max sum-rate and the max sum-power IA solutions is studied in this paper by means of simulation results. In comparison to the conventional IA solution proposed by Cadambe and Jafar in [1], the relative advantage of the proposed schemes increases with the number of transmitted streams. The impact of channel correlation has also been considered.

This article is organized as follows: the system model, the principle of interference alignment and the closed-form solution for the 3-user channel are reviewed in Sections 2 and 3. The max sum-rate and max sum-power solutions are described in Section 4, while Section 5 presents simulations and performance comparisons.

2 Interference Alignment in the K-User Deterministic MIMO Channel

Consider the K-user interference channel, comprised of K transmitter - receiver pairs (links) that interfere with each other. We assume that each user wishes to achieve d degrees of freedom and is equipped with N antennas at both sides of

[1] The notation $(n_T \times n_R, d)^K$ means that every transmitter has n_T antennas, every receiver has n_R antennas and each one of the K users wishes to achieve d DoF. These interference channels are called symmetric.

[2] The term sum-power refers to the total power received in the interference-free subspaces by all the users.

the link. Also, let $\mathbf{V}^{[k]} \in \mathbb{C}^{N \times d}$ be an orthonormal basis of the transmitted signal space for user k. The discrete-time signal received at receiver k at a given time instant is the superposition of the signals transmitted by the K transmitters, weighted by their respective channel gains and affected by noise. It can be written as

$$\mathbf{y}^{[k]} = \mathbf{H}^{[kk]}\mathbf{V}^{[k]}\mathbf{s}^{[k]} + \sum_{l \neq k} \mathbf{H}^{[kl]}\mathbf{V}^{[l]}\mathbf{s}^{[l]} + \mathbf{w}^{[k]}, \tag{1}$$

where $\mathbf{H}^{[kl]} \in \mathbb{C}^{N \times N}$ is the flat-fading MIMO channel from transmitter l to receiver k, $\mathbf{s}^{[l]} \in \mathbb{C}^{d \times 1}$ is the signal transmitted by the l-th user and $\mathbf{w}^{[k]}$ is the additive and spatially white Gaussian noise at receiver k.

In this context, interference alignment is achieved when we are able to find a set of unitary precoding matrices $\mathbf{V}^{[k]}$ and unitary interference filtering matrices $\mathbf{U}^{[k]}$ such that, for $k = 1, \ldots, K$

$$\mathbf{U}^{[k]H}\mathbf{H}^{[kl]}\mathbf{V}^{[l]} = 0, \quad \forall l \neq k, \tag{2}$$

and

$$\mathrm{rank}(\mathbf{U}^{[k]H}\mathbf{H}^{[kk]}\mathbf{V}^{[k]}) = d. \tag{3}$$

When an IA solution exists, the signal received at user k after projecting $\mathbf{y}^{[k]}$ onto the orthogonal subspace of the interference, using $\mathbf{U}^{[k]}$, yields

$$\begin{aligned} \mathbf{r}^{[k]} &= \mathbf{U}^{[k]H}\mathbf{H}^{[kk]}\mathbf{V}^{[k]}\mathbf{s}^{[k]} + \sum_{l \neq k} \mathbf{U}^{[k]H}\mathbf{H}^{[kl]}\mathbf{V}^{[l]}\mathbf{s}^{[l]} + \mathbf{U}^{[k]H}\mathbf{w}^{[k]} \\ &= \mathbf{U}^{[k]H}\mathbf{H}^{[kk]}\mathbf{V}^{[k]}\mathbf{s}^{[k]} + \mathbf{n}^{[k]}. \end{aligned} \tag{4}$$

According to (3) and (4), the effective channel $\mathbf{U}^{[k]H}\mathbf{H}^{[kk]}\mathbf{V}^{[k]}$ is now $d \times d$ dimensional. In summary, by applying IA the MIMO interference channel has been transformed into a set of Gaussian parallel $d \times d$ MIMO channels.

3 Closed-Form IA Solutions for the 3-User Case

In this paper, we focus on the case where the number of users is $K = 3$, the number of antennas is $N = 2d$ at the transmitter and receiver sides, and each user wishes to send d streams. All the interference alignment solutions (as proposed in [1]) can be obtained as follows:

1. The precoder for the user 1, $\mathbf{V}^{[1]}$, is formed by taking any subset of d eigenvectors of the following $2d \times 2d$ matrix, not necessarily the main eigenvectors

$$\mathbf{E} = (\mathbf{H}^{[31]})^{-1}\mathbf{H}^{[32]}(\mathbf{H}^{[12]})^{-1}\mathbf{H}^{[13]}(\mathbf{H}^{[23]})^{-1}\mathbf{H}^{[21]}. \tag{5}$$

