A Minimum BER Loading Algorithm for OFDM in Access Power Line Communications

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Abstract. In this paper, we investigate the resource allocation problem for access broadband power line communications (PLC) according to the OPERA specification for power grids monitoring and control. The proposed loading algorithm attempts to minimize the BER while guaranteeing a certain throughput for Amplitude Differential Phase Shift Keying (ADPSK) modulation, when the attenuation varies significantly in function of the frequency. A performance comparison between the proposed algorithm and the Fischer-Huber algorithm is also discussed. Numerical simulations show that the proposed method can improve the average BER performance.

Keywords: Broadband Power line communications, high attenuation variance, loading algorithm.

1 Introduction

The concept of data transmission through power lines has attracted a lot of interest during the last decades. One of the main advantages is that a significant cost is saved by operating over the already existing power network infrastructure [1-4]. However, due to the fact that the power supply grids are originally designed for energy delivery rather than high speed communication, the power line network turns out to be rather hostile. The line impedance, (high) attenuation and phase shift may vary with frequency, time, location, and distance [1-4]. The appearance of OFDM has been proved to be an excellent solution to suppress these problems. In an OFDM system, additional significant gains can be achieved by allocating more bits to subcarriers with larger margins, less or even no bits to seriously faded carriers, i.e. using a bit loading algorithm. Over the past years, different power and bit allocation schemes with diverse optimization objectives have been studied [5-8]. Nevertheless, during the earlier period, the power line communications provide much lower speed compared to other alternatives, consequently the purpose of the bit loading research in the past decades is to maximize the overall throughput while guaranteeing a target bit error rate. In 2006, the latest standards of HomePlug and OPERA announced data rates up to 200 Mbps in the physical layer [9], which makes it competitive enough with other technologies, such as Ethernet in its 100 Mbps version, wireless in 100 Mbps at most,

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ADSL and so on. The high speed also releases the purpose of bit loading algorithm in PLC from maximizing the throughput.

On the other hand, in practice there is class of reliability-demand applications in PLC, which desires to transmit fixed data with a fixed power at the lowest bit error rate. For power grid monitoring and control in our case, the idea is that all the end-users in one network transfer certain status information periodically to the master in a fixed speed at the best reliability, the master collects the status report and responds to the emergency if there is an abnormal status reported. This requires the algorithm to guarantee the reliability rather than improve the throughput. The minimum BER bit loading algorithm appears as the solution of these problems.

The OPERA specifications announced on June 2007 are considered in this paper [10]. The OPERA system, supporting raw data up to 200 Mbps, employs OFDM over a bandwidth from 2 to 30 MHz, using 1536 sub-carriers. The sub-carriers adopt Amplitude Differential Phase Shift Keying (ADPSK) for modulation, which is a good solution for power line channels [11-12]. The information is assigned to the phase change and the actual amplitude because (1) the carrier is assumed to be unknown and uniformly distributed; (2) the amplitude of the received signal still providesinformation on the transmitted amplitude even if no channel state information is available at the receiver. The possible modulations per sub-carrier are: DPSK, 4DPSK, 2A4DPSK, 2A8DPSK, 4A8DPSK, 4A16DPSK, 8A16DPSK, 8A32DPSK, 16A32DPSK, 16A64DPSK.

The remainder of this paper is organized as follows: in section 2 the considered system is described and the bit-loading algorithm is presented in section 3. Numerical results are reported in section 4. Finally, conclusions and further research are shown in section 5.

2 Optimal Bit Allocation Problem

2.1 System Model

Fig.1 shows the considered OFDM system for power line communications. The entire bandwidth is divided into 1536 equal parts, the transfer function of each subcarrier is supposed to be constant during a control interval. With a symbol interval of T and data rate R_b, the number of bits per symbol is then $b=R_bT$ bits. The Encoder translates raw input data into coded data with certain gain; here we suppose there are not additional bits generated after encoding. According to the sub-carrier attenuation H estimated from feedback, the data bits b are allocated to sub-carriers respectively; subsequently the allocated bits of each sub-carrier are modulated in a predefined constellation.

The time domain symbols are obtained by the inverse discrete Fourier transform and transmitted via a power line channel, the sub-carrier's transmission coefficients H are obtained from measurements [13]. After Discrete Fourier Transform (DFT), the received samples are demodulated into binary bit streams and then are multiplexed into one flow. The system contains a feedback channel to transfer the absolute value of the transmission coefficient H, as in [5-8], the feedback is assumed to be noiseless and to have no delays, i.e., perfect feedback.

In this paper, the effect of the notch or frequency-selective fade is not taken into consideration since the sub-carrier's channel has been assumed to be flat. In practice, the notch part could be excluded from the bandwidth and this does not affect the process of bit-loading algorithm.

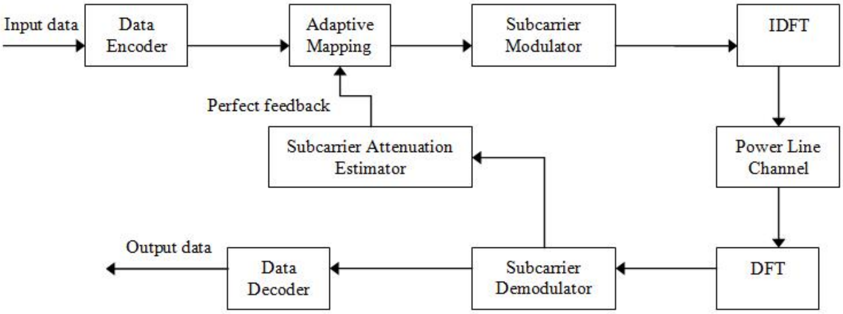


Fig. 1. Simplified Block diagram of the OPERA OFDM system

2.2 Power Line Channel

As described before, the power line channel is rather hostile for high speed data transmission, earlier studies on the characteristics of power lines [1-4] have revealed the situations met in power line communications.

First, the high frequency dependent attenuation is serious; the attenuation (in dB) theoretically increases linearly with frequency and distance. A model for the attenuation as a function of frequency and distance is proposed in [13]. By selecting the parameter of *c, d, e, g,* the attenuation, which represents the amplitude of the transfer function, can be defined by the formula:

$$A(f,l) = c * f * l + d * f + e * l + g \quad (dB) \tag{1}$$

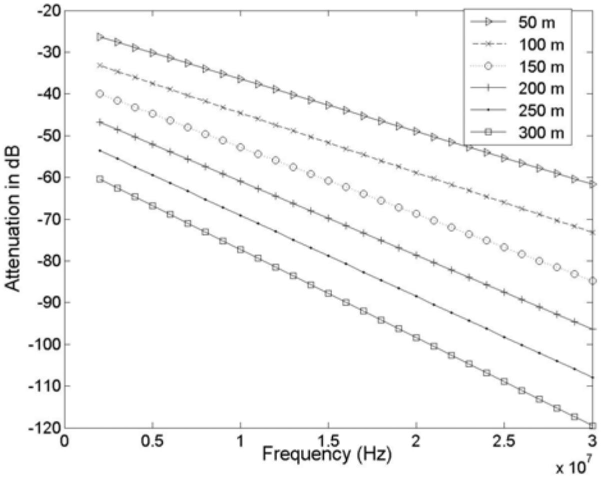


Fig. 2. A sample Power line channel attenuation with cable length neglecting the impact of notches

The dynamic range D of the sub-carrier transmission coefficient H, which shows the attenuation varying with frequency, is expressed as

$$D = 20\log(|H_{\max}| / |H_{\min}|) \tag{2}$$

where $H_{max} = max\,(H_i)$ and $H_{min} = min\,(H_i)$ for i=1…M.

From Fig.2, D is getting higher with increasing length; for 50 meters, D is around 40 dB; for 300 meters, D could even reach 60 dB.