2. The precoders for users 2 and 3, $\mathbf{V}^{[2]}$ and $\mathbf{V}^{[3]}$, are obtained respectively as

$$\mathbf{V}^{[2]} = (\mathbf{H}^{[32]})^{-1}\mathbf{H}^{[31]}\mathbf{V}^{[1]}, \tag{6}$$

and

$$\mathbf{V}^{[3]} = (\mathbf{H}^{[23]})^{-1}\mathbf{H}^{[21]}\mathbf{V}^{[1]}. \tag{7}$$

Since $\mathbf{E}$ is a full-rank $2d \times 2d$ matrix, there are $C_{2d}^{d} = \binom{2d}{d}$ ways to choose the d-dimensional unitary precoder for the first user. Each one of these precoders for user 1 yields a distinct IA solution.

An interesting fact of the 3-user interference channel is that it induces a permutation structure which makes that starting the process described above with a different user yields exactly the same set of the IA solutions. In other words, the IA solutions do not depend on which user is picked first.

To clarify this point let us, for example, start the procedure with user 2 instead of user 1. Now, $\mathbf{V}^{[2]}$ is formed by taking any subset of d eigenvectors of the matrix

$$\mathbf{E}' = (\mathbf{H}^{[12]})^{-1}\mathbf{H}^{[13]}(\mathbf{H}^{[23]})^{-1}\mathbf{H}^{[21]}(\mathbf{H}^{[31]})^{-1}\mathbf{H}^{[32]} \tag{8}$$

and the precoder for the user 1, can be calculated as

$$\mathbf{V}^{[1]} = \mathbf{H}^{[31]}(\mathbf{H}^{[32]})^{-1}\mathbf{V}^{[2]}. \tag{9}$$

The following relationship holds between the eigenvectors of $\mathbf{E}$ and $\mathbf{E}'$ (this is due to the permutation structure induced by the 3-user channel)

$$\nu(\mathbf{E}) = \mathbf{H}^{[31]}(\mathbf{H}^{[32]})^{-1}\nu(\mathbf{E}'), \tag{10}$$

and therefore it is clear that the same solutions are obtained starting with user 1 or 2. Obviously, this also occurs when we start the alignment procedure from user 3. In conclusion, the number of IA solutions for the $(2d \times 2d, d)^3$ systems is exactly $C_{2d}^{d} = \binom{2d}{d}$.

The main implication of this result is that for reasonable values of d (e.g., from 1 to 5), it is feasible to find the best IA solution (in terms of sum-rate, for instance) by exhaustive search over the finite set of $\binom{2d}{d}$ solutions. The additional computation cost is moderate and, as we will show in Section 5, a significant improvement can be achieved.

4 Max Sum-Rate and Max Sum-Power IA Solutions

The most straightforward figure of merit for measuring the system performance is the sum-rate, which is given by

$$R = \sum_{k=1}^{K} \log \left| \mathbf{I}_N + \left(\sigma^2 \mathbf{I}_N + \sum_{l \neq k} \mathbf{Q}^{[kl]} \right)^{-1} \mathbf{Q}^{kk} \right|, \tag{11}$$

where $\mathbf{Q}^{[kl]}$ denotes the $N \times N$ covariance matrix of the signal from the l-th transmitter to the k-th receiver, and σ^2 is the variance of the additive white Gaussian noise.

When the interference is perfectly aligned (i.e. (2) and (3) are satisfied), the interference channel is decoupled into a set of parallel Gaussian MIMO channels and the sum-rate in (11) reduces to

$$R = \sum_{k=1}^{K} \log \left| \mathbf{I}_N + \frac{1}{\sigma^2} \mathbf{U}^{[k]H} \mathbf{H}^{[kk]} \mathbf{V}^{[k]} \mathbf{V}^{[k]H} \mathbf{H}^{[kk]H} \mathbf{U}^{[k]} \right|, \tag{12}$$

which simply adds up the achievable rates in each interference-free $d \times d$ MIMO channel given by $\overline{\mathbf{H}}^{[k]} = \mathbf{U}^{[k]H}\mathbf{H}^{[kk]}\mathbf{V}^{[k]}$, for $k = 1, \ldots, K$. Moreover, it is clear that after the channel has been block-diagonalized better throughputs can be obtained by using non-unitary precoders and decoders, or by applying power waterfilling among the eigenmodes of the equivalent non-interfering $d \times d$ MIMO channels. Maximizing the sum-rate in (11) subject to the unitary constraints $\mathbf{V}^{[k]H}\mathbf{V}^{[k]} = \mathbf{I}_d$, $\mathbf{U}^{[k]H}\mathbf{U}^{[k]} = \mathbf{I}_d$ and also to the zero-interference constraint (2) is a challenging problem that has not been solved yet.