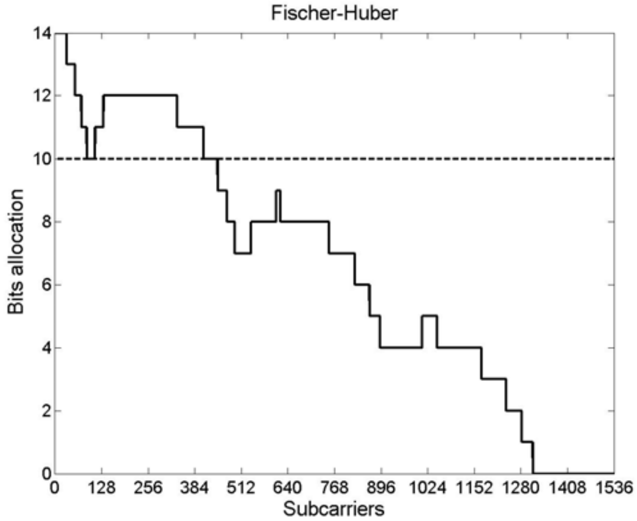
Second, the noise in the power line channel is not AWGN noise as normal [1-4]. It can be typified into three categories: the colored noise has a relatively low power spectral density which decreases with increasing frequency, the narrowband background noise and the impulsive noise (synchronous and asynchronous with the main frequency). The impulsive noise asynchronous with the main's frequency is the most detrimental type of noise for data transmission. Its duration varies from a few microseconds to milliseconds and has a random inter-arrival time.

2.3 State of the Art

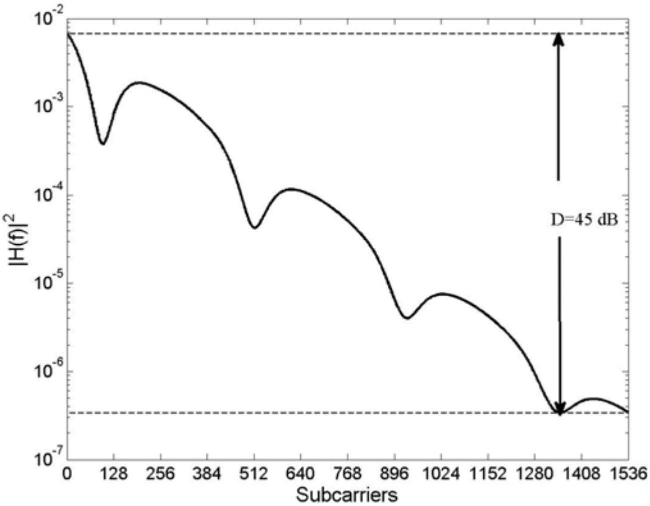
Fischer and Huber proposed in [5] the first minimum BER bit loading algorithm, in this paper, the minimization problem of BER is solved for Quadrature Amplitude Modulation (QAM) constellations with the restrictions that the total energy is constant and the total bit rate is also constant. The optimal solution is obtained from the equation:

$$b_i = b / M + 1 / M \cdot \log(\prod_{k=1}^{M} \frac{N_k}{N_i^M}) \tag{3}$$

where b_i are the bits allocated to the i^{th} subcarrier; for negative b_i the corresponding subcarrier will be turned off. M is the total number of subcarriers, b is the bit rate target and N_i is the noise variance at the i^{th} subcarrier. The equation can be applied iteratively until all b_i of the remaining subcarriers are positive. Finally, energy is distributed flatly along the remaining subcarriers. This algorithm has the advantage that the BER of M-QAM could be represented by an analytical formula, which makes it simpler than Chow's algorithm [6]. However, the algorithm suffers challenges in a high D channel. When D is high, the bits allocated to each subcarrier could vary within a large range. As is shown in Fig.3, one possible situation is that some of the subcarriers have reached the maximum bits it could allow when there are still bits left to be allocated. These fully occupied subcarriers should be turned off and excluded from the set of next allocations; the over allocated bits should be withdrawn and reallocated. The reallocation will be performed under two restrictions: 1), turn off the negative subcarriers; 2), keep the b_i lower than $bmax$, and withdraw the b_i- b_{max} bits when b_i is larger than b_{max}. Since some of the subcarriers have been turned off at the beginning without considering the second restriction, to get the optimal result, some of the removed subcarriers have to be turned on again. That is to say, the two restrictions are not independent in this algorithm, which makes the iterative operation more complicated than the original one with one restriction. Furthermore, the BER of ADPSK modulation could not be expressed with an analytical formula; the algorithm needs to perform bit round off as well, because it does not allocate integer number of bits on each carrier.



a) Allocation Result



b) Attenuation of the subcarriers

Fig. 3. A Fischer bits allocation result in high D channel without considering the modulation limitation

Lev Golgfeld introduced a minimum BER power loading algorithm for OFDM in a fading channel in [7]; the optimal power allocation is obtained assuming the same signal constellation in each subcarrier. The aggregate BER is calculated as a function of the average SNR defined as

$$SNR_{av} = \sum_{i=1}^{M} \frac{SNR_i}{M} \tag{4}$$

This algorithm takes the dynamic range D of subcarrier transmission coefficients H_i into account. It offers the best power allocation in some situations but not all the time as we know.

3 Bit Loading Algorithm

The minimum BER loading algorithms try to solve the following problem:

$$\begin{cases} \min \quad p_e = 1 - \prod_{i=1}^{M} (1 - p_e(SNR_i)), \qquad i = 1, 2....M \\ s.t. \qquad \begin{cases} \sum_{i=1}^{M} b_i = b \\ \sum_{i=1}^{M} P_i = P_b \end{cases} \end{cases} \tag{5}$$

where p_e is the aggregate BER of all the subcarriers, here we suppose the BER of each subcarrier is independent.

$$SNR_i = |H_i|^2 \left(\frac{E_i}{N_i} \right) \tag{6}$$

is the SNR in the ith subcarrier, E_i=P_i*T is the allocated energy for the subcarrier, N_i is the spectral density of the noise, P_i is the power allocated for the i^{th} subcarrier and P_b is the total power could be used.

By considering the upper boundary of bits allocated to each subcarrier, the bit and power allocation formulated above by Eq.3 can be solved by an improved bit-add way. We start bit allocation under constant transmission power per bit. With certain units of power, the subcarriers which have smaller BER than the threshold, which could be the *per_median* or *per_quarter*, get one more bit and stop when the subcarrier reaches the modulation's upper boundary. When bit allocation is done, the allocated power is adjusted for sets of subcarriers with the same number of allocated bits to equalize the subcarriers SNR. The detail of this algorithm is as follows:

1) Sort all the subcarriers according to the value of attenuation over noise power: $|H_i|^2/\sigma_i^2$.
2) For all the $i \in S=\{1,2,....M\}$, set $b_i = 0$, $E_i = 0$, $p_{er}^i = 0$ and $E_{\_allo} = P_b*T/b$.
3) For all subcarriers in set S, set $E_i = E_{\_allo.}$ Find the median BER for all these subcarriers, p_{er}_median = median (p_{er}^i | $i \in (1,2,\ldots M)$), for all the subcarriers that $p_{er}^i < p_{er}$_median, add one more bit to b_i and keep the changed $E_i = E_i + E_{\_allo}$. For the first allocation, the algorithm could be simplified by finding the

median value of $|H_i|^2/\sigma_i^2$ instead of calculating and finding the median BER for each subcarrier since they are one-to-one correspondence with each other.

4) If b_i ($i \in S$) reaches the upper boundary of the modulation, then remove the corresponding subcarriers from S.
5) Repeat step 3, 4 while $\prod_{i=0}^{M} b_i < b$. When the total number of allocated bits is equal to the number of bits to be transmitted, namely $\prod_{i=0}^{M} b_i = b$, then terminated the bit allocation and go to step 7, if $\prod_{i=0}^{M} b_i > b$, go to step 6.
6) Set b_remove= $\prod_{i=0}^{M} b_i - b$, sort the subcarriers according to the last BER; remove one bit and $E_{\_allo}$ per subcarrier from the *b_remove* subcarriers with highest BER.
7) Sort the b_i according to the following rule: $S(n,i)=\{(n,i)|b_i=k\}$, where *n* keeps the number of subcarriers with same allocated bits, *i* keeps the corresponding index of the subcarrier. $k \in (0,1,2\ldots10)$.
8) For the set of subcarriers with the same allocated bits, reallocate the power: $E_{i\_new}=E_i*|H_i|^{-2}/\left(\sum_{j=1}^{n}\left|H_j\right|^{-2}\right)$.
9) End.

4 Performance Evaluation

In this section, we evaluate the performance of the loading algorithm by computer simulations. In order to compare the proposed algorithm with that of Fischer et al, we add an extra limitation of the modulation level in Fischer's algorithm and the performance under QAM modulation is compared.

The channel model is obtained from measurement of OPERA as in [13]. The modeled noise in the simulation is Additive White Gaussian Noise plus impulsive noise for two reasons: 1), the colored noise could be converted into white noise by a pre-whitening filter; 2) the impulsive noise is the most detrimental noise for data transmission. The noise PSD is

$$N_i=N_0+P_{im}N_{im} \tag{7}$$

where N_i is the PSD of the overall noise which includes AWGN N_0 and the impulsive noise N_{im}. P_{im} is the total average occurrence of the impulsive noise duration in time T and the impulsive noise is given by Bernoulli-Gaussian process, i.e., a product of a real Bernoulli process with expected value p and a complex Gaussian process with mean zero and variance $\sigma_{im}^2 >> \sigma_o^2$. Hence, when considering the effect of impulsive noise on the BER performance of OFDM system, the signal to noise ratio should be:

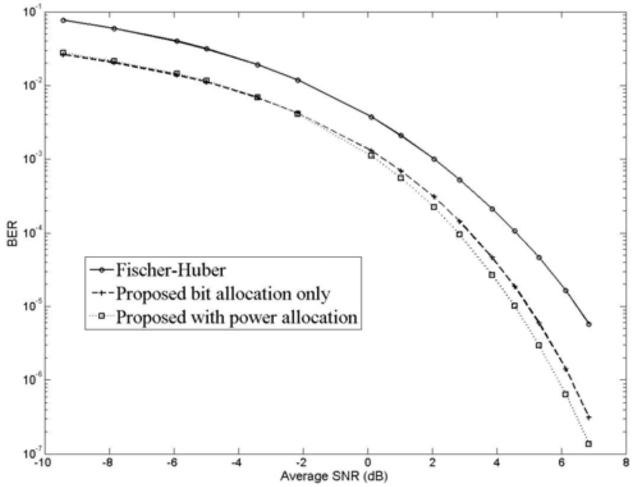
$$SNR_i = \left|H_i\right|^2\left(\frac{E_i}{N_i}\right) = \left|H_i\right|^2\left(\frac{E_i}{N_0+P_{im}N_{im}}\right) \tag{8}$$

The power line length is assumed to be 200 meters; the number of modulated subcarriers is 1536; p is set to be 0.01, the variance of impulsive noise σ_{im}^2 is one hundred times larger than σ_o^2.