As an alternative to the sum-rate criterion, in this paper we will also consider the maximization of the power received in the interference-free subspaces, which is given by

$$P = \sum_{k=1}^{K} \operatorname{tr}\left(\mathbf{U}^{[k]H}\mathbf{H}^{[kk]}\mathbf{V}^{[k]}\mathbf{V}^{[k]H}\mathbf{H}^{[kk]H}\mathbf{U}^{[k]}\right), \tag{13}$$

where $\operatorname{tr}(\mathbf{A})$ denotes the trace of matrix $\mathbf{A}$.

As it has been shown in Section 3, for $K = 3$ users, a finite number of closed-form interference alignment solutions exist. Therefore, in principle it is possible to choose the solution that maximizes (12) or (13) by exhaustive search over all IA solutions.

In networks with $K > 3$ users, closed-form IA solutions are yet to be found. For this reason, one has to resort to iterative algorithms which alternatively optimize the precoders at the transmitters and the decoders at the receivers [3], [4]. These alternating minimization algorithms obtain an IA solution that do not optimize neither the sum-rate nor the sum-power. Although they can be modified to maximize certain cost function (i.e. the sum-rate in (12)), this requires the computation of quite complex derivatives [5]. The advantage of maximizing the sum-power is that it requires less complex derivatives. In Section 5 the usefulness of using the sum-power as a proxy for the sum-rate is assessed.

5 Simulation Results

In this section, numerical simulation results are presented to evaluate the performance of the maximum sum-rate (MaxSR-IA) and the maximum sum-power (MaxPower-IA) solutions. These solutions are also compared with the conventional IA solution (proposed in [1]) which randomly selects a subset of d eigenvectors of the matrix $\mathbf{E}$ in (5). All the results presented in this section consider a $(2d \times 2d, d)^3$ systems with $d = 1, \ldots, 4$.

5.1 Rayleigh Interference MIMO Channel

For this example, the matrices $\mathbf{H}^{[kl]}$ represent independent and identically distributed (i.i.d.) Rayleigh fading MIMO channels with unit-variance entries. The average sum-rate has been evaluated for 10000 channel realizations for different values of d and signal-to-noise ratio (SNR) values ranging from 0 to 40 dB. The obtained results are depicted in Fig. 1.

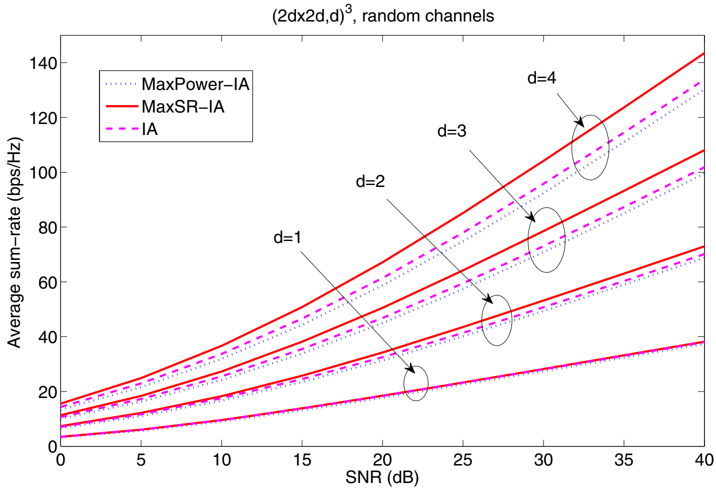

Fig. 1. Average sum-rate achieved by the MaxSR-IA, MaxPower-IA and the conventional IA solutions

As it can be seen in Fig. 1, the MaxSR-IA solution provides a considerable improvement with respect to the other two IA solutions for all values of d. MaxPower-IA seems to be a good approximation of the MaxSR-IA for low values of d. However, as d becomes larger its performance gets considerably lower than the MaxSR-IA solution.

In turn, the average sum-rate improvement with respect to the conventional IA solution normalized by the number of streams is depicted in Fig. 2. It shows that the relative sum-rate difference between the MaxSR-IA and the conventional IA solution is increased when the value of d increases.