Fig. 4 shows the comparison of the BER performance with three different data rates under AWGN plus impulsive noise, *D* is chosen to be 40 dB, which is normal in PLC. The median value is chosen in the case of 6000 bits and 8000 bits, while the quarter value is chosen for 4000 bits. According to Fig.4, for the case of 6000 bits per symbol, the proposed power allocation scheme has a 2 dB better SNR than the Fischer-Huber algorithm at almost every BER value as shown in b); in the case of 8000 bits in c), the improvement is about 1 dB, which is smaller than in the case of 6000 bits, but is still outstanding. In the case of 4000 bits, the proposed algorithm adopts *per_quarter* as the threshold for adding one more bit, it is fortunate that when the number of bits is smaller, the change from per_median to *per_quarter* just induces a slight complexity. For D equals 40 dB, the best break point is around 4800 bits, when the number of bits to be allocated is smaller than 4800 bits, it is better to adopt *per_quarter* rather than *per_median*, a tradeoff between BER performance and complexity.

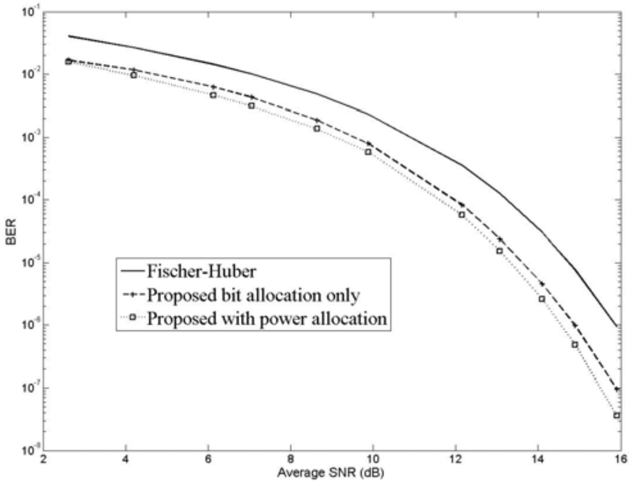
A comparison of the proposed algorithm with or without power allocation is also shown in Fig.4. It is clearly shown that power allocation can further improve the performance in our case.

In addition, by introducing the median or quarter parameter, the iterations can be reduced sharply compared to bit-add algorithm [8], which makes the computation time becomes *o(M)* in the worst case, a big improvement over H-H algorithm.

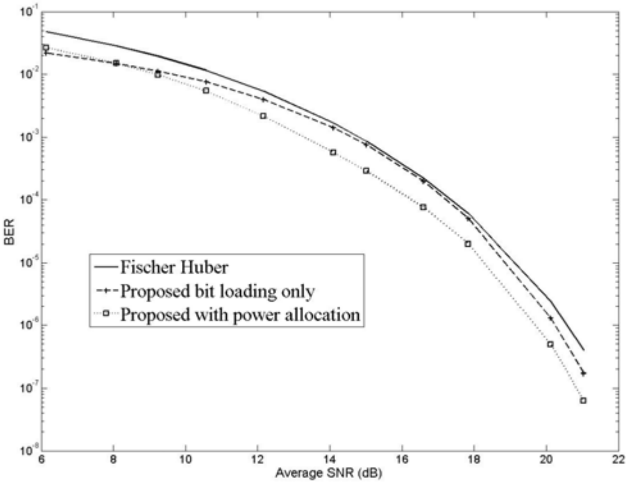


a) D=40dB, b=4000 bits, M=1536.

Fig. 4. Performance comparison (BER versus Average SNR) between the Algorithms



b) D=40dB, b=6000 bits, M=1536.



c) D=40dB, b=8000 bits, M=1536.

Fig. 4. *(continued)*

5 Conclusion

In this paper, we proposed a fast loading algorithm for power line carriers with a high variation in attenuation. In this kind of carriers, loading algorithms without considering the modulation limitation are not good choices any more. The proposed algorithm in this paper is an improved bit-add algorithm, it minimizes the BER based on the criterion that the subcarrier which has the smallest BER with unit power gets one more bit, by introducing the median value, the algorithm could simplify the computational effort of bit-add algorithm.

The OPERA specification also offers a HURTO mode for control information or data information that needs high reliability without losing efficiency [10]. As a future work, we will investigate the mixed system for data transmission with different priorities.

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