5.2 Correlated MIMO Channel

In this subsection we evaluate the impact of MIMO channel correlation using the well-known Kronecker model

$$\mathbf{H}_c^{[kl]} = \mathbf{R}_{rx}^{[k]1/2}\mathbf{H}^{[kl]}\mathbf{R}_{tx}^{[l]1/2} \tag{14}$$

where $\mathbf{H}_c^{[kl]}$ is the correlated MIMO channel matrix for transmitter l and receiver k, $\mathbf{R}_{rx}^{[k]}$ and $\mathbf{R}_{tx}^{[l]}$ are the receiver k and transmitter l antenna correlations. All transmitter and receiver correlation matrices are assumed equal, that is $\mathbf{R}_{tx}^{[l]} = \mathbf{R}_{rx}^{[k]} = \mathbf{R} \quad \forall\, l, k \in \{1, 2, 3\}$. The ij entry of $\mathbf{R}$ is given by the Jakes model [6] which assumes that the antennas are uniformly arranged along a line (each pair is separated a distance L),

$$\mathbf{R}_{ij} = J_0\left(2\pi\frac{L|i-j|}{\lambda}\right), \tag{15}$$

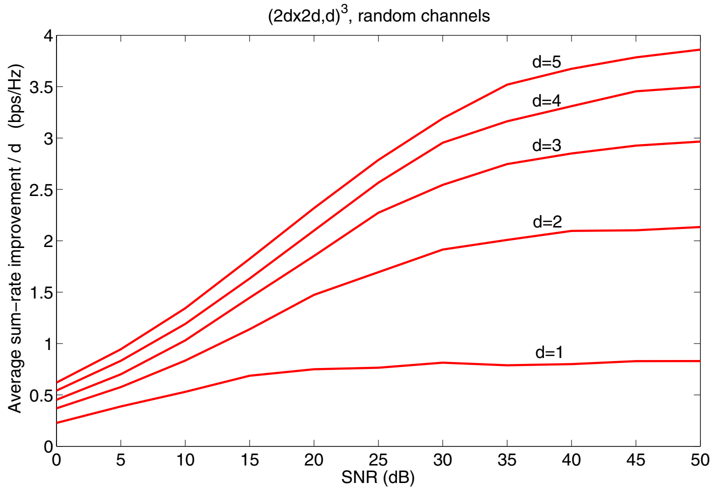

Fig. 2. Sum-rate improvement per stream achieved by the MaxSR-IA solution over the conventional IA solution from $d = 1$ to $d = 5$ streams per user

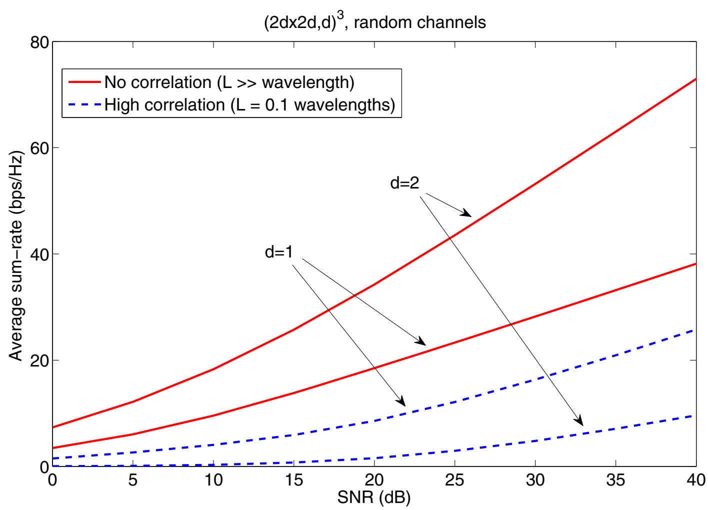

Fig. 3. Impact of correlation in the average sum-rate for 1 and 2 streams per user

where $J_0(\cdot)$ is the zeroth order Bessel function. The correlation effect has been studied by assuming an antenna separation $L = 0.1\lambda$. Obviously, such a small separation gives a highly correlated channel. Fig. 3 shows the MaxSR-IA solution performance for $d = 1$ and $d = 2$.

As it can be seen in Fig. 3, an increasing correlation is detrimental for the sum-rate. However, it is interesting to notice that the average sum-rate for $d = 1$

is better than for $d = 2$. This effect is due to the fact that highly correlated scenarios result in almost rank-deficient MIMO channels, which do not allow to transmit more than one stream per user. In other words, with strong correlation it is more difficult to exploit the different eigenmodes of the MIMO channels.

6 Conclusion

In this paper the number of interference alignement solutions for the 3-user symmetric channel has been analyzed. Exploiting the fact that a finite number of solutions exist, we have derived the maximum sum-rate and the maximum sum-power IA solutions for this scenario. The obtained results have shown that the improvement of the sum-rate solution increases when the number of streams per user increases. As expected, both proposed solutions overcome the conventional IA solution (in terms of sum-rate). This justifies the need to find algorithms that optimize the sum-rate while cancelling the interference leakage.

